DAE / IIA - 2020

MATH - 113 APPLIED MATHEMATICS - I

PAPER 'B' PART - A (OBJECTIVE)

Time : 30 Minutes Marks : 15

Q.1: *Encircle the correct answer.*

1. The unit vector of $\mathbf{i-2j-2k}$ is:

[a] $i-2j-2k$ [b] $\frac{1}{3}(i-2j-2k)$

[c] $\frac{1}{\sqrt{3}}(i-2j-2k)$ [d] $\frac{1}{2}(i-2j-2k)$

2. If $\vec{a}=\mathbf{i+j+k}$ and $\vec{b}=\mathbf{-i-j-mk}$ are perpendicular then 'm' will be equal to:

[a] 1 [b] -2 [c] ±1 [d] ±3

3. If $\hat{n}$ is the unit vector in the direction of $\vec{a}\times\vec{b}$, then $\hat{n}$ is:

[a] $\frac{\vec{a}\times\vec{b}}{|\vec{a}|\,|\vec{b}|}$ [b] $\frac{\vec{a}\,.\,\vec{b}}{|\vec{a}|\,|\vec{b}|}$

[c] $\frac{|\vec{a}\times\vec{b}|}{|\vec{a}|\,|\vec{b}|\sin\theta}$ [d] $\frac{\vec{a}\times\vec{b}}{|\vec{a}\times\vec{b}|}$

4. If the order of the matrix $\mathbf{A}$ is $\mathbf{p\times q}$ and order of $\mathbf{B}$ is $\mathbf{q\times r}$, then order of $\mathbf{AB}$ will be:

[a] $p\times q$ [b] $q\times p$

[c] $p\times r$ [d] $r\times p$

5. ~~If two rows of a determinant are identical then its value is;~~

[a] 1 [b] 0 [c] -1 [d] 2

6. The value of 'm' for which matrix $\begin{bmatrix}\mathbf{k} & \mathbf{6}\\ \mathbf{4} & \mathbf{3}\end{bmatrix}$ is singular.

[a] -8 [b] 24 [c] 8 [d] -24

7. In an equilateral triangle, each angle is:

[a] $30°$ [b] $45°$

[c] $60°$ [d] None of these

8. If a = 4cm, b = 2cm are adjacent sides of triangle and $\theta=30°$ is the included angle then area is:

[a] 2 sq.cm [b] 4 sq.cm

[c] 8 sq.cm [d] 12 sq.cm

9. Area of parallelogram having 'a' and 'b' as adjacent sides and θ is the included angles is:

[a] $ab\cos\theta$ [b] $\frac{1}{2}ab\sin\theta$

[c] $ab\sin\theta$ [d] $a\sin\theta$

10. Area of a regular hexagon of side 'a' is:

[a] $\frac{\sqrt{r}}{4}$ [b] $\frac{2}{\sqrt{3}}a^2$ [c] $\frac{3\sqrt{3}}{4}a^2$ [d] $\frac{3\sqrt{3}}{2}a^2$

11. Area of circle with diameter 'd' is:

[a] $\frac{\sqrt{r}}{4}$ [b] $\frac{2}{\sqrt{3}}a^2$ [c] $\frac{3\sqrt{3}}{4}a^2$ [d] $\frac{3\sqrt{3}}{2}a^2$

12. If volume of a cube is 27, then side of cube is:

[a] $\frac{\pi}{2}r^2$ [b] $\frac{\pi}{2}d^2$ [c] $\frac{\pi}{4}d^2$ [d] None

13. Lateral surface area of right circular cylinder is:

[a] πr^2 [b] πrh [c] $2\pi rh$ [d] $2\pi r^2$

14. If area of base of pyramid is 'A' and height 'h', then volume of pyramid.

[a] $\frac{1}{3}Ah$ [b] $\frac{1}{2}Ah$ [c] $\frac{1}{6}Ah$ [d] Ah

15. Volume of cone of radius of base 3cm and height 12cm is;

[a] 108π cu.cm [b] 36π cu.cm

[c] 12π cu.cm [d] 54π cu.cm

Answer Key

1	b	2	b	3	d	4	c	5	b
6	c	7	b	8	d	9	b	10	c
11	c	12	a	13	c	14	a	15	b

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DAE / IIA - 2020

MATH - 113 APPLIED MATHEMATICS - I

PAPER 'B' PART - B (SUBJECTIVE)

Time : 2 : 30 Hrs Marks : 60

Section - I

1. **Define isosceles triangle.**

Sol. A triangle whose two sides are equal and third side is different is called Isosceles triangle.

2. **Write the area of an equilateral triangle with side 'a'.**

Sol. Area of equilateral triangle

$$= \boxed{\frac{\sqrt{3}}{4}a^2 \text{ sq.unit}}$$

3. **The area of a rectangle is 20 sq.cm and one of its side is 4cm long. Find its breadth and the perimeter of the rectangle.**

Sol. As, Area of rectangle = $20cm^2$

$$\text{Length} \times \text{Breadth} = 20$$

$$4 \times \text{Breadth} = 20$$

$$\text{Breadth} = 20/4 = \boxed{5\,cm}$$

$$\text{Perimeter} = 2(a+b)$$

$$= 2(4+5) = 2(9) = \boxed{18\,cm}$$

4. **The perimeter of a Rhombus is 140cm and one of the opposite angle is $30°$. Find its area.**

Sol. Let, length of one side of a Rhombus = a

As, Perimeter of Rhombus = 140cm

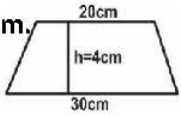
$$4a = 140$$

$$a = \frac{140}{4} \Rightarrow \boxed{a = 35\,cm}$$

Area of Rhumbus $= a^2 \sin\theta$

$$= (35)^2 \sin 30° = \boxed{612.50\,sq.cm}$$

5. **Find the area of trapezoid whose parallel sides are 20cm and 30cm and perpendicular distance between them is 4cm.**



Sol. Area of Trapezoid

$$= \frac{\text{Sum of parallel sides}}{2} \times \text{height}$$

$$= \frac{20+30}{2} \times 4 = \boxed{100\,sq.cm}$$

6. **Define inscribed polygon.**

Sol. If a circle passes through the corners of a polygon, then this polygon is called inscribed polygon.

7. **The perimeter of a regular hexagon is 12cm, find its area.**

Sol. Perimeter of hexagon = 12 cm

$$6a = 12$$

$$\Rightarrow \quad a = 12/6 = 2cm$$

$$\text{Area} = \frac{na^2}{4}\cot\left(\frac{180°}{n}\right)$$

$$A = \frac{6(2)^2}{4}\cot\left(\frac{180°}{6}\right)$$

$$A = 6\cot 30°$$

$$A = \frac{6}{\tan 30°} = \boxed{10.39\,sq.cm}$$

8. **What is the area and circumference of circle of radius 'r'.**

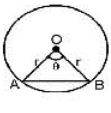
Sol. Area of Circle = A = $\boxed{\pi r^2 \text{ sq.unit}}$

Circumference of Circle = C = $\boxed{2\pi r \text{ unit}}$

9. **Write the formula of Area of the minor segment when angle is 'θ' and radius 'r' are given.**

Sol. Area of minor Segment

$$= \frac{1}{2}r^2(\theta - \sin\theta)$$



10. **The base of a right prism is an equilateral triangle with a side of**

4cm and its height is 25cm, find its volume.

Sol. Here: $a = 4cm,\ h = 25cm\ \&\ V = ?$

Area of base (equilateral triangle)

$$= \frac{\sqrt{3}}{4}a^2 = \frac{\sqrt{3}}{4}(4)^2 = 6.92cm^2$$

Volume = Area of base × height

$$\text{Volume} = 6.92 \times 25$$

$$\text{Volume} = \boxed{173.2cm^3}$$

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11. **The volume of the cube is 95 cu.cm. Find the surface area and the edge of the cube.**

Sol. Let 'a' be edge of cube

As, volume = 95

$$a^3 = 95$$

$$(a^3)^{1/3} = (95)^{1/3}$$

$$a = 4.56$$

Edge of cube = $a = \boxed{4.56cm}$

Surface area of cube = $6a^2$

$$S.A. = 6(4.56)^2 = \boxed{124.92cm^2}$$

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12. **A rectangular cuboid 9cm long and 7cm wide given that the volume of the cuboid is 315cm³. Find the height of the cuboid.**

Sol. Let, $\ell = 9cm,\ b = 7cm,\ V = 315\ cm^3$ & $h = ?$

As, Volume of cuboid = $315\ cm^3$

$$\ell b h = 315$$

$$9 \times 7 \times h = 315$$

$$h = \frac{315}{63}$$

$$\boxed{h = 5cm}$$

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13. **Find the cost of digging a well 3m in diameter and 24m in depth at the rate of Rs.10 per cu. m.**

Sol. Here: $d = 3m,\ h = 24m$

As, $d = 3m \Rightarrow r = \frac{d}{2} = \frac{3}{2} = 1.5m$

Volume of well = $V = \pi r^2 h$

$$= \pi(1.5)^2(24) = 169.646m^3$$

Total cost of digging the well

= Volume × Rate

$$= 169.646 \times 10 = \boxed{Rs.1696.46}$$

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14. **Find the diameter of the cylinder if its volume is 704cm³ and height is 14cm.**

Sol. Here: $d = ?, V = 704cm^3\ \&\ h = 14cm$

As, Volume of Cylinder = $704cm^3$

$$\Rightarrow \quad \pi r^2 h = 704$$

$$\Rightarrow \quad r^2 = \frac{704}{\pi h}$$

$$\Rightarrow \quad r^2 = \frac{704}{\pi(14)}$$

$$\Rightarrow \quad r^2 = 16$$

$$\Rightarrow \quad \sqrt{r^2} = \sqrt{16}$$

$$\Rightarrow \quad r = 4\,cm$$

Diameter = $d = 2r = 2(4) = \boxed{8\,cm}$

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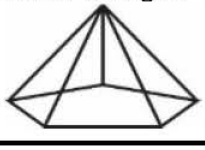
15. **Find the volume of a pentagonal based pyramid whose area of base is 15 sq. cm and height is 15cm.**

Sol. Here V=? Area of base = $15\ cm^2$ & height = 15cm

Volume = $\frac{1}{3}$ × area of base × height

$$V = \frac{1}{3} \times 15 \times 15$$

$$V = \boxed{75cm^3}$$



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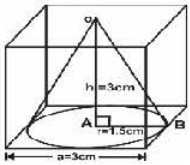
16. **Find the volume of the largest cone that can be cut out of a cube whose edge is 3cm.**

Sol. Let 'a' = edge of the cube = 3cm

Then h = 3cm &

$$r=\frac{a}{2}=\frac{3}{2}=1.5\text{cm}$$



$$\text{Volume}=\frac{1}{3}\pi r^2 h$$

$$V=\frac{1}{3}\pi(1.5)^2(3)=\boxed{7.069\,\text{cm}^3}$$

17. **How many cu. ft. of gas are necessary to inflate a spherical balloon to a diameter of 60 inch?**

Sol. $\text{Here: } d=60\,\text{inch}=\frac{60}{12}\text{ft}=5\,\text{ft}$

$$\text{Gas require}=\frac{\pi}{6}d^3$$

$$=\frac{\pi}{6}(5)^3=\boxed{65.45\,\text{ft}^3}$$

18. **What is unit vector.**

Sol. A vector whose magnitude is unity is called a unit vector.

19. **Find the magnitude of the vector $-2\mathbf{i}-4\mathbf{j}+3\mathbf{k}$**

Sol. Let, $\vec{a}=-2i-4j+3k$

$$|\vec{a}|=\sqrt{(-2)^2+(-4)^2+(3)^2}$$

$$|\vec{a}|=\sqrt{4+16+9}$$

$$|\vec{a}|=\boxed{\sqrt{29}}$$

20. **Find the unit vector parallel to the sum of the vector. $\vec{a}=[2,4,-5]$ and $\vec{b}=[1,2,3]$**

Sol. Let $\vec{v}$ = sum of the vectors

$$\vec{a}\ \&\ \vec{b}=\vec{a}+\vec{b}=[2,4,-5]+[1,2,3]$$

$$\vec{v}=[3,6,-2]=3i+6j-2k$$

$$|\vec{v}|=\sqrt{(3)^2+(6)^2+(-2)^2}$$

$$|\vec{v}|=\sqrt{9+36+4}$$

$$|\vec{v}|=\sqrt{49}=7$$

$$\text{Unit vector}=|\hat{v}|=\frac{\vec{v}}{|\vec{v}|}=\boxed{\frac{3i+6j-2k}{7}}$$

21. **Find a vector whose magnitude is 2, and is parallel to $5\hat{i}+3\hat{j}+2\hat{k}$**

Sol. Let $\vec{a}$ be a require vector, so

$|\vec{a}|=2$ & Let, $\vec{b}=5\hat{i}+3\hat{j}+2\hat{k}$

$$|\vec{b}|=\sqrt{(5)^2+(3)^2+(2)^2}$$

$$|\vec{b}|=\sqrt{25+9+4}=\sqrt{38}$$

As $\vec{a}$ and $\vec{b}$ are parallel vectors:

So $\hat{a}=\hat{b}$

$$\frac{\vec{a}}{|\vec{a}|}=\frac{\vec{b}}{|\vec{b}|}$$

$$\frac{\vec{a}}{2}=\frac{5i+3j+2k}{\sqrt{38}}$$

$$\vec{a}=\frac{2(5i+3j+2k)}{\sqrt{38}}=\boxed{\frac{10i+6j+4k}{\sqrt{38}}}$$

22. **For what value of λ, the vectors $2\mathbf{i}-\mathbf{j}+2\mathbf{k}$ & $3\mathbf{i}+2\lambda\mathbf{j}$ are perpendicular.**

Sol. Let, $\vec{a}=2i-j+2k$ & $\vec{b}=3i+2\lambda j$

As given vectors are perpendicular.

So, $\vec{a}.\vec{b}=0$

$\Rightarrow (2i-j+2k).(3i+2\lambda j)=0$

$\Rightarrow (2)(3)+(-1)(2\lambda)+(2)(0)=0$

$\Rightarrow 6-2\lambda+0=0$

$\Rightarrow -2\lambda=-6$

$\Rightarrow \lambda=\frac{-6}{-2}\quad\Rightarrow\quad\boxed{\lambda=3}$

23. **Define scalar matrix.**

Sol. A diagonal matrix in which all diagonal elements are same is called scalar matrix.

24. **Show that** $\begin{vmatrix} b & -1 & a \\ a & b & 0 \\ 1 & a & b \end{vmatrix}=b^3+a^3$

Sol. $\text{L.H.S.} = \begin{vmatrix} b & -1 & a \\ a & b & 0 \\ 1 & a & b \end{vmatrix}$

$= b\begin{vmatrix} b & 0 \\ a & b \end{vmatrix} - (-1)\begin{vmatrix} a & 0 \\ 1 & b \end{vmatrix} + a\begin{vmatrix} a & b \\ 1 & a \end{vmatrix}$

$= b(b^2 - 0) + 1(ab - 0) + a(a^2 - b)$

$= b^3 - 0 + ab - 0 + a^3 - ab$

$= b^3 + a^3 = \text{R.H.S.}$ **Proved.**

25. **Find x and y if**

$$\begin{bmatrix} 2 & 1 \\ -3 & 2 \end{bmatrix} = \begin{bmatrix} x+3 & 1 \\ -3 & 3y-4 \end{bmatrix}$$

Sol. $\begin{bmatrix} 2 & 1 \\ -3 & 2 \end{bmatrix} = \begin{bmatrix} x+3 & 1 \\ -3 & 3y-4 \end{bmatrix}$

Comparing corresponding elements of both matrices :

$x + 3 = 2$ | $3y - 4 = 2$

$x = 2 - 3$ | $3y = 2 + 4$

$\boxed{x = -1}$ | $3y = 6$

$y = \frac{6}{3}$

$\boxed{y = 2}$

26. **Find 'k' if** $A = \begin{bmatrix} 4 & k & 3 \\ 7 & 3 & 6 \\ 2 & 3 & 1 \end{bmatrix}$ **is a singular matrix.**

Sol. As, A is singular so $|A| = 0$

$$\begin{vmatrix} 4 & k & 3 \\ 7 & 3 & 6 \\ 2 & 3 & 1 \end{vmatrix} = 0$$

$$4\begin{vmatrix} 3 & 6 \\ 3 & 1 \end{vmatrix} - k\begin{vmatrix} 7 & 6 \\ 2 & 1 \end{vmatrix} + 3\begin{vmatrix} 7 & 3 \\ 2 & 3 \end{vmatrix} = 0$$

$4(3 - 18) - k(7 - 12) + 3(21 - 6) = 0$

$4(-15) - k(-5) + 3(15) = 0$

$-60 + 5k + 45 = 0$

$5k - 15 = 0$

$5k = 15$

$k = \frac{15}{5}$

$\boxed{k = 3}$

27. **Find** A^{-1} **if** $A = \begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix}$

Sol. $A = \begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix}$

$|A| = \begin{vmatrix} 1 & 3 \\ 2 & -1 \end{vmatrix}$

$|A| = -1 - 6 = -7$

$\text{Adj } A = \begin{bmatrix} -1 & -3 \\ -2 & 1 \end{bmatrix}$

$A^{-1} = \frac{\text{Adj}(A)}{|A|}$

$A^{-1} = \frac{\begin{bmatrix} -1 & -3 \\ -2 & 1 \end{bmatrix}}{-7}$

$$A^{-1} = \begin{bmatrix} \frac{1}{7} & \frac{3}{7} \\ \frac{2}{7} & -\frac{1}{7} \end{bmatrix}$$

Section - II

Note : **Attemp any three (3) questions** $3 \times 8 = 24$

Q.2.(a) **From the point within an equilateral triangle perpendicular are drawn to the three sides are 6, 7 and 8cm respectively. Find the area of triangle.**

Sol. See Q.2 of Ex# 10 (Page # 462)

(b) Regular polygons of 15 sides are inscribed in and circumscribed about a circle whose radius is 12cm. Show that the difference of their areas is nearly 20 square cm.

Sol. See Q.4 of Ex # 12 (Page # 487)

Q.3.(a) Following ordinates of equal intervals are drawn in a plot of base 1200 meters. Find the area by Simpson's rule if these ordinates are 50, 60, 80, 90, 30, 50, 60, 80, 70, 90, 100, 120, 130meters.

Sol. See Q.7 of Ex # 14 (Page # 510)

(b) A 10cm cube of cast iron is melted and cast into a hexagonal prism with a height of 12cm. Find the side of the base of prism.

Sol. See Q.3 of Ex # 15 (Page # 518)

Q.4.(a) Find the whole surface of a pyramid whose base is an equilateral triangle of side 3m and its slant height is 6m.

Sol. See Q.7 of Ex # 17[A] (Page # 548)

(b) ~~Find the cost of canvas, at the rate of Rs.5 per square meter, required to make a tent in the form of a frustum of a square pyramid. The sides of the base and top are 6m and 4m respectively and the height is 8m, taking no account of waste.~~

Sol. See Q.5 of Ex # 17[B] (Page # 556)

Q.5.(a) Given the vectors $\vec{a} = 3\underline{i} - 2\underline{j} + 4\underline{k}$ and $\vec{b} = 2\underline{i} + \underline{j} + 3\underline{k}$ find the magnitude and direction cosines of $3\vec{a} - 2\vec{b}$.

Sol. See Q.9(ii) of Ex # 8.1 (Page # 374)

(b) Find the Sine of the angle between the vectors: $\vec{a} = \underline{i} + \underline{j} + \underline{k}$ and $\vec{b} = 2\underline{i} + 3\underline{j} - \underline{k}$

Sol. See Q.19(i) of Ex # 8.2 (Page # 388)

Q.6. Use Cramer's rule to solve the following system of equation

$$x - 2y + z = -1$$
$$3x + y - 2z = 4$$
$$y - z = 1$$

Sol. See Q.8(iii) of Ex # 9.2 (Page # 429)

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