DAE / IIA - 2020

MATH-123 APPLIED MATHEMATICS - I

PAPER 'B' PART - A (OBJECTIVE)

Time : 30 Minutes Marks : 15

Q.1: *Encircle the correct answer.*

1. Conjugate of $(2+3i)+(1-i)$ is:

[a] $3-2i$ [b] $3+4i$
[c] $3-4i$ [d] $3+2i$

2. $(1+2i)(3-5i)$ is equal to:

[a] $13+i$ [b] $-2-i$
[c] $-4+3i$ [d] $3i$

3. Modulus of $3+4i$ is:

[a] 47 [b] 16
[c] 5 [d] 3

4. The Fractions $\frac{2x+5}{x^2+5x+6}$ is known as:

[a] Proper [b] Improper
[c] Neither proper nor improper
[d] None of these

5. Numbers of digit in a binary system are:

[a] 2 [b] 7
[c] 10 [d] 8

6. Conversion of 9 to binary number system is:

[a] $(1001)_2$ [b] $(101)_2$
[c] $(11)_2$ [d] None of these

7. ~~In Boolean algebra~~ $\overline{X+Y}$ ~~is equal to:~~

[a] $\overline{X}+\overline{Y}$ [b] $\overline{X}.\overline{Y}$
[c] XY [d] $X+Y$

8. If switch is off it is represented by:

[a] 0 [b] 1
[c] OR [d] NOT

9. Slope of the line $\frac{x}{a}+\frac{y}{b}=1$ is:

[a] $\frac{a}{b}$ [b] $\frac{b}{a}$
[c] $\frac{-b}{a}$ [d] $\frac{-a}{b}$

10. Point $(-4,-5)$ lies in the quadrant:

[a] 1^{st} [b] 2^{nd}
[c] 3^{rd} [d] 4^{th}

11. The points are collinear if their slopes are:

[a] Equal [b] Unequal
[c] $m_1m_2=-1$ [d] $m_1=-\frac{1}{m_2}$

12. $y=2$ is a line parallel to:

[a] $x-$axis [b] $y-$axis
[c] $y=x$ [d] $x=3$

13. Center of the circle $x^2+y^2-2x-4y=8$ is:

[a] $(1,2)$ [b] $(2,4)$
[c] $(1,3)$ [d] None of these

14. Radius of the circle $x^2+y^2+2gx+2fy+c=0$ is:

[a] c [b] c^2
[c] $\sqrt{g^2+f^2-c}$ [d] None of these

15. Straight line from center to the circumference is:

[a] Circle [b] Radius
[c] Diameter [d] None of these

Answer Key

1	a	2	a	3	c	4	a	5	a
6	a	7	b	8	a	9	d	10	c
11	a	12	a	13	a	14	c	15	b

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DAE / IIA - 2020

MATH - 123 APPLIED MATHEMATICS - I

PAPER 'B' PART - B (SUBJECTIVE)

Time: 2:30 Hrs Marks: 60

Section - I

1. Find z such that $|z| = \sqrt{2}$ and $\arg(z) = \frac{\pi}{4}$

Sol. $z = r\ \text{cis}\ \theta = \sqrt{2}\,\text{cis}\left(\frac{\pi}{4}\right)$

$z = \sqrt{2}\left[\cos 45^\circ + i\sin 45^\circ\right]$

$z = \sqrt{2}\left[\frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}\right] = \boxed{1+i}$

2. Find the value of 'x' and 'y' such that $(2x-3y)+i(x-y)6 = 2-i(2x-y+3)$

Sol. $(2x-3y)+i(x-y)6 = 2-i(2x-y+3)$

Comparing real & imaginary parts:

$2x-3y = 2 \rightarrow (i),$

$(x-y)6 = -(2x-y+3)$

$6x-6y = -2x+y-3$

$6x-6y+2x-y = -3$

$8x-7y = -3 \rightarrow (ii)$

Multiply eq. (i) by 4 & subtracting eq. (ii)

$8x-12y = 8$

$\pm 8x \mp 7y = \mp 3$

$-5y = 11 \Rightarrow \boxed{y = -\frac{11}{5}}$

Put $y = -\frac{11}{5}$ in eq. (i), we have:

$2x - 3\left(-\frac{11}{5}\right) = 2 \Rightarrow 2x + \frac{33}{5} = 2$

$2x = 2 - \frac{33}{5} \Rightarrow 2x = \frac{10-33}{5}$

$2x = -\frac{23}{5} \Rightarrow \boxed{x = -\frac{23}{10}}$

3. Show that $\left|\frac{1+2i}{2-i}\right| = 1$

Sol. $\text{L.H.S.} = \frac{1+2i}{2-i}$

$= \frac{\sqrt{(1)^2+(2)^2}}{\sqrt{(2)^2+(-1)^2}}$

$= \frac{\sqrt{1+4}}{\sqrt{4+1}} = \frac{\sqrt{5}}{\sqrt{5}} = 1 = \text{R.H.S.}$ **Proved.**

4. Factorize $36a^2 + 100b^2$

Sol. $36a^2 + 100b^2$

$= 36a^2 - 100b^2 i^2$

$= (6a)^2 - (10bi)^2$

$= \boxed{(6a-10bi)(6a+10bi)}$

5. Show that $z^2 + \bar{z}^2$ is a real number.

Sol. Let, $z = a+bi$, then $\bar{z} = a-bi$

Take $z^2 + \bar{z}^2$

$= (a+bi)^2 + (a-bi)^2$

$= (a)^2 + (bi)^2 + 2(a)(bi) + (a)^2 + (bi)^2 - 2(a)(bi)$

$= a^2 - b^2 + \cancel{2abi} + a^2 - b^2 - \cancel{2abi}$

$= 2a^2 - 2b^2 = 2(a^2 - b^2) \in \mathbb{R}$

Hence $z^2 + \bar{z}^2$ is real number. **Proved.**

6. What is partial fractions.

Sol. The process, which convert a single rational fraction, into the sum of two or more single rational fractions is called partial fractions.

7. Resolve into partial fractions $\frac{1}{x^2-1}$

Sol. $\frac{1}{x^2-1} = \frac{1}{(x)^2-(1)^2} = \frac{1}{(x-1)(x+1)}$

$$\frac{1}{(x-1)(x+1)}=\frac{A}{x-1}+\frac{B}{x+1}\rightarrow(i)$$

$$1=A(x+1)+B(x-1)\rightarrow(ii)$$

Put $x=1$ in eq.(ii)

$$1=A(1+1)+B(1-1)$$

$$1=A(2)+B(0)$$

$$1=2A+0 \Rightarrow \boxed{A=\tfrac{1}{2}}$$

Put $x=-1$ in eq.(ii)

$$1=A(-1+1)+B(-1-1)$$

$$1=A(0)+B(-2)$$

$$1=0-2B \Rightarrow \boxed{B=-\tfrac{1}{2}}$$

Put values of A, & B in eq. (i),

we get: $\boxed{\frac{1}{2(x-1)}-\frac{1}{2(x+1)}}$

8. ~~**Form of partial fractions of**~~

~~$$\frac{8x^2}{(1-x^2)(1+x^2)^2}$$~~

Sol.

$$\frac{8x^2}{(1-x^2)(1+x^2)^2}$$

$$=\frac{8x^2}{(1-x)(1+x)(1+x^2)^2}$$

$$=\boxed{\frac{A}{(1-x)}+\frac{B}{(1+x)}+\frac{Cx+D}{(1+x^2)}+\frac{Ex+F}{(1+x^2)^2}}$$

9. Add the binary numbers

$$(1101)_2+(1011_2)$$

Sol.

$$\begin{array}{r} 1101 \\ +1011 \\ \hline 11000 \end{array}$$

$$\boxed{(11000)_2}$$

10. Convert Binary number $(101101)_2$ to octal number.

Sol. $\overbrace{101}\ \overbrace{101}$

101	101
$1\times2^2+0\times2^1+1\times2^0$	$1\times2^2+0\times2^1+1\times2^0$
$4+0+1=5$	$4+0+1=5$

$$(101101)_2=\boxed{(55)_8}$$

11. Prove by Boolean Algebra rules:

$$\mathbf{X(X+Z)=X}$$

Sol. $L.H.S.=X(X+Z)$

$=XX+XZ$

$=X+XZ \qquad \because XX=X$

$=X(1+Z)$

$=X(1) \qquad \because 1+Z=1$

$=X=R.H.S.$ **Proved.**

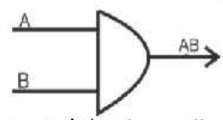
12. Prepare a truth table for

$$\mathbf{X(\overline{X}+Y)=X\cdot Y}$$

Sol. $X(\overline{X}+Y)=X\cdot Y$

X	Y	$\overline{X}$	$\overline{X}+Y$	L.H.S. $X(\overline{X}+Y)$	R.H.S. XY
0	0	1	1	0	0
0	1	1	1	0	0
1	0	0	0	0	0
1	1	0	1	1	1

13. Define AND Gate.



Sol. The OR gate is an electronic circuit that gives a high output (1) when all Of its inputs are 1.

14. Prove that:

$$\mathbf{AB+AC+ABC=AB+AC}$$

Sol. $L.H.S.=AB+AC+ABC$

$=AB+AC(1+B)$

$=AB+AC(1) \quad \because 1+B=1$

$=AB+AC=R.H.S.$ **Proved.**

15. **Find the coordinates of the mid-point of the segment $P_1(3, 7)$ and $P_2(-2, 3)$.**

Sol. $\text{Mid-point} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$$= \left(\frac{3+(-2)}{2}, \frac{7+3}{2}\right) = \left(\frac{3-2}{2}, \frac{10}{2}\right) = \boxed{\left(\frac{1}{2}, 5\right)}$$

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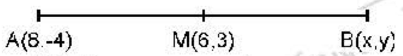
16. **If the mid-point of a segment is (6, 3) and one end point is (8, − 4), what are the coordinates of the other end point.**

Sol. Let B(x, y) be require end point.



As, $\text{Mid-point} = (6, 3)$

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = (6, 3)$$

$$\left(\frac{8+x}{2}, \frac{-4+y}{2}\right) = (6, 3)$$

Comparing both order pairs, we have :

$$\frac{8+x}{2} = 6 \text{ and } \frac{-4+y}{2} = 3$$

$8+x = 12$ | $-4+y = 6$

$x = 12-8 = 4$ | $y = 6+4 = 10$

Hence other end point $= \boxed{(4, 10)}$

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17. ~~**Find the equation of a line through the point $(3, -2)$ and slope is $\frac{3}{4}$.**~~

Sol. Equation of line in point-slope form:

$$y - y_1 = m(x - x_1)$$

$$y-(-2) = \frac{3}{4}(x-3) \Rightarrow 4(y+2) = 3(x-3)$$

$$4y+8 = 3x-9 \Rightarrow 4y+8-3x+9 = 0$$

$$-3x+4y+17 = 0 \Rightarrow \boxed{3x-4y-17=0}$$

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18. ~~**Find the equation of a line whose perpendicular distance from the origin is 2 and inclination of the perpendicular is 225°.**~~

Sol. Here $P = 2$ and $\theta = 225°$

Normal form of equation of line is

$$x\cos\theta + y\sin\theta = P$$

$$x\cos 225° + y\sin 225° = 2$$

$$x\left(-\frac{1}{\sqrt{2}}\right) + y\left(-\frac{1}{\sqrt{2}}\right) = 2$$

Multipling each term by $\sqrt{2}$ we get :

$$-x-y = 2\sqrt{2} \Rightarrow \boxed{x+y+2\sqrt{2}=0}$$

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19. ~~**Find the distance from the point $(-2, 1)$ to the line $3x+4y-12=0$**~~

Sol. Distance between point & line

$$D = \frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}} = \frac{|3(-2)+4(1)-12|}{\sqrt{(3)^2+(4)^2}}$$

$$D = \frac{|-6+4-12|}{\sqrt{9+16}} = \frac{|-14|}{\sqrt{25}} = \boxed{\frac{14}{5}}$$

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20. ~~**Find the angle between the lines having slopes −3 and 2.**~~

Sol. Let, $m_1 = -3$ and $m_2 = 2$

$$\theta = \tan^{-1}\left(\frac{m_2-m_1}{1+m_2m_1}\right)$$

$$\theta = \tan^{-1}\left(\frac{2-(-3)}{1+(2)(-3)}\right)$$

$$\theta = \tan^{-1}\left(\frac{2+3}{1-6}\right) = \tan^{-1}\left(\frac{5}{-5}\right)$$

$$\theta = \tan^{-1}(-1) = \boxed{135°}$$

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21. **Show that the points $(1, 2)$, $(7, 6)$ and $(4, 4)$ are collinear.**

Sol. $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 2 & 1 \\ 7 & 6 & 1 \\ 4 & 4 & 1 \end{vmatrix}$

$= 1\begin{vmatrix} 6 & 1 \\ 4 & 1 \end{vmatrix} - 2\begin{vmatrix} 7 & 1 \\ 4 & 1 \end{vmatrix} + 1\begin{vmatrix} 7 & 6 \\ 4 & 4 \end{vmatrix}$

$= 1(6-4) - 2(7-4) + 1(28-24)$

$= 1(2) - 2(3) + 1(4) = 2 - 6 - 4 = 0$

Hence given points are collinear. **Proved.**

22. ~~**Find the equation of the perpendicular bisector of the line segment joining the points**~~ $(2, 4)$ ~~**and**~~ $(6, 8)$.

Sol. Let, $A(2, 4)$ & $B(6, 8)$

Slope of $\overline{AB}$

$= m' = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{8-4}{6-2} = \dfrac{4}{4} = 1$

As, given line and require line are **perpendicular**

So, Slope of require line $= m$

$= -\dfrac{1}{m'} = -1$

Midpoint of $\overline{AB}$

$= \left(\dfrac{2+6}{2}, \dfrac{4+8}{2}\right) = \left(\dfrac{8}{2}, \dfrac{12}{2}\right) = (4,6)$

Equation of line through (4,6) having slope -1 is:

$y - y_1 = m(x - x_1)$

$y - 6 = -1(x-4) \Rightarrow y - 6 = -x + 4$

$y - 6 + x - 4 = 0 \Rightarrow \boxed{x + y - 10 = 0}$

23. **Find the equation of circle with center $(-1, 2)$ and radius $r = \sqrt{2}$.**

Sol. Standard form of equation of circle:

$(x-h)^2 + (y-k)^2 = r^2$

Put $h = -1,\ k = 2\ \&\ r = \sqrt{2}$

$(x-(-1))^2 + (y-2)^2 = (\sqrt{2})^2$

$(x+1)^2 + (y-2)^2 = 2$

$x^2 + 2x + 1 + y^2 - 4y + 4 - 2 = 0$

$\boxed{x^2 + y^2 + 2x - 4y + 3 = 0}$

24. **Find center and radius of the circle $x^2 + y^2 - 6x + 6y = 0$**

Sol. $x^2 + y^2 - 6x + 6y = 0$

Comparing with general equation of circle.

$x^2 + y^2 + 2gx + 2fy + c = 0$

$2g = -6$	$2f = 6$	$c = 0$
$g = -\dfrac{6}{2} = -3$	$f = \dfrac{6}{2} = 3$	

Center $= (-g, -f)$

Center $= (-(-3), -3) = \boxed{(3, -3)}$

Radius $= r = \sqrt{g^2 + f^2 - c}$

$r = \sqrt{(-3)^2 + (3)^2 - 0} = \sqrt{9+9}$

$r = \sqrt{18} = \sqrt{9 \times 2} = \boxed{3\sqrt{2}}$

25. **What type of circle is represented by $x^2 + y^2 + 2x - 4y + 8 = 0$**

Sol. $x^2 + y^2 + 2x - 4y + 8 = 0$

Comparing this equation with general form of equation of circle:

$x^2 + y^2 + 2gx + 2fy + c = 0$

$2g = 2$	$2f = -4$	$c = 8$
$g = \dfrac{2}{2} = 1$	$f = -\dfrac{4}{2} = -2$	

Radius $= r = \sqrt{g^2 + f^2 - c}$

$r = \sqrt{(-1)^2 + (2)^2 - 8}$

$r = \sqrt{1 + 4 - 8} = \sqrt{-3} = \boxed{\sqrt{3}i}$

So, it is an **Imaginary circle**.

26. **Find the equation of a circle with center at $(3, 0)$ and tangent to y-axis.**

Sol. Here : Centre $= (h, k) = (3, 0)$

& Radius = r = 3

Standard form of eq. of circle:

$(x-h)^2+(y-k)^2=r^2$

Put $h=3,\ k=0\ \&\ r=3$

$(x-3)^2+(y-0)^2=(3)^2$

$(x)^2-2(x)(3)+(3)^2+(y)^2=9$

$x^2-6x+\cancel{9}+y^2-\cancel{9}=0$

$\boxed{x^2+y^2-6x=0}$

27. **Find the equation of the circle having $(-3,\ 7)$ and $(2,\ -1)$ as the end points of its diameter.**

Sol. Let, $A(-3,\ 7)\ \&\ B(2,\ -1)$

Center = Midpoint of So, $r=\frac{d}{2}=\boxed{\frac{\sqrt{89}}{2}}$

Standard form of equation of circle:

$(x-h)^2+(y-k)^2=r^2$

$A(-3,\ 7)\ \&\ B(2,\ -1)$

Centre $=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$

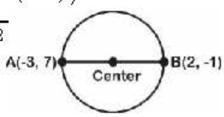
Centre $=\left(\frac{-3+2}{2},\frac{7-1}{2}\right)=\boxed{\left(-\frac{1}{2},3\right)}$

Diameter = Distance between $A(-3,\ 7)\ \&\ B(2,\ -1)$

$d=\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$

$d=\sqrt{(-3-2)^2+(7-(-1))^2}$

$d=\sqrt{(-5)^2+(7+1)^2}$



$d=\sqrt{25+64}=\sqrt{89}$

Put: $h=-\frac{1}{2},\ k=3\ \&\ r=\frac{\sqrt{89}}{2}$

$\left(x+\frac{1}{2}\right)^2+(y-3)^2=\left(\frac{\sqrt{89}}{2}\right)^2$

$(x)^2+2(x)\left(\frac{1}{2}\right)+\left(\frac{1}{2}\right)^2+(y)^2-2(y)(3)+(3)^2=\frac{89}{4}$

$x^2+x+\frac{1}{4}+y^2-6y+9-\frac{89}{4}=0$

$x^2+y^2+x-6y+\frac{1}{4}+9-\frac{89}{4}=0$

$x^2+y^2+x-6y+\frac{1+36-89}{4}=0$

$x^2+y^2+x-6y-\frac{52}{4}=0$

$\boxed{x^2+y^2+x-6y-13=0}$

Section - II

Note: **Attemp any three (3) questions $3\times8=24$**

Q.2. **Find the quotient of $1+\sqrt{3}i$ and $1+i$.**

Sol. See example Q.20 of chapter 08

Q.3. **Resolve into partial fractions**

$$\frac{3x+7}{(x^2+x+1)(x^2-4)}$$

Sol. See Q.6 of Ex # 9.3 (Page # 374)

Q.4. **Minimize the expression by use of Boolean Rules**

$$AB+\overline{A}C+BC=AB+\overline{A}C$$

Sol. See example Q.5[c] of chapter 11

Q.5. **Find the equation of two lines parallel to the line $x-6y+8=0$ & a distance of $\frac{18}{\sqrt{37}}$ units from it.**

Sol. See Q.6 of Ex # 12.6 (Page # 504)

Q.6. **Find the equation of the circle through $(2,\ -1)$ and $(-2,\ 0)$ with centre on $2x-y-1=0$.**

Sol. See Q.3[d] of Ex # 13 (Page # 528)

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