DAE / IIA - 2019

MATH - 113 APPLIED MATHEMATICS - I

PAPER 'B' PART - A (OBJECTIVE)

Time : 30 Minutes Marks : 15

Q.1: *Encircle the correct answer.*

1. **Magnitude of the vector $2\mathbf{i}-2\mathbf{j}-\mathbf{k}$ is:**

[a] 4 [b] 3 [c] 2 [d] 1

2. **Unit vector $\mathbf{i}+\mathbf{j}+\mathbf{k}$ is;**

[a] $i+j+k$ **[b]** $\frac{1}{3}(i+j+k)$

[c] $\frac{1}{\sqrt{3}}(i+j+k)$ **[d]** $\frac{1}{2}(i+j+k)$

3. $\vec{a}.\vec{b}$ **is a:**

[a] Vector quantity

[b] Scalar quantity

[c] Unity **[d]** None of these

4. **The order of the matrix [1 2 3] is:**

[a] 1×3 **[b]** 3×1

[c] 3×3 **[d]** 2×3

5. ~~**If all the elements of a row or a column are zero then value of the determinant is:**~~

[a] 1 [b] 2

[c] zero [d] None of these

6. **If A and B are symmetric, then $(\mathbf{AB})^t = ?$**

[a] BA **[b]** A^tB^t

[c] B^tA^t **[d]** Both a and c

7. **A right triangle has one angle is:**

[a] 30º [b] 90º

[c] 60º [d] 45º

8. **If a = 2 cm, b = 3cm, c = 5cm sides of triangle, the perimeter of triangle is:**

[a] 8cm [b] 6cm

[c] 18cm [d] 10cm

9. **The diagonals of rhombus are 6cm and 5cm then area is:**

[a] 15 sq cm [b] 30 sq cm

[c] 10 sq cm [d] 11 sq.cm

10. **Decagon have number of sides:**

[a] 8 [b] 9

[c] 10 [d] 12

11. **The area of circle is 16π then radius 'r' is:**

[a] 4 [b] 2

[c] 16 [d] 8

12. **Is Simpson's rule the number of ordinates are:**

[a] Odd [b] Even

[c] In fraction [d] None of these

13. **Volume of the cube with side 2m is:**

[a] 4 [b] 8

[c] 16 [d] 2

14. **Volume of right circular cylinder of height 10cm and diameter is 4cm is:**

[a] 40π [b] 160π

[c] 80π [d] 4π

15. **Volume of a cone of height 'h' and base radius 'r' is:**

[a] $\frac{1}{3}\pi r^2 h$ **[b]** $\frac{1}{3}\pi rh$

[c] $\pi r^3 h$ **[d]** $\frac{1}{2}\pi r^2 h$

Answer Key

1	b	2	b	3	b	4	a	5	c
6	c	7	b	8	d	9	a	10	c
11	a	12	a	13	b	14	a	15	a

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DAE / IIA - 2019

MATH - 113 APPLIED MATHEMATICS - I

PAPER 'B' PART - B (SUBJECTIVE)

Time: 2:30Hrs Marks: 60

Section - I

Q.1. Write short answers to any Eighteen (18) questions.

1. Find unit vector along the vector $4i - 3j - 5k$

Sol. Let $\vec{a} = 4i - 3j - 5k$

$$|\vec{a}| = \sqrt{(4)^2 + (-3)^2 + (-5)^2}$$

$$|\vec{a}| = \sqrt{16 + 9 + 25} = \sqrt{50}$$

$$|\vec{a}| = \sqrt{25 \times 2} = 5\sqrt{2}$$

$$\text{Unit Vector} = \hat{a} = \frac{\vec{a}}{|\vec{a}|} = \boxed{\frac{4i - 3j - 5k}{5\sqrt{2}}}$$

18. Find 'α' so that $\left|\alpha i + (\alpha+1)j + 2k\right| = 3$

Sol. $\left|\alpha\hat{i} + (\alpha+1)\hat{j} + 2\hat{k}\right| = 3$

$$\sqrt{(\alpha)^2 + (\alpha+1)^2 + (2)^2} = 3$$

$$\sqrt{\alpha^2 + \alpha^2 + 2\alpha + 1 + 4} = 3$$

$$\sqrt{2\alpha^2 + 2\alpha + 5} = 3$$

Squaring both sides, we get :

$$2\alpha^2 + 2\alpha + 5 = 9$$

$$2\alpha^2 + 2\alpha + 5 - 9 = 0$$

$$2\alpha^2 + 2\alpha - 4 = 0$$

$$2\alpha^2 + 4\alpha - 2\alpha - 4 = 0 \ \{\text{By Factorization}\}$$

$$2\alpha(\alpha + 2) - 2(\alpha + 2) = 0$$

$$(\alpha + 2)(2\alpha - 2) = 0$$

Either	OR
$\alpha + 2 = 0$	$2\alpha - 2 = 0$
$\boxed{\alpha = -2}$	$2\alpha = 2 \Rightarrow \boxed{\alpha = 1}$

3. Find a vector whose magnitude is 2, and is parallel to $5\hat{i} + 3\hat{j} + 2\hat{k}$

Sol. Let $\vec{a}$ be a require vector, so

$|\vec{a}| = 2$ & Let, $\vec{b} = 5\hat{i} + 3\hat{j} + 2\hat{k}$

$$|\vec{b}| = \sqrt{(5)^2 + (3)^2 + (2)^2}$$

$$|\vec{b}| = \sqrt{25 + 9 + 4} = \sqrt{38}$$

As $\vec{a}$ and $\vec{b}$ are parallel vectors:

So $\hat{a} = \hat{b}$

$$\frac{\vec{a}}{|\vec{a}|} = \frac{\vec{b}}{|\vec{b}|}$$

$$\frac{\vec{a}}{2} = \frac{5i + 3j + 2k}{\sqrt{38}} = \frac{2(5i + 3j + 2k)}{\sqrt{38}}$$

$$\vec{a} = \boxed{\frac{10i + 6j + 4k}{\sqrt{38}}}$$

4. For what value of λ, the vectors $2i - j + 2k$ & $3i + 2\lambda j$ are perpendicular.

Sol. Let, $\vec{a} = 2i - j + 2k$ & $\vec{b} = 3i + 2\lambda j$

As given vectors are perpendicular.

So, $\vec{a}.\vec{b} = 0$

$$\Rightarrow (2i - j + 2k).(3i + 2\lambda j) = 0$$

$$\Rightarrow (2)(3) + (-1)(2\lambda) + (2)(0) = 0$$

$$\Rightarrow 6 - 2\lambda + 0 = 0$$

$$\Rightarrow -2\lambda = -6$$

$$\Rightarrow \lambda = \frac{-6}{-2} \Rightarrow \boxed{\lambda = 3}$$

5. Find scalar x and y such that $x(i + 2j) + y(3i + 4j) = 7i + 9j$

Sol. $x(i + 2j) + y(3i + 4j) = 7i + 9j$

$$xi + 2xj + 3yi + 4yj = 7i + 9j$$

$$(x + 3y)i + (2x + 4y)j = 7i + 9j$$

Comparing coefficients of i & j, we have :

$x + 3y = 7 \rightarrow (i)$ | $2x + 4y = 9 \rightarrow (ii)$

Multiplying eq. (i) by 2:

$2x+6y=14 \rightarrow (iii)$

Subtracting eq. (iii) & eq. (ii)

$$\begin{array}{r} 2x+6y=14 \\ -2x \pm 4y=-9 \\ \hline 2y=5 \end{array} \Rightarrow \boxed{y=\tfrac{5}{2}}$$

Put $y=\frac{5}{2}$ in eq. (i)

$$x+3\left(\frac{5}{2}\right)=7$$

$$x=7-\frac{15}{2}=\frac{14-15}{2} \Rightarrow x=\boxed{-\frac{1}{2}}$$

6. ~~**Without expansion, show that :**~~

$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix}=0$$

Sol. $\text{L.H.S.}=\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix}$

$$=\begin{vmatrix} 1+3 & 2 & 3 \\ 4+6 & 5 & 6 \\ 7+9 & 8 & 9 \end{vmatrix} \quad \text{By } C_1+C_3$$

$$=\begin{vmatrix} 4 & 2 & 3 \\ 10 & 5 & 6 \\ 16 & 8 & 9 \end{vmatrix}$$

$$=2\begin{vmatrix} 2 & 2 & 3 \\ 5 & 5 & 6 \\ 8 & 8 & 9 \end{vmatrix} \quad \text{Taking 2 common from } C_1$$

$=2(0) \qquad \because C_1=C_2$

$=0=\text{R.H.S.}$ **Proved.**

7. **Find the inverse of** $\begin{bmatrix} \mathbf{2} & \mathbf{1} \\ \mathbf{6} & \mathbf{3} \end{bmatrix}$

Sol. Let $A=\begin{bmatrix} 2 & 1 \\ 6 & 3 \end{bmatrix}$

$$|A|=\begin{vmatrix} 2 & 1 \\ 6 & 3 \end{vmatrix}=(2)(3)-(6)(1)$$

$$|A|=6-6=0$$

As, $|A|=0$ so inverse of A does not exits.

8. **Solve by using Cramer's Rule**
$\mathbf{x-y=2}$, $\mathbf{x+4y=5}$

Sol. First write in the form of matrices:

$$A=\begin{bmatrix} 1 & -1 \\ 1 & 4 \end{bmatrix}, X=\begin{bmatrix} x \\ y \end{bmatrix}, B=\begin{bmatrix} 2 \\ 5 \end{bmatrix}$$

$$|A|=\begin{vmatrix} 1 & -1 \\ 1 & 4 \end{vmatrix}=4-(-1)=4+1=5$$

$$|A_x|=\begin{vmatrix} 2 & -1 \\ 5 & 4 \end{vmatrix}=8-(-5)=8+5=13$$

$$|A_y|=\begin{vmatrix} 1 & 2 \\ 1 & 5 \end{vmatrix}=5-2=3$$

$$x=\frac{|A_x|}{|A|}=\frac{13}{5} \quad \& \quad y=\frac{|A_y|}{|A|}=\frac{3}{5}$$

$$\text{S.S.}=\boxed{\left\{\left(\frac{13}{5},\frac{3}{5}\right)\right\}}$$

9. **Find 'k' if** $\begin{vmatrix} \mathbf{k-2} & \mathbf{1} \\ \mathbf{5} & \mathbf{k+2} \end{vmatrix}=\mathbf{0}$

Sol. $\begin{vmatrix} k-2 & 1 \\ 5 & k+2 \end{vmatrix}=0$

$$(k-2)(k+2)-(1)(5)=0$$

$$k^2+\cancel{2k}-\cancel{2k}-4-5=0$$

$$k^2-9=0$$

$$k^2=9 \Rightarrow \sqrt{k^2}=\pm\sqrt{9} \Rightarrow \boxed{k=\pm 3}$$

10. **Define a co-factor of an element of a matrix.**

Sol. The number $C_{ij}=(-1)^{i+j}M_{ij}$ Where M_{ij} is the minor of element a_{ij} is called the cofactor of element a_{ij}.

11. **Find the area of a triangle whose base is 12cm and hypotenuse is 20cm.**

Sol. By Pythagoruse Theorem:

$$(\text{Base})^2 + (\text{Perp})^2 = (\text{Hyp})^2$$

$$(12)^2 + (h)^2 = (20)^2$$

$$h^2 = 400 - 144$$

$$h^2 = 256 \Rightarrow h = 16\text{cm}$$

$$\text{Area of triangle} = \frac{1}{2}(\text{Base} \times \text{Height})$$

$$\text{Area} = \frac{1}{2}(12 \times 16) = \boxed{96 \text{ sq.cm}}$$

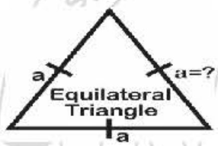
12. What is the side of the equilateral triangle whose area is $9\sqrt{3}$ sq.cm.

Sol. Let 'a' be length of each side of an equilateral triangle.

As, Area of equilateral triangle $= 9\sqrt{3}$ sq.cm

$$\Rightarrow \frac{\sqrt{3}}{4}a^2 = 9\sqrt{3}$$

$$\Rightarrow a^2 = 9\sqrt{3}\left(\frac{4}{\sqrt{3}}\right)$$



$$\Rightarrow a^2 = 36 \Rightarrow \sqrt{a^2} = \sqrt{36} \Rightarrow \boxed{a = 6\text{ cm}}$$

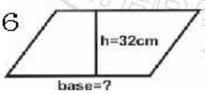
13. Find the base of a parallelogram whose area is 256sq.cm and height 32cm.

Sol. Here: base = ? & height = 32cm

Area of parallelogram = 256

Base × Height = 256

Base × 32 = 256



$$\text{Base} = \frac{256}{32} \Rightarrow \boxed{\text{Base} = 8\text{cm}}$$

14. Find the area of trapezoid whose parallel sides are 20cm and 30cm and perpendicular distance between them is 4cm.

Sol. Area of Trapezoid

$$= \frac{\text{Sum of parallel sides}}{2} \times \text{height}$$

$$= \frac{20+30}{2} \times 4 = \boxed{100\text{sq.cm}}$$

15. The perimeter of a regular hexagon is 12cm, find its area.

Sol. Perimeter of hexagon = 12 cm

$$6a = 12 \Rightarrow a = 12/6 = 2\text{cm}$$

$$\text{Area} = \frac{na^2}{4}\cot\left(\frac{180°}{n}\right)$$

$$A = \frac{6(2)^2}{4}\cot\left(\frac{180°}{6}\right) = 6\cot 30°$$

$$A = \frac{6}{\tan 30°} = \boxed{10.39\text{sq.cm}}$$

16. What are concentric circles.

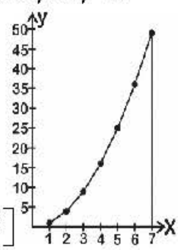
Sol. The circles with common center are called concentric circles.

17. Find the area of the curve $y = x^2$ between the values x = 1 and x = 7?

Sol. For values of x = 1, 2, 3, . . . , 7, the corresponding values of y are in the given table.

x	1	2	3	4	5	6	7
y	1	4	9	16	25	36	49

Ordinates are 1, 4, 9, 16, 25, 36, 49



$S = 1$

$A = 1 + 49 = 50$

$D = 9 + 25 = 34$

$E = 4 + 16 + 36 = 56$

By Simpson's Rule

$$\text{Area} = \frac{S}{3}[A + 2D + 4E]$$

$$A = \frac{1}{3}[50 + 2(34) + 4(56)]$$

$$A = \frac{1}{3}[50 + 68 + 224]$$

$$A = \frac{1}{3}[342]$$

$$A = \boxed{114 \text{ sq.unit}}$$

18. Find surface area of cube of volume 64cm^3.

Sol. Let 'a' be length of one side of cube

As, Volume of cube $= 64\text{cm}^3$

$a^3 = 64$

$(a^3)^{1/3} = (64)^{1/3} \Rightarrow \boxed{a = 4\text{cm}}$

Surface area of cube $= 6a^2$

$S.A. = 6(4)^2 = \boxed{96\,\text{cm}^2}$

19. The curve surface of a cylinder is 100sq.m and diameter of the base is 20m. Find height and volume of cylinder.

Sol. Here: $h = ?\ V = ?\ L.S.A. = 100\text{m}^2\ \&\ d = 20\text{m}$

As, $r = \dfrac{d}{2} = \dfrac{20}{2} = 10\text{m}$

As, L.S.A. $= 100\text{m}^2$

$\Rightarrow \quad 2\pi rh = 100$

$\Rightarrow \quad 2\pi(10)h = 100$

$h = \dfrac{100}{20\pi} \Rightarrow \boxed{h = 15.9\,\text{m}}$

Volume of Cylinder $= \pi r^2 h$

$V = \pi(10)^2(15.9) = \boxed{4995.13\text{m}^3}$

20. Find the height of the cylinder if volume is 528cm³ and diameter is 4cm.

Sol. Here : $h = ?\ V = 528\text{cm}^3\ \&\ d = 4\text{cm}$

As, $r = \dfrac{d}{2} = \dfrac{4}{2} = 2\text{cm}$

As, Volume of Cylinder $= 528\text{cm}^3$

$\Rightarrow \quad \pi r^2 h = 528$

$h = \dfrac{528}{\pi(2)^2} \Rightarrow \boxed{h = 42\,\text{cm}}$

21. Define pyramid.

Sol. A pyramid is a solid, whose base is a plane polygon and sides being triangles that meet in a common vertex.

22. Find the volume of a pyramid with a square base of side 10cm and height 15cm.

Sol. Here : $V = ?,\ a = 10\text{cm}\ \&\ h = 15\text{cm}$

Area of base (square) $= a^2$

$= (10)^2 = 100\text{cm}^2$

Volume $= \dfrac{1}{3} \times$ Area of base $\times$ height

$V = \dfrac{1}{3} \times 100 \times 15 = \boxed{500\,\text{cm}^3}$

23. Write the formula for volume of a cone and total surface area of a cone.

Sol. Volume of cone $= V = \boxed{\dfrac{1}{3}\pi r^2 h \text{ cu.unit}}$

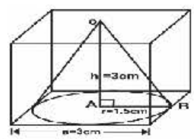
Total surface Area of Cone

$(T.S.A) = \boxed{\pi r\ell + \pi r^2 \text{ sq.unit}}$

24. Find the volume of the largest cone that can be cut out of a cube whose edge is 3cm.

Sol. Let 'a' = edge of the cube = 3cm

Then $h = 3\text{cm}$



$\& \ r = \dfrac{a}{2} = \dfrac{3}{2} = 1.5\text{cm}$

$V = \dfrac{1}{3}\pi r^2 h = \dfrac{1}{3}\pi(1.5)^2(3) = \boxed{7.069\text{cm}^3}$

25. Write formula of curved surface area of cone and slant height of cone.

Sol.

Curved surface area $= \boxed{\pi r\ell \text{ sq.unit}}$

Slant height $= \ell = \boxed{\sqrt{r^2 + h^2} \text{ unit}}$

26. Define spherical shell.

Sol. A spherical shell is the region between two concentric spheres of different radius.

27. How many lead balls, each of radius 1cm can be made from a sphere whose radius is 8cm.

Sol. Let r_1 = Radius of sphere = 8cm

& r_2 = Radius of ball = 1cm

$V_1 = \text{Volume of sphere} = \frac{4}{3}\pi r_1^3$

$V_1 = \frac{4}{3}\pi(8)^3 = 2144.66\,\text{cm}^3$

$V_2 = \text{Volume of Ball} = \frac{4}{3}\pi r_2^3$

$V_2 = \frac{4}{3}\pi(1)^3 = 4.19\,\text{cm}^3$

$\text{No. of balls made} = \frac{V_1}{V_2}$

$= \frac{2144.66}{4.19} = \boxed{512\,\text{balls}}$

Section - II

Note : Attemp any three (3) questions 3×8 = 24

Q.2.(a) If $\vec{a} = 3i - 2j + 5k$ and $\vec{b} = -2i - j + k$, Find $2\vec{a} - 3\vec{b}$ and also its unit vector.

Sol. See example # 04 of Ch# 08

(b) Find the sine of the angle and the unit vector perpendicular to each: $\vec{a} = i + j + k,\ \vec{b} = 2i + 3j - k$

Sol. See Q.19(i) of Ex # 8.2 (Page # 388)

Q.3.(a) If $A = \begin{vmatrix} 2 & -2\sqrt{2} \\ \sqrt{2} & 2 \end{vmatrix}$ and $B = \begin{vmatrix} 2 & 2\sqrt{2} \\ -\sqrt{2} & 2 \end{vmatrix}$ show that A and B commute.

Sol. See Q.9 of Ex # 9.1 (Page # 413)

(b) ~~Show that:~~

$$\begin{vmatrix} \ell & a & a \\ a & \ell & a \\ a & a & \ell \end{vmatrix} = (2a+\ell)(\ell-a)^2$$

Sol. See Q.5(i) of Ex # 9.2 (Page # 415)

Q.4 The sides of a triangular lawn are proportional to the numbers 5,12 and 13. The cost of fencing it at the rate of Rs.2 per meter is Rs. 120. Find the sides, also find the cost of turning the lawn at 25 paisa per square meter.

Sol. See Q.6 of Ex # 10 (Page # 465)

Q.5.(a) Find the area of an irregular plane figure whose ordinates are 20, 23, 28, 32, 34, 37 and 40m respectively and the width of each strip is 7 meter.

Sol. See Q.4 of Ex # 14 (Page # 508)

(b) The diameter of a right circular cylinder is 38cm and its length is 28cm. Find its volume and total surface area.

Sol. See Q.4 of Ex # 14 (Page # 508)

Q.6.(a) Find the volume of a pyramid whose base is an equilateral triangle of side 3m and height 4m.

Sol. See Q.1 of Ex # 17[A] (Page # 545)

(b) A circular disc of lead 3cm in thickness and 12cm diameter is wholly converted into shots of radius 0.5cm. Find the number of shots.

Sol. See Q.4 of Ex # 19[A] (Page # 586)

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