DAE / IIA - 2019

MATH - 123 APPLIED MATHEMATICS - I

PAPER 'A' PART - A (OBJECTIVE)

Time : 30 Minutes   Marks : 15

Q.1: *Encircle the correct answer.*

1. The second degree equation is known as:
[a] Linear   [b] Quadratic
[c] Cubic   [d] Reciprocal

2. If the discriminant $b^2 - 4ac$ is negative, then roots are:
[a] Real   [b] Rational
[c] Irrational   [d] Imaginary

3. ~~The sum of the roots of~~ $2x^2 - 3x - 5 = 0$ ~~is:~~
[a] $-\frac{3}{2}$ [b] $\frac{3}{2}$
[c] $\frac{2}{3}$ [d] $-\frac{2}{3}$

4. Value of $\binom{6}{4}$ is:
[a] 10 [b] 15 [c] 20 [d] 25

5. In the expansion of $(a+b)^n$, then $\binom{n}{r}a^{n-r}b^r$ will be:
[a] nth term   [b] rth term
[c] $(r+1)$th term
[d] $(r-1)$th term

6. The second last term in the expansion of $(a+b)^7$ is:
[a] $7a^6b$ [b] $7ab^6$
[c] $7b^7$ [d] b^7

7. ~~One degree is equal to:~~
[a] πrad   [b] $\frac{\pi}{180}$rad
[c] $\frac{180}{\pi}$rad   [d] $\frac{\pi}{360}$rad

8. ~~If~~ $\ell = 12$cm ~~and r = 3cm, then θ is equal to:~~
[a] 36rad   [b] 4rad
[c] $\frac{1}{4}$rad   [d] 18rad

9. π rad ~~is equal to:~~
[a] 360º   [b] 270º
[c] 180º   [d] 90º

10. $\cos\left(\frac{\pi}{2}+\theta\right)$ is equal to:
[a] $\cos\theta$ [b] $-\cos\theta$
[c] $\sin\theta$ [d] $-\sin\theta$

11. $\sin 2\alpha$ is equal to:
[a] $\cos^2\alpha - \sin^2\alpha$
[b] $\cos 2\alpha$
[c] $1-\cos^2\alpha$ [d] $2\sin\alpha\cos\alpha$

12. If in a triangle ABC, B = 2, C = 2, A = 60º then side 'a' is:
[a] 2 [b] 3 [c] 4 [d] 5

13. The magnitude of a vector $\underline{i} - 3\underline{j} + 5\underline{k}$ is:
[a] 3   [b] 25
[c] 35   [d] $\sqrt{35}$

14. $\vec{a}\cdot\vec{a}$ is equal to:
[a] 1   [b] a^2
[c] $|\overline{a}|$ [d] 0

15. If $\vec{a} = 2\underline{i} - 3\underline{j} + \underline{k}$ and $\vec{b} = -\underline{i} + 2\underline{j} + 7\underline{k}$ then $\vec{a}\cdot\vec{b}$ is equal to:
[a] -1   [b] -2
[c] -3   [d] -4

Answer Key

1	b	2	d	3	b	4	b	5	c
6	d	7	b	8	b	9	c	10	d
11	d	12	a	13	d	14	b	15	a

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DAE / IIA - 2019

MATH - 123 APPLIED MATHEMATICS - I

PAPER 'A' PART - B (SUBJECTIVE)

Time: 2:30 Hrs Marks: 60

Section - I

Q.1. Write short answers to any Eighteen (18) questions.

1. Solve the equation $x^2 - x = 2$

Sol. $x^2 - x = 2$

$x^2 - x - 2 = 0$

$x^2 - 2x + x - 2 = 0$

$x(x-2) + 1(x-2) = 0$

$(x-2)(x+1) = 0$

Either $x - 2 = 0$, $x = 2$ OR $x + 1 = 0$, $x = -1$

$S.S. = \{-1, 2\}$

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2. Solve the equation $(2x+3)(x+1) = 1$ by factorization.

Sol. $(2x+3)(x+1) = 1$

$2x^2 + 2x + 3x + 3 - 1 = 0$

$2x^2 + 5x + 2 = 0$

$2x^2 + 4x + x + 2 = 0$

$2x(x+2) + 1(x+2) = 0$

$(x+2)(2x+1) = 0$

Either $x + 2 = 0$, $x = -2$ OR $2x + 1 = 0$, $2x = -1 \Rightarrow x = -\frac{1}{2}$

$S.S. = \left\{-\frac{1}{2}, -2\right\}$

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3. For what value of 'k' the roots of the given equation $2x^2 + 5x + k = 0$ are equal.

Sol. Here: $a = 2$, $b = 5$, $c = k$

As, Roots are equal. So, Disc. $= 0$

$\Rightarrow b^2 - 4ac = 0$

$\Rightarrow (5)^2 - 4(2)(k) = 0$

$\Rightarrow 25 - 8k = 0$

$\Rightarrow -8k = -25$

$\Rightarrow k = \frac{-25}{-8} \Rightarrow \boxed{k = \frac{25}{8}}$

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4. ~~Form the quadratic equation whose roots are~~ $i\sqrt{3}$ & $-i\sqrt{3}$

Sol.

$S = i\sqrt{3} + (-i\sqrt{3})$

$S = i\sqrt{3} - i\sqrt{3}$

$S = 0$

$P = (i\sqrt{3})(-i\sqrt{3})$

$P = -(i)^2(\sqrt{3})^2$

$P = -(-1)(3) = 3$

$x^2 - Sx + P = 0$

$x^2 - 0x + 3 = 0 \Rightarrow \boxed{x^2 + 3 = 0}$

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5. ~~Find the sum and the product of the roots of equation~~ $5x^2 + x - 7 = 0$

Sol. Here: $a = 5$, $b = 1$, $c = -7$

Sum of the roots $= -\frac{b}{a} = -\frac{1}{5}$,

Product of the roots $= \frac{c}{a} = \frac{-7}{5}$

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6. Expand $(x+y)^4$ by Binomial Theorem.

Sol. $(x+y)^4$

$= \binom{4}{0}(x)^4(y)^0 + \binom{4}{1}(x)^3(y)^1 + \binom{4}{2}(x)^2(y)^2 + \binom{4}{3}(x)^1(y)^3 + \binom{4}{4}(x)^0(y)^4$

$= (1)(x^4)(1) + (4)(x^3)(y) + (6)(x^2)(y^2) + (4)(x)(y^3) + (1)(1)(y^4)$

$= \boxed{x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4}$

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7. **Find the 7^{th} term in the expansion of $\left(x-\frac{1}{x}\right)^9$**

Sol. Here: $a=x$, $b=-\frac{1}{x}$, $n=9$ & $r=6$

Using general term formula:

$$T_{r+1}=\binom{n}{r}a^{n-r}b^r$$

$$T_{6+1}=\binom{9}{6}(x)^{9-6}\left(-\frac{1}{x}\right)^6$$

$$T_7=84x^3\left(\frac{1}{x^6}\right)=\frac{84}{x^3}$$

8. ~~**Expand $\frac{1}{\sqrt{1+x}}$ upto three terms.**~~

Sol. $\frac{1}{\sqrt{1+x}}=(1+x)^{-1/2}$

$$=1+\left(-\frac{1}{2}\right)(x)+\frac{\left(-\frac{1}{2}\right)\left(-\frac{1}{2}-1\right)}{2!}(x)^2+\dots$$

$$=1-\frac{x}{2}+\frac{1}{2}\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)x^2+\dots$$

$$=\boxed{1-\frac{x}{2}+\frac{3}{8}x^2+\dots}$$

9. **Calculate $(1.04)^5$ by binomial theorem up to two decimal places.**

Sol. $(1.04)^5=(1+0.04)^5$

$$=\binom{5}{0}(1)^5(0.04)^0+\binom{5}{1}(1)^4(0.04)^1+\binom{5}{2}(1)^3(0.04)^2+\dots$$

$$=(1)(1)(1)+5(1)(0.04)+10(1)(0.0016)+\dots$$

$$=1+0.2+0.016+\dots=1.2160=\boxed{1.22}$$

10. **Write the general term in binomial expansion $(a+b)^n$.**

Sol. General Term $=T_{r+1}=\binom{n}{r}a^{n-r}b^r$

11. ~~**Find the missing element ℓ, r, θ when**~~ **$r=620m$, $\theta=32°$**

Sol. We know that: $\ell=r\theta$

$$\ell=r\theta=(620)(0.56)=\boxed{346.27\,m}$$

12. **Prove that**

$4\tan 60°\tan 30°\tan 45°\sin 30°\cos 60°=1$

Sol.

$$\text{L.H.S.}=4\tan 60°\tan 30°\tan 45°\sin 30°\cos 60°$$

$$=4(\sqrt{3})\left(\frac{1}{\sqrt{3}}\right)(1)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)=1=\text{R.H.S.}$$ **Proved**

13. **Prove that:**

$(1+\sin\theta)(1-\sin\theta)=\frac{1}{\sec^2\theta}$

Sol. $\text{L.H.S.}=(1+\sin\theta)(1-\sin\theta)$

$$=(1)^2-(\sin\theta)^2$$

$$=1-\sin^2\theta$$

$$=\cos^2\theta$$

$$=\frac{1}{\sec^2\theta}=\text{R.H.S.}$$ **Proved.**

14. **Prove that:**

$\cos 2\alpha=\cos^2\alpha-\sin^2\alpha$

Sol. $\text{L.H.S.}=\cos 2\alpha=\cos(\alpha+\alpha)$

$$=\cos\alpha\cos\alpha-\sin\alpha\sin\alpha$$

$$=\cos^2\alpha-\sin^2\alpha=\text{R.H.S.}$$ **Proved.**

15. **Find the value of $\sin 15°$, without using calculator.**

Sol. $\sin 15°$

$= \sin(45° - 30°)$

$= \sin 45° \cos 30° - \cos 45° \sin 30°$

$= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right)$

$= \boxed{\frac{\sqrt{3}-1}{2\sqrt{2}}}$

16. **Prove that:**

$$(\sin\theta - \cos\theta)^2 = 1 - \sin 2\theta$$

Sol. $\text{L.H.S.} = (\sin\theta - \cos\theta)^2$

$= \sin^2\theta + \cos^2\theta - 2\sin\theta\cos\theta$

$= 1 - \sin 2\theta = \text{R.H.S.}$ **Proved.**

17. ~~**Express**~~ $2\sin 3\theta \cos\theta$ ~~**as sum or difference.**~~

Sol. $2\sin 3\theta \cos\theta$

$= \sin(3\theta + \theta) + \sin(3\theta - \theta)$

$= \boxed{\sin 4\theta + \sin 2\theta}$

18. **Prove that:**

$$\tan(45° - \theta) = \frac{1 - \tan\theta}{1 + \tan\theta}$$

Sol. $\text{L.H.S.} = (45° - \theta)$

$= \frac{\tan 45 - \tan\theta}{1 + \tan 45 . \tan\theta}$

$= \frac{1 - \tan\theta}{1 + (1).\tan\theta}$

$= \frac{1 - \tan\theta}{1 + \tan\theta} = \text{R.H.S}$ **Proved.**

19. **In any triangle ABC, if a = 5, c = 6, a = 45º, find γ.**

Sol. By using law of sines:

$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$

Here we take : $\frac{a}{\sin\alpha} = \frac{c}{\sin\gamma}$

$a\sin\gamma = c\sin\alpha$

$5\sin\gamma = 6\sin 45°$

$\sin\gamma = \frac{6\sin 45°}{5}$

$\sin\gamma = 0.8485$

$\gamma = \sin^{-1}(0.8485)$

$\boxed{\gamma = 58° 3'}$

20. **Write the law of Sines.**

Sol. In any triangle ABC, with usual notations.

$$\boxed{\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}}$$

21. **In any triangle ABC in which b = 45, c = 34, ∝ = 52º, find a.**

Sol. By using Law of cosines:

$a^2 = b^2 + c^2 - 2bc\cos\alpha$

$a^2 = (45)^2 + (34)^2 - 2(45)(34)\cos 52°$

$a^2 = 2025 + 1156 - 1883.92$

$a^2 = 1297.07$

$\sqrt{a^2} = \sqrt{1297.07}$

$\boxed{a = 36.01}$

22. ~~**How far is a man from the foot of tower 150 meters high, if the measure of angle of elevation of its top as observed by him is 40º30′.**~~

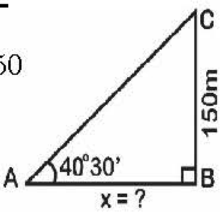
Sol. Let the distance of man from the foot of the tower $= x = ?$

$$\tan 40^\circ 30' = \frac{150}{x}$$

$$x \tan 40^\circ 30' = 150$$

$$x = \frac{150}{\tan 40^\circ 30'}$$

$$x = \boxed{175.63\text{m}}$$



23. **If vectors $3i + j - k$ & $\lambda i - 4j + 4k$ are parallel, find the value of λ.**

Sol. As $\vec{a}$ and $\vec{b}$ are parallel,

so $\vec{a} \times \vec{b} = 0$

$$\Rightarrow \begin{vmatrix} i & j & k \\ 3 & 1 & -1 \\ \lambda & -4 & 4 \end{vmatrix} = 0$$

$$\Rightarrow i\begin{vmatrix} 1 & -1 \\ -4 & 4 \end{vmatrix} - j\begin{vmatrix} 3 & -1 \\ \lambda & 4 \end{vmatrix} + k\begin{vmatrix} 3 & 1 \\ \lambda & -4 \end{vmatrix} = 0$$

$$\Rightarrow i(4-4) - j(12+\lambda) + k(-12-\lambda) = 0$$

$$\Rightarrow i(0) - (12+\lambda)j + (-12-\lambda)k = 0i + 0j + 0k$$

Comparing coeffients of j both sides, we get:

$$12 + \lambda = 0$$

$$\boxed{\lambda = -12}$$

24. **Find $\vec{a} \times \vec{b}$, if**

$$\vec{a} = 2i + 3j + 4k \ \ \& \ \ \vec{b} = i - j + k$$

Sol.
$$\vec{a} \times \vec{b} = \begin{vmatrix} i & j & k \\ 2 & 3 & 4 \\ 1 & -1 & 1 \end{vmatrix}$$

$$= i\begin{vmatrix} 3 & 4 \\ -1 & 1 \end{vmatrix} - j\begin{vmatrix} 2 & 4 \\ 1 & 1 \end{vmatrix} + k\begin{vmatrix} 2 & 3 \\ 1 & -1 \end{vmatrix}$$

$$= i(3+4) - j(2-4) + k(-2-3)$$

$$= i(7) - j(-2) + k(-5)$$

$$= \boxed{7i + 2j - 5k}$$

25. **Find 'x' so that $\vec{a}$ and $\vec{b}$ are perpendicular. $\vec{a} = 2i + 4j - 7k$ and $\vec{b} = 2i + 6j + xk$**

Sol. As $\vec{a}$ & $\vec{b}$ are perpendicular, so

$$\vec{a} \cdot \vec{b} = 0$$

$$(2i + 4j - 7k) \cdot (2i + 6j + xk) = 0$$

$$(2)(2) + (4)(6) + (-7)(x) = 0$$

$$4 + 24 - 7x = 0$$

$$28 - 7x = 0$$

$$-7x = -28$$

$$x = \frac{-28}{-7}$$

$$\boxed{x = 4}$$

26. **Find the unit vector along the vector $4i - 3j - 5k$**

Sol. Let $\vec{a} = 4i - 3j - 5k$

$$|\vec{a}| = \sqrt{(4)^2 + (-3)^2 + (-5)^2}$$

$$|\vec{a}| = \sqrt{16 + 9 + 25}$$

$$|\vec{a}| = \sqrt{50}$$

$$|\vec{a}| = \sqrt{25 \times 2}$$

$$|\vec{a}| = 5\sqrt{2}$$

$$\text{Unit Vector} = \hat{a} = \frac{\vec{a}}{|\vec{a}|}$$

$$\hat{a} = \boxed{\frac{4i - 3j - 5k}{5\sqrt{2}}}$$

27. Find the magnitude and direction cosines of the vector $3i+7j-4k$

Sol. Let, $\vec{a}=3i+7j-4k$

Magnitude of $\vec{a}=|\vec{a}|$

$=\sqrt{(3)^2+(7)^2+(-4)^2}$

$=\sqrt{9+49+16}$

$=\sqrt{74}$

Direction Cosines of $\vec{a}$:

$\frac{3}{\sqrt{74}},\frac{7}{\sqrt{74}},\frac{-4}{\sqrt{74}}$

Section - II

Note: Attemp any three (3) questions $3\times 8=24$

Q.2.(a) Solve the equation.

$$\frac{a}{ax-1}+\frac{b}{bx-1}=a+b$$

by factorization.

Sol. See Q.1(ix) of Ex # 1.1 (Page # 10)

(b) For what value of K the roots of given equation are equal

$x^2+3(k+1)x+4k+5=0$

Sol. See Q.2(i) of Ex # 1.2 (Page # 30)

Q.3. Find the term involving x^5 in the expansion of $\left(2x^2-\frac{3}{x}\right)^{10}$

Sol. See Q.7(i) of Ex # 2.1 (Page # 76)

Q.4.(a) If $\sin\theta=\frac{2}{3}$ and the terminal side of the angle lies in the second quadrant, find the remaining trigonometric ratios of θ.

Sol. See Q.3 of Ex # 3.2 (Page # 122)

(b) Prove that:

$$(\operatorname{cosec}\theta-\cot\theta)^2=\frac{1-\cos\theta}{1+\cos\theta}$$

Sol. See Q.6 of Ex # 3.3 (Page # 133)

Q.5.(a) If $\sin\alpha=\frac{4}{5}$ and $\sin\beta=\frac{12}{13}$, both α and β are in the 1st quadrant find $\sin(\alpha-\beta)$.

Sol. See Q.7(i) of Ex # 4.1 (Page # 161)

(b) Express $\sin3\theta+\sin5\theta+\sin7\theta+\sin9\theta$ as a product

Sol. See Q.3 of Ex # 4.3 (Page # 185)

Q.6.(a) If $\vec{a}=3i-j-4k$, $\vec{b}=-2i+4j-3k$ & $\vec{c}=i+2j-k$ Find unit vector parallel to $3\vec{a}-2\vec{b}+4\vec{c}$

Sol. See Q.1 of Ex # 6.1 (Page # 247)

(b) Find the cosine of the angle between the vectors $\bar{a}=4\underline{i}+2\underline{j}\_\underline{k},\ \bar{b}=2\underline{i}+4\underline{j}-\underline{k}$

Sol. See Q.3(iii) of Ex # 6.2 (Page # 256)

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