DAE / IIA - 2020

MATH - 113 APPLIED MATHEMATICS - I

PAPER 'A' PART - A (OBJECTIVE)

Time : 30 Minutes Marks : 15

Q.1: *Encircle the correct answer.*

1. A second degree equation is known as:

[a] Linear [b] Quadratic
[c] Cubic [d] None of these

2. If the discriminant $b^2 - 4ac$ is negative, the roots are:

[a] Real [b] Rational
[c] Irrational [d] Imaginary

3. The 10th term in $7, 17, 27, ...$ is:

[a] 97 [b] 98 [c] 99 [d] 100

4. The 5th term of a G.P. $1, \frac{1}{2}, \frac{1}{4}, ...$ is:

[a] $\frac{1}{8}$ [b] $-\frac{1}{8}$ [c] $\frac{1}{16}$ [d] $\frac{1}{32}$

5. If a, b, c are in A.P; then:

[a] $b - a = c - b$ [b] $\frac{b}{a} = \frac{c}{b}$
[c] $a + b = b + c$ [d] $\frac{a}{c} = \frac{b}{a}$

6. Third term of $(x + y)^4$ is:

[a] $4x^2y^2$ [b] $4x^3y$
[c] $6x^2y^2$ [d] $6x^3y$

7. $\binom{6}{4}$ will have the value:

[a] 10 [b] 15
[c] 20 [d] 25

8. The fraction $\frac{(x-1)(x-2)(x-3)}{(x-4)(x-5)(x-6)}$ is called:

[a] Proper [b] Improper
[c] Both [a] & [b] [d] None of these

9. One radian is equal to:

[a] $90°$ [b] $\left(\frac{90}{\pi}\right)°$
[c] $180°$ [d] $\left(\frac{180}{\pi}\right)°$

10. The relation between arc 'ℓ' central angle 'θ' in radian and radius 'r' is:

[a] $\ell = \frac{\theta}{r}$ [b] $\ell = \frac{r}{\theta}$
[c] $\ell = r\theta$ [d] $\ell = r^2\theta$

11. Angle made at the centre of a circle of a circle by an arc equal to the radius of the circle is called:

[a] Right angle [b] Degree
[c] Radian [d] Acute angle

12. $\cos(\alpha - \beta)$ is equal to:

[a] $\cos\alpha\cos\beta - \sin\alpha\sin\beta$
[b] $\cos\alpha\cos\beta + \sin\alpha\sin\beta$
[c] $\cos\alpha\sin\beta - \sin\alpha\cos\beta$
[d] $\sin\alpha\cos\beta + \cos\alpha\sin\beta$

13. $\cos(\alpha + \beta) - \cos(\alpha - \beta)$ is equal to:

[a] $2\sin\alpha\cos\beta$ [b] $2\cos\alpha\sin\beta$
[c] $2\cos\alpha\cos\beta$ [d] $-2\sin\alpha\sin\beta$

14. In a triangle ABC, , $\angle A = 70°$, $\angle B = 60°$ then $\angle C$ is:

[a] $30°$ [b] $40°$ [c] $50°$ [d] $60°$

15. In $\angle B = 90°$; $b = 2$; $\angle A = 30°$ then side 'a' of the triangle ABC is equal to:

[a] 4 [b] 3 [c] 2 [d] 1

Answer Key

1	b	2	d	3	b	4	c	5	a
6	c	7	b	8	b	9	d	10	c
11	c	12	b	13	d	14	c	15	d

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DAE / IIA - 2020

MATH - 113 APPLIED MATHEMATICS - I

PAPER 'A' PART - B (SUBJECTIVE)

Time: 2:30 Hrs   Marks: 60

Section - I

1. Solve the quadratic equation $6x^2 - 5x = 4$ by factorization.

Sol.
$$6x^2 - 5x = 4$$
$$6x^2 - 5x - 4 = 0$$
$$6x^2 - 8x + 3x - 4 = 0$$
$$2x(3x-4) + 1(3x-4) = 0$$
$$(3x-4)(2x+1) = 0$$

Either: $3x - 4 = 0$, $3x = 4$, $x = \frac{4}{3}$

OR: $2x + 1 = 0$, $2x = -1$, $x = -\frac{1}{2}$

$$S.S. = \left\{-\frac{1}{2}, \frac{4}{3}\right\}$$

2. ~~Find the sum and product of the roots of the equation~~ $9x^2 + 6x + 1 = 0$

Sol. Here: $a = 9,\ b = 6,\ c = 1$

Sum of Roots: $S = -\frac{b}{a} = -\frac{6}{9} = \boxed{-\frac{2}{3}}$

Product of Roots: $P = \frac{c}{a} = \boxed{\frac{1}{9}}$

3. ~~Form the quadratic equation whose roots are~~ $-2+\sqrt{3},\ -2-\sqrt{3}$

Sol.
$$S = -2+\sqrt{3} + \left(-2-\sqrt{3}\right)$$
$$S = -2+\sqrt{3} - 2 - \sqrt{3} = -4$$
$$P = \left(-2+\sqrt{3}\right)\left(-2-\sqrt{3}\right)$$
$$P = (-2)^2 - \left(\sqrt{3}\right)^2 = 4 - (3) = 1$$
$$x^2 - Sx + P = 0$$
$$x^2 - (-4)x + 1 = 0 \Rightarrow \boxed{x^2 + 4x + 1 = 0}$$

4. Define a sequence.

Sol. A set of numbers arranged in order by some fixed rule is called a sequence. For examples:
(i) $2, 4, 6, \ldots$ **(ii)** $3, 9, 27, \ldots$

5. Find the 7th term of an A.P. $1, 4, 7, \ldots$

Sol. Here: $a_1 = 1$ & $d = 4 - 1 = 3$
$$a_7 = a_1 + 6d$$
$$a_7 = 1 + 6(3)$$
$$a_7 = 1 + 18 \Rightarrow \boxed{a_7 = 19}$$

6. Find the A.M between $\sqrt{5} - 4$ and $\sqrt{5} + 4$

Sol. Let $a = \sqrt{5} - 4$ and $b = \sqrt{5} + 4$
$$A.M. = A = \frac{a+b}{2}$$
$$A = \frac{\sqrt{5} - 4 + \sqrt{5} + 4}{2} = \frac{2\sqrt{5}}{2} = \boxed{\sqrt{5}}$$

7. ~~Write the formula of sum of the first 'n' terms of a G.P. for~~ $|r| < 1$ and for $|r| > 1$.

Sol. Formulas:
$$(i)\ S_n = \frac{a(1-r^n)}{1-r} \text{ if } |r| < 1$$
$$(ii)\ S_n = \frac{a(r^n-1)}{r-1} \text{ if } |r| > 1$$

8. Find the geometric mean between 8 and 72.

Sol. Let: $a = 8$ & $b = 72$
$$\text{As, } G = \pm\sqrt{ab}$$
$$G = \pm\sqrt{8 \times 72} = \pm\sqrt{576} = \boxed{\pm 24}$$

9. ~~Find the sum of infinite geometric series in which~~ $a = 128$ and $r = -\frac{1}{2}$.

Sol.
$$S_\infty = \frac{a}{1-r}$$

$$S_\infty = \frac{128}{1-\left(-\frac{1}{2}\right)} = \frac{128}{1+\frac{1}{2}} = \frac{128}{\frac{2+1}{2}}$$

$$S_\infty = \frac{128}{3/2} = 128 \times \frac{2}{3} = \boxed{\frac{256}{3}}$$

10. **Expand $\left(\frac{x}{2}-\frac{2}{y}\right)^4$ by using binomial theorem.**

Sol. $\left(\frac{x}{2}-\frac{2}{y}\right)^4$

$$= \binom{4}{0}\left(\frac{x}{2}\right)^4\left(\frac{2}{y}\right)^0 - \binom{4}{1}\left(\frac{x}{2}\right)^3\left(\frac{2}{y}\right)^1 + \binom{4}{2}\left(\frac{x}{2}\right)^2\left(\frac{2}{y}\right)^2 - \binom{4}{3}\left(\frac{x}{2}\right)^1\left(\frac{2}{y}\right)^3 + \binom{4}{4}\left(\frac{x}{2}\right)^0\left(\frac{2}{y}\right)^4$$

$$= (1)\left(\frac{x^4}{16}\right)(1) - 4\left(\frac{x^3}{8}\right)\left(\frac{2}{y}\right) + 6\left(\frac{x^2}{4}\right)\left(\frac{4}{y^2}\right) - 4\left(\frac{x}{2}\right)\left(\frac{8}{y^3}\right) + (1)(1)\left(\frac{16}{y^4}\right)$$

$$= \boxed{\frac{x^4}{16} - \frac{x^3}{y} + 6\frac{x^2}{y^2} - 16\frac{x}{y^3} + \frac{16}{y^4}}$$

11. **Calculate $(1.02)^{10}$ by binomial theorem up to two decimal places.**

Sol. $(1.02)^{10} = (1+0.02)^{10}$

$$= \binom{10}{0}(1)^{10}(0.02)^0 + \binom{10}{1}(1)^9(0.02)^1 + \binom{10}{2}(1)^8(0.02)^2 + \ldots$$

$$= (1)(1)(1) + 10(1)(0.02) + 45(1)(0.0004) + \ldots$$

$$= 1 + 0.2 + 0.018 + \ldots = 1.2180 = \boxed{1.22}$$

12. ~~**Expand $\frac{1}{\sqrt{1+x}}$ to three terms.**~~

Sol. $\frac{1}{\sqrt{1+x}} = (1+x)^{-1/2}$

$$= 1 + \left(-\frac{1}{2}\right)(x) + \frac{\left(-\frac{1}{2}\right)\left(-\frac{1}{2}-1\right)}{2!}(x)^2 + \ldots$$

$$= 1 - \frac{x}{2} + \frac{1}{2}\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)x^2 + \ldots$$

$$= \boxed{1 - \frac{x}{2} + \frac{3}{8}x^2 + \ldots}$$

13. **Resolve $\frac{1}{x^2-x}$ into partial fractions.**

Sol. $\frac{1}{x^2-x} = \frac{1}{x(x-1)}$

$$\frac{1}{x(x-1)} = \frac{A}{x} + \frac{B}{x-1} \rightarrow (i)$$

$$1 = A(x-1) + Bx \rightarrow (ii)$$

Put $x = 0$ in eq. (ii)

$$1 = A(0-1) + B(0)$$

$$1 = -A \Rightarrow \boxed{A = -1}$$

Put $x = 1$ in eq. (ii)

$$1 = A(1-1) + B(1)$$

$$1 = A(0) + B \Rightarrow \boxed{B = 1}$$

Put values of A, & B in eq. (i), we get: $\boxed{-\frac{1}{x} + \frac{1}{x-1}}$

14. **Write identity equation of $\frac{x-5}{(x+1)(x^2+3)}$**

Sol. $$\frac{x-5}{(x+1)(x^2+3)} = \boxed{\frac{A}{(x+1)} + \frac{Bx+C}{(x^2+3)}}$$

15. **Form of partial fractions of $\frac{1}{(x+1)^2(x-2)}$ is ________.**

Sol. $$\frac{1}{(x+1)^2(x-2)} = \boxed{\frac{A}{(x+1)} + \frac{B}{(x+1)^2} + \frac{C}{(x-2)}}$$

16. ~~**What is the length of an arc of a circle of radius 5 cm whose central angle is 140º.**~~

Sol. Here: $\ell = ?,\ r = 5\text{cm},\ \theta = 140^\circ$

$$\theta = 140^\circ = 140 \times \frac{\pi}{180} = 2.44\text{rad.}$$

By using formula: $\ell = r\theta$

$$\ell = r\theta = (5)(2.44) = \boxed{12.22\text{cm}}$$

17. ~~**Find the radius of the circle, when ℓ = 8.4m, θ = 2.8 rad.**~~

Sol. We know that: $\ell = r\theta$

$$\Rightarrow r = \frac{\ell}{\theta} = \frac{8.4}{2.8} = \boxed{3\text{m}}$$

18. **Prove that:**

$$(1+\sin\theta)(1-\sin\theta) = \frac{1}{\sec^2\theta}$$

Sol. $\text{L.H.S.} = (1+\sin\theta)(1-\sin\theta)$

$$= (1)^2 - (\sin\theta)^2 = 1 - \sin^2\theta$$

$$= \cos^2\theta = \frac{1}{\sec^2\theta} = \text{R.H.S.}$$ **Proved.**

19. **Prove that:**

$$\cos^4\theta - \sin^4\theta = 1 - 2\sin^2\theta$$

Sol. $\text{L.H.S.} = \cos^4\theta - \sin^4\theta$

$$= (\cos^2\theta)^2 - (\sin^2\theta)^2$$

$$= (\cos^2\theta - \sin^2\theta)(\cos^2\theta + \sin^2\theta)$$

$$= (1 - \sin^2\theta - \sin^2\theta)(1)$$

$$= 1 - 2\sin^2\theta = \text{R.H.S.}$$ **Proved.**

20. **Prove that:** $\sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta$

Sol. $\text{L.H.S.} = \sin\left(\frac{\pi}{2} - \theta\right)$

$$= \sin(90^\circ - \theta) \quad \because \left\{\frac{\pi}{2} \times \frac{180^\circ}{\pi} = 90^\circ\right\}$$

$$= \sin 90^\circ \cos\theta - \cos 90^\circ \sin\theta$$

$$= (1)\cos\theta - (0)\sin\theta \quad \because \left\{\begin{array}{c}\text{Using calculator}\\ \cos 90^\circ = 0\ \&\ \sin 90^\circ = 1\end{array}\right\}$$

$$= \cos\theta - 0 = \cos\theta = \text{R.H.S.}$$ **Proved.**

21. **Show that:**

$$\sin(\alpha+\beta) + \sin(\alpha-\beta) = 2\sin\alpha\cos\beta$$

Sol. $\text{L.H.S.} = \sin(\alpha+\beta) + \sin(\alpha-\beta)$

$$= \sin\alpha\cos\beta + \cancel{\cos\alpha\sin\beta} + \sin\alpha\cos\beta - \cancel{\cos\alpha\sin\beta}$$

$$= 2\sin\alpha\cos\beta = \text{R.H.S.}$$ **Proved.**

22. **Express $\cos(a+b)\cos(a-b) - \sin(a+b)\sin(a-b)$ as single term.**

Sol.

$$\cos(a+b)\cos(a-b) - \sin(a+b)\sin(a-b)$$

$$= \cos(a+b+a-b)$$

$$= \boxed{\cos 2a}$$

23. ~~**Express the sum $\cos\theta - \cos 4\theta$ as product.**~~

Sol. $\cos\theta - \cos 4\theta$

$$= -2\sin\left(\frac{\theta+4\theta}{2}\right)\sin\left(\frac{\theta-4\theta}{2}\right)$$

$$= -2\sin\left(\frac{5\theta}{2}\right)\sin\left(\frac{-3\theta}{2}\right) = \boxed{2\sin\left(\frac{5\theta}{2}\right)\sin\left(\frac{3\theta}{2}\right)}$$

24. **Define the law of Sine.**

Sol. In any triangle ABC, with usual notations.

$$\boxed{\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}}$$

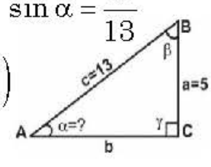
25. ~~**In right triangle ABC, γ = 90°, a = 5, c = 13 then find value of angle α.**~~

Sol. We know that, from figure:

$$\sin\alpha = \frac{a}{c} \Rightarrow \sin\alpha = \frac{5}{13}$$

$$\alpha = \sin^{-1}\left(5/13\right)$$

$$\boxed{\alpha = 22^\circ 37'}$$



26. **The sides of a triangle are 16, 20 and 33 meters respectively. Find its greatest angle.**

Sol. Let a = 16, b = 20, c = 33
As side 'c' is greatest so, we will find angle 'γ'.

Using law of cosines: $\cos\gamma = \dfrac{a^2+b^2-c^2}{2ab}$

$$\cos\gamma = \frac{(16)^2+(20)^2-(33)^2}{2(16)(20)} = -0.6765$$

$$\gamma = \cos^{-1}(-0.6765) \Rightarrow \boxed{\gamma = 132^\circ 34'}$$

27. **In any triangle ABC in which a = 16, b = 17, γ = 25º, find 'c'.**

Sol. By using law of cosines:

$$c^2 = a^2+b^2-2ab\cos\gamma$$

$$c^2 = (16)^2+(17)^2-2(16)(17)\cos 25^\circ$$

$$c^2 = 256+289-493.03$$

$$c^2 = 51.97 \Rightarrow \sqrt{c^2} = \sqrt{51.97} \Rightarrow \boxed{c = 7.2}$$

Section - II

Note: Attemp any three (3) questions 3 × 8 = 24

Q.2.(a) Solve the equation $\dfrac{1}{x+1}+\dfrac{1}{x+2}+\dfrac{1}{x+3}=\dfrac{3}{x}$ **by using quadratic formula.**

Sol. See Q.3(iii) of Ex # 1.1 (Page # 20)

(b) **Show that the roots of the equation $(mx+c)^2 = 4ax$ will be equal; is $c = \dfrac{a}{m}$**

Sol. See Q.3(ii) of Ex # 1.2 (Page # 33)

Q.3.(a) The 9th term of an A.P is 30 and the 17th term is 50. Find the first three terms.

Sol. See Q.8 of Ex # 2.1 (Page # 80)

(b) **Find 'n' so that $\dfrac{a^{n+1}+b^{n+1}}{a^n+b^n}$ may be the A.M's between a and b.**

Sol. See Q.7 of Ex # 2.2 (Page # 86)

Q.4.(a) Find the 5th term in the expansion of $\left(2x^2-\dfrac{3}{x}\right)^{10}$.

Sol. See Q.5(i) of Ex # 3.1 (Page # 148)

(b) **Resolve $\dfrac{3x^2-2x-5}{(x-2)(x+2)(x+3)}$ into partial fractions.**

Sol. See Q.3 of Ex # 4.1 (Page # 182)

Q.5.(a) Prove that:

$$\sqrt{\frac{1+\cos\theta}{1-\cos\theta}} = \operatorname{cosec}\theta + \cot\theta$$

Sol. See Q.10 of Ex # 5.3 (Page # 256)

(b) **Show that:**

$$\sqrt{3}\cos\theta - \sin\theta = 2\cos(\theta+30^\circ)$$

Sol. See Q.5(ii) of Ex # 6.1 (Page # 279)

Q.6.(a) Prove that:

$$\sin 3\theta = 3\sin\theta - 4\sin^3\theta$$

Sol. See Triple angles proof (Page # 294)

(b) **From a point on the ground the measure of angle of elevation of the top of a tower is 30°. On walking 100 meters towards the tower the measure of the angle if found to be 45°. Find the height of the tower.**

Sol. See example # 03 of Ch # 07

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