DAE / IIA - 2020

MATH-123 APPLIED MATHEMATICS-I

PAPER 'A' PART - A (OBJECTIVE)

Time: 30 Minutes Marks: 15

Q.1: *Encircle the correct answer.*

1. The quadratic formula is:

[a] $\frac{b \pm \sqrt{b^2 - 4ac}}{2a}$

[b] $\frac{-b \pm \sqrt{b^2 + 4ac}}{2a}$

[c] $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

[d] $\frac{b \pm \sqrt{b^2 + 4ac}}{2a}$

2. ~~The sum of the roots of~~ $2x^2 - 3x - 5 = 0$ ~~is:~~

[a] $-\frac{3}{2}$ [b] $\frac{3}{2}$

[c] $\frac{2}{3}$ [d] $-\frac{2}{3}$

3. ~~If~~ ± 3 ~~are the roots of the equation, then the equation is:~~

[a] $x^2 - 3 = 0$ [b] $x^2 - 9 = 0$

[c] $x^2 + 3 = 0$ [d] $x^2 + 9 = 0$

4. The number of terms in the expansion of $(a+b)^{13}$ are:

[a] 12 [b] 13 [c] 14 [d] 15

5. The middle term in the expansion of $(a+b)^6$ is:

[a] $15a^4b^2$ [b] $20a^3b^3$

[c] $15a^2b^4$ [d] $6ab^5$

6. $\binom{3}{0}$ will have the value:

[a] 0 [b] 1 [c] 2 [d] 3

7. ~~π rad is equal to:~~

[a] $360°$ [b] $270°$

[c] $180°$ [d] $90°$

8. The relation between arc 'ℓ' central angle 'θ' in radian and radius 'r' is:

[a] $\ell = \frac{\theta}{r}$ [b] $\ell = \frac{r}{\theta}$

[c] $\ell = r\theta$ [d] $\ell = r^2\theta$

9. $\cos(\pi + \theta)$ is equal to:

[a] $\cos\theta$ [b] $-\sin\theta$

[c] $-\cos\theta$ [d] $\sin\theta$

10. $\sin 2\alpha$ is equal to:

[a] $\cos^2\alpha - \sin^2\alpha$

[b] $\cos 2\alpha$

[c] $1 - \cos^2\alpha$ [d] $2\sin\alpha\cos\alpha$

11. If $b = 2,\ A = 30°,\ B = 45°,$ then 'a' is equal to:

[a] 2 [b] $\sqrt{2}$ [c] $\frac{\sqrt{3}}{2}$ [d] $\frac{2}{\sqrt{3}}$

12. In right triangle if one angle is $45°$, then the other will be:

[a] $45°$ [b] $50°$

[c] $60°$ [d] $75°$

13. Magnitude of the vector $2i - 2j - k$ is:

[a] 4 [b] 3 [c] 2 [d] 1

14. $\vec{a} \cdot \vec{b}$ is a:

[a] Vector quantity

[b] Scalar quantity

[c] Unity [d] None of these

15. If $\vec{a}$ and $\vec{b}$ are collinear vectors then:

[a] $\bar{a} \times \bar{b} = 0$ [b] $\bar{a} \cdot \bar{b} = 0$

[c] $\bar{a} - \bar{b} = 0$ [d] $\bar{a} + \bar{b} = 0$

Answer Key

1	c	2	b	3	b	4	c	5	b
6	b	7	c	8	c	9	c	10	d
11	b	12	a	13	b	14	b	15	a

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DAE / IIA - 2020

MATH-123 APPLIED MATHEMATICS-I

PAPER 'A' PART-B (SUBJECTIVE)

Time: 2:30 Hrs Marks: 60

Section-I

1. Solve the equation by factorization: $3x^2 + 5x = 2$

Sol. $3x^2 + 5x = 2$

$3x^2 + 5x - 2 = 0$

$3x^2 + 6x - x - 2 = 0$

$3x(x+2) - 1(x+2) = 0$

$(x+2)(3x-1) = 0$

Either | OR

$x + 2 = 0$ | $3x - 1 = 0$

$x = -2$ | $3x = 1 \Rightarrow x = \frac{1}{3}$

$$\text{S.S.} = \boxed{\left\{-2,\ \frac{1}{3}\right\}}$$

2. Solve the quadratic equation: $x^2 - 3x - 18 = 0$

Sol. $x^2 - 3x - 18 = 0$

$x^2 - 6x + 3x - 18 = 0$

$x(x-6) + 3(x-6) = 0$

$(x-6)(x+3) = 0$

Either | OR

$x - 6 = 0$ | $x + 3 = 0$

$x = 6$ | $x = -3$

$$\text{S.S.} = \boxed{\{-3,\ 6\}}$$

3. For what value of 'k' the roots of the equation $kx^2 + 4x + 3 = 0$ are equal.

Sol. Here: $a = k,\ b = 4,\ c = 3$

As Roots are equal, So

$Disc = 0 \Rightarrow b^2 - 4ac = 0$

$(4)^2 - 4(k)(3) = 0$

$16 - 12k = 0$

$-12k = -16$

$k = \frac{-16}{-12} \Rightarrow \boxed{k = \frac{4}{3}}$

4. ~~If the sum of the roots of~~ $4x^2 + kx - 7 = 0$ ~~is 3. Find the value of~~ 'k'.

Sol. $4x^2 + kx - 7 = 0$

Here: $a = 4,\ b = k,\ c = -7$

As, Sum of Roots $= 3$

$-\frac{b}{a} = 3$

$-\frac{k}{4} = 3$

$-k = 12 \Rightarrow \boxed{k = -12}$

5. ~~Form the quadratic equation whose roots are~~ $-2+\sqrt{3},\ -2-\sqrt{3}$

Sol. $S = -2+\sqrt{3} + \left(-2-\sqrt{3}\right)$

$S = -2+\sqrt{3} - 2 - \sqrt{3} = -4$

$P = \left(-2+\sqrt{3}\right)\left(-2-\sqrt{3}\right)$

$P = (-2)^2 - \left(\sqrt{3}\right)^2$

$P = 4 - (3) = 1$

$x^2 - Sx + P = 0$

$x^2 - (-4)x + 1 = 0 \Rightarrow \boxed{x^2 + 4x + 1 = 0}$

6. Expand by Binomial theorem $\left(x + \frac{1}{x}\right)^4$.

Sol. By using binomial theorem.

$$= \binom{4}{0}(x)^4\left(\frac{1}{x}\right)^0 + \binom{4}{1}(x)^3\left(\frac{1}{x}\right)^1 + \binom{4}{2}(x)^2\left(\frac{1}{x}\right)^2 + \binom{4}{3}(x)^1\left(\frac{1}{x}\right)^3 + \binom{4}{4}(x)^0\left(\frac{1}{x}\right)^4$$

$$=(1)(x^4)(1)+(4)(x^3)\left(\frac{1}{x}\right)+(6)(x^2)\left(\frac{1}{x^2}\right)$$
$$+(4)(x)\left(\frac{1}{x^3}\right)+(1)(1)\left(\frac{1}{x^4}\right)$$
$$=\boxed{x^4+4x^2+6+\frac{4}{x^2}+\frac{1}{x^4}}$$

7. State Binomial Theorem for positive integer n.

Sol. The rule for expansion of $(a+b)^n$, where 'n' is any positive integral power, is called binomial theorem, and defined as:

$$(a+b)^n=\binom{n}{0}a^nb^0+\binom{n}{1}a^{n-1}b^1+\binom{n}{2}a^{n-2}b^2+\ldots+\binom{n}{n}a^0b^n$$

8. Find the 5th term in the expansion of $\left(2x-\frac{x^2}{4}\right)^7$

Sol. Here: $a=2x$, $b=-\frac{x^2}{4}$, $n=7$ & $r=4$

$$T_{r+1}=\binom{n}{r}a^{n-r}b^r\Rightarrow T_{4+1}=\binom{7}{4}(2x)^{7-4}\left(-\frac{x^2}{4}\right)^4$$
$$T_5=(35)(8x^3)\left(\frac{x^8}{256}\right)\Rightarrow\boxed{T_5=\frac{35}{32}x^{11}}$$

9. ~~Expand to three terms~~ $\frac{1}{(1+x)^2}$

Sol. $$\frac{1}{(1+x)^2}=(1+x)^{-2}$$

Put $b=x$ & $n=-2$ in Binomial series Formula, we have:

$$=1+(-2)(x)+\frac{(-2)(-2-1)}{2!}(x)^2+\ldots$$
$$=1-2x+\frac{(-2)(-3)}{2}x^2+\ldots$$
$$=\boxed{1-2x+3x^2+\ldots}$$

10. Which term is the middle term in $(a+b)^n$ when n is odd.

Sol. When n is odd

Then, there are two middle terms:

$$\text{Middle term}=\left(\frac{n+1}{2}\right)^{th}+\left(\frac{n+3}{2}\right)^{th}\text{ terms.}$$

11. ~~Convert into degree measure: 0.726 radian.~~

Sol. 0.726 rad

$$=0.726\times\frac{180}{\pi}=\boxed{41^\circ35'48''}$$

12. ~~Find 'r' when ℓ = 33 cm and θ = 6 radian.~~

Sol. By using formula: $\ell=r\theta$

$$r=\frac{\ell}{\theta}=\frac{33}{6}=\boxed{5.5\text{cm}}$$

13. Prove that:

$$\cos30^\circ\cos60^\circ-\sin30^\circ\sin60^\circ=0$$

Sol. $\text{L.H.S.}=\cos30^\circ\cos60^\circ-\sin30^\circ\sin60^\circ$

$$=\left(\frac{\sqrt3}{2}\right)\left(\frac{1}{2}\right)-\left(\frac{1}{2}\right)\left(\frac{\sqrt3}{2}\right)$$
$$=\frac{\sqrt3}{4}-\frac{\sqrt3}{4}$$
$$=0=\text{R.H.S.}$$ **Proved.**

14. Prove that:

$$\cos^4\theta-\sin^4\theta=1-2\sin^2\theta$$

Sol. $\text{L.H.S.}=\cos^4\theta-\sin^4\theta$

$$=(\cos^2\theta)^2-(\sin^2\theta)^2$$
$$=(\cos^2\theta-\sin^2\theta)(\cos^2\theta+\sin^2\theta)$$
$$=(1-\sin^2\theta-\sin^2\theta)(1)$$
$$=1-2\sin^2\theta=\text{R.H.S.}$$ **Proved.**

15. Prove that:

$$\tan(45^\circ+\theta)\tan(45^\circ-\theta)=1$$

Sol. $\text{L.H.S.}=\tan(45^\circ+\theta)\tan(45^\circ-\theta)$

$$= \frac{\tan 45^\circ + \tan\theta}{1-\tan 45^\circ \tan\theta} \times \frac{\tan 45^\circ - \tan\theta}{1+\tan 45^\circ \tan\theta}$$

$$= \frac{1+\tan\theta}{1-(1)\tan\theta} \times \frac{1-\tan\theta}{1+(1)\tan\theta} \quad \because \left\{ \begin{array}{l} \text{Using calculator} \\ \tan 45^\circ = 1 \end{array} \right\}$$

$$= \frac{1+\tan\theta}{1-\tan\theta} \times \frac{1-\tan\theta}{1+\tan\theta} = 1 = \text{R.H.S.}$$ **Proved.**

16. **Show that:**

$$\sin(\alpha+\beta) + \sin(\alpha-\beta) = 2\sin\alpha\cos\beta$$

Sol. $\text{L.H.S.} = \sin(\alpha+\beta) + \sin(\alpha-\beta)$

$$= \sin\alpha\cos\beta + \cancel{\cos\alpha\sin\beta} + \sin\alpha\cos\beta - \cancel{\cos\alpha\sin\beta}$$

$$= 2\sin\alpha\cos\beta = \text{R.H.S.}$$ **Proved.**

17. ~~**Express as product:**~~

$\cos 12\theta + \cos 4\theta$

Sol. $\cos 12\theta + \cos 4\theta$

$$= 2\cos\left(\frac{12\theta+4\theta}{2}\right)\cos\left(\frac{12\theta-4\theta}{2}\right)$$

$$= \boxed{2\cos 8\theta \cos 4\theta}$$

18. **Find $\cos\theta$ if $\sin\theta = \frac{7}{25}$ and angle θ is an acute angle.**

Sol. $\sin\theta = 7/25$

$$\cos^2\theta = 1 - \sin^2\theta$$

$$\cos^2\theta = 1 - \left(\frac{7}{25}\right)^2$$

$$\cos^2\theta = 1 - \frac{49}{625}$$

$$\cos^2\theta = \frac{625-49}{625}$$

$$\cos^2\theta = \frac{576}{625}$$

$$\sqrt{\cos^2\theta} = \pm\sqrt{\frac{576}{625}}$$

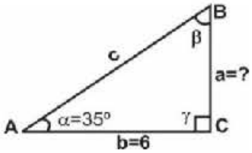
$$\cos\theta = \pm\frac{24}{25}$$

As θ is acute angle, so $\boxed{\cos\theta = \frac{24}{25}}$

19. ~~**In right triangle ABC, b = 6, α = 35º, γ = 90º, Find side 'a'.**~~

Sol. We know that, from figure:

$$\tan\alpha = \frac{a}{b}$$



$$\tan 35^\circ = \frac{a}{6}$$

$$6\tan 35^\circ = a \Rightarrow \boxed{a = 4.2}$$

20. **In any triangle ABC, if a = 20, c = 32 and γ = 70º find angle α.**

Sol. By using law of sines:

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$$

$$\text{we take: } \frac{a}{\sin\alpha} = \frac{c}{\sin\gamma}$$

$$\frac{\sin\alpha}{a} = \frac{\sin\gamma}{c}$$

$$\sin\alpha = \frac{a\sin\gamma}{c}$$

$$\sin\alpha = \frac{20\sin 70^\circ}{32}$$

$$\sin\alpha = 0.5873$$

$$\alpha = \sin^{-1}(0.5873)$$

$$\boxed{\alpha = 35^\circ 57' 58''}$$

21. **In any triangle ABC if A = 16, b = 17, γ = 25º, Find c.**

Sol. By using law of cosines:

$$c^2 = a^2 + b^2 - 2ab\cos\gamma$$

$$c^2 = (16)^2 + (17)^2 - 2(16)(17)\cos 25^\circ$$

$$c^2 = 256 + 289 - 493.03$$

$$c^2 = 51.97$$

$$\sqrt{c^2} = \sqrt{51.97} \Rightarrow \boxed{c = 7.2}$$

22. **Define the laws of cosines.**

Sol.
i. $a^2 = b^2 + c^2 - 2bc\cos\alpha$
ii. $b^2 = c^2 + a^2 - 2ca\cos\beta$
iii. $c^2 = a^2 + b^2 - 2ab\cos\gamma$

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23. **Find the magnitude of vector** $\mathbf{-2i - 4j + 3k}$

Sol. Let, $\vec{a} = -2i - 4j + 3k$

$|\vec{a}| = \sqrt{(-2)^2 + (-4)^2 + (3)^2}$

$|\vec{a}| = \sqrt{4 + 16 + 9} = \boxed{\sqrt{29}}$

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24. **Given the vectors:** $\vec{a} = 3i - 2j + k$, $\vec{b} = 2i - 4j - 3k$, $\vec{c} = -i + 2j + 2k$

Find $\vec{a} + \vec{b} + \vec{c}$

Sol. $\vec{a} + \vec{b} + \vec{c}$

$= 3i - 2j + k + 2i - 4j - 3k - i + 2j + 2k$

$= \boxed{4i - 4j + 0k}$

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25. **Find** $(\vec{a} + \vec{b}).(\vec{a} - \vec{b})$ **if**

$\vec{a} = 2i + 2j + 3k$ & $\vec{b} = 2i - j + k$

Sol. $\vec{a} + \vec{b} = (2i + 2j + 3k) + (2i - j + k)$

$\vec{a} + \vec{b} = 2i + 2j + 3k + 2i - j + k$

$\vec{a} + \vec{b} = 4i + j + 4k$

$\vec{a} - \vec{b} = (2i + 2j + 3k) - (2i - j + k)$

$\vec{a} - \vec{b} = 2i + 2j + 3k - 2i + j - k$

$\vec{a} - \vec{b} = 3j + 2k$

$(\vec{a} + \vec{b}) \bullet (\vec{a} - \vec{b})$

$= (4i + j + 4k) \bullet (3j + 2k)$

$= (4)(0) + (1)(3) + (4)(2)$

$= 0 + 3 + 8 = \boxed{11}$

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26. **Find the area of parallelogram with adjacent sides,**

$\vec{a} = 7i - j + k$ & $\vec{b} = 2j - 3k$

Sol. $\vec{a} \times \vec{b} = \begin{vmatrix} i & j & k \\ 7 & -1 & 1 \\ 0 & 2 & -3 \end{vmatrix}$

$= i\begin{vmatrix} -1 & 1 \\ 2 & -3 \end{vmatrix} - j\begin{vmatrix} 7 & 1 \\ 0 & -3 \end{vmatrix} + k\begin{vmatrix} 7 & -1 \\ 0 & 2 \end{vmatrix}$

$= i(3 - 2) - j(-21 - 0) + k(14 + 0)$

$= i + 21j + 14k$

$|\vec{a} \times \vec{b}| = \sqrt{(1)^2 + (21)^2 + (14)^2}$

$|\vec{a} \times \vec{b}| = \sqrt{1 + 441 + 196} = \sqrt{638}$

Area of parallelogram

$= |\vec{a} \times \vec{b}| = \boxed{\sqrt{638}\text{ sq. unit}}$

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27. **Find scalar x and y such that**

$\mathbf{x(i + 2j) + y(3i + 4j) = 7i + 9j}$

Sol. $x(i + 2j) + y(3i + 4j) = 7i + 9j$

$xi + 2xj + 3yi + 4yj = 7i + 9j$

$(x + 3y)i + (2x + 4y)j = 7i + 9j$

Comparing coefficients of i & j, we have:

$x + 3y = 7 \rightarrow (i)$ | $2x + 4y = 9 \rightarrow (ii)$

Multiplying eq. (i) by 2:

$2x + 6y = 14 \rightarrow (iii)$

Subtracting eq. (iii) & eq. (ii)

$$\begin{array}{r} 2x + 6y = 14 \\ -2x \pm 4y = -9 \\ \hline 2y = 5 \end{array} \Rightarrow \boxed{y = 5/2}$$

Put $y = 5/2$ in eq. (i)

$x + 3\left(\frac{5}{2}\right) = 7$

$x = 7 - \frac{15}{2}$

$x = \frac{14 - 15}{2} \Rightarrow x = \boxed{-\frac{1}{2}}$

Section - II

Note : Attemp any three (3) questions $3 \times 8 = 24$

Q.2.(a) Solve by factorization:

$$abx^2 + \left(b^2 - ac\right)x - bc = 0$$

Sol. See Q.1(vii) of Ex# 1.1 (Page # 9)

(b) ~~Find the value of~~ 'k' ~~if the product of the roots of~~

$$(k+1)x^2 + (4k+3)x + (k-1) = 0$$

is $\frac{7}{2}$.

Sol. See Q.2(i) of Ex# 1.3 (Page # 41)

Q.3. If 'x' is nearly equal to unity, prove that: $\frac{mx^n - nx^m}{x^n - x^m} = \frac{1}{1-x}$

Sol. See Q.4 of Ex# 2.2 (Page # 104)

Q.4.(a) A circular wire of radius 6cm is cut straightened and then bend so as to lie along the circumference of a hoop of radius 24cm. Find the measure of the angle which it subtends at the center of the hoop.

Sol. See Q.9 of Ex# 3.1 (Page # 118)

(b) Prove that :

$$\sqrt{\frac{1-\sin\theta}{1+\sin\theta}} = \sec\theta - \tan\theta$$

Sol. See Q.9 of Ex# 3.3 (Page # 134)

Q.5.(a) Prove that :

$$\tan(45° + \theta) = \frac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta}$$

Sol. See Q.5(iv) of Ex# 4.1 (Page # 157)

(b) In ΔABC if $\alpha = 60°$, $\beta = 45°$. Find ratio of b to c.

Sol. See Q.10 of Ex# 5.3 (Page # 225)

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