DAE / IIA - 2019

MATH-113 APPLIED MATHEMATICS-I

PAPER 'A' PART-A (OBJECTIVE)

Time: 30 Minutes Marks: 15

Q.1: *Encircle the correct answer.*

1. To make x^2-5x a complete square, we should add:

[a] 25 [b] $\frac{25}{4}$ [c] $\frac{25}{9}$ [d] $\frac{25}{16}$

2. If ± 3 are the roots of the equation, then the equation is:

[a] $x^2-3=0$ [b] $x^2-9=0$
[c] $x^2+3=0$ [d] $x^2+9=0$

3. The 10th term is 7, 17, 27, ... is:

[a] 97 [b] 98
[c] 99 [d] 100

4. The sum of the series $1+2+3+\ldots+100$ is:

[a] 100 [b] 5000
[c] 5050 [d] 500

5. The 6th term of G.P. $1, \sqrt{2}, \sqrt{4}, \ldots$ is:

[a] $4\sqrt{2}$ [b] 4
[c] $\sqrt{2}$ [d] $\sqrt{4}$

6. The 2nd last term in the expansion of $(a+b)^7$ is:

[a] $7a^6b$ [b] $7ab^6$
[c] $7b^7$ [d] b^7

7. In the expansion of $(1+x)^n$ the co-efficient of 3rd terms is:

[a] $\binom{n}{0}$ [b] $\binom{n}{1}$

[c] $\binom{n}{2}$ [d] $\binom{n}{3}$

8. The number of partial fraction of $\frac{6x+27}{4x^3-9x}$ are:

[a] 2 [b] 3
[c] 4 [d] None of these

9. $15°$ is equal to:

[a] $\frac{\pi}{6}\text{rad}$ [b] $\frac{\pi}{3}\text{rad}$

[c] $\frac{\pi}{12}\text{rad}$ [d] $\frac{\pi}{15}\text{rad}$

10. An angle subtended at the center of a circle by an arc equal to the radius of the circle is called:

[a] Right angle [b] Degree
[c] Radian [d] Acute angle

11. If $\sin\theta=\frac{3}{5}$ and the terminal side of angle lies in 2nd quadrant, then $\tan\theta$ is equal to:

[a] $\frac{4}{5}$ [b] $\frac{-4}{5}$

[c] $\frac{5}{4}$ [d] $\frac{-3}{4}$

12. $\cos\left(\frac{\pi}{2}+\theta\right)$ is equal to:

[a] $\cos\theta$ [b] $-\cos\theta$
[c] $\sin\theta$ [d] $-\sin\theta$

13. $\sin(A+B)-\sin(A-B)$ is equal to:

[a] $2\sin A\cos B$ [b] $2\cos A\sin B$
[c] $-2\sin A\sin B$ [d] $2\cos A\cos B$

14. If b = 2, A = 30º, B = 45º, then side a = ________

[a] 2 [b] $\sqrt{2}$

[c] $\frac{\sqrt{3}}{2}$ [d] $\frac{2}{\sqrt{3}}$

15. If in a triangle ABC, a = 3, b = 4, c = 2, then $\cos\gamma=?$

[a] $\frac{1}{2}$ [b] $\frac{3}{4}$ [c] $\frac{7}{8}$ [d] 3

Answer Key

1	b	2	b	3	a	4	c	5	a
6	a	7	c	8	b	9	c	10	c
11	d	12	d	13	d	14	b	15	c

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DAE / IIA - 2019

MATH - 113 APPLIED MATHEMATICS - I

PAPER 'A' PART - B (SUBJECTIVE)

Time: 2:30 Hrs Marks: 60

Section - I

Q.1. Write short answers to any Eighteen (18) questions.

1. **Solve the quadratic equation $\mathbf{x(x+7)=(2x-1)(x+4)}$ by factorization.**

Sol. $x(x+7)=(2x-1)(x+4)$

$x^2+7x=2x^2+8x-x-4$

$x^2+7x-2x^2-8x+x+4=0$

$-x^2+4=0 \Rightarrow x^2-4=0$

$(x)^2-(2)^2=0$

$(x-2)(x+2)=0$

Either $x-2=0$, $x=2$ OR $x+2=0$, $x=-2$

$S.S.=\{-2, 2\}$

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2. **Discuss the nature of roots of equation $\mathbf{x^2+x+1=0}$.**

Sol. Here : $a=1,\ b=1,\ c=1$

$\text{Disc.}=b^2-4ac$

$=(1)^2-4(1)(1)=1-4=-3$

∴ The roots are **Imaginary.**

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3. ~~**Find the value of 'k', if the sum of roots of equation**~~

$\mathbf{(2k-1)x^2+(4k-1)x+k+3=0}$

~~**Is**~~ $\mathbf{5/2}$.

Sol. Here: $a=2k-1,\ b=4-1k,\ c=k+3$

As, Sum of Roots $=\frac{5}{2}$

$$\Rightarrow \frac{-b}{a}=\frac{5}{2} \Rightarrow \frac{-(4k-1)}{2k-1}=\frac{5}{2}$$

$-2(4k-1)=5(2k-1)$

$-8k+2=10k-5$

$-8k-10k=-5-2$

$-18k=-7$

$$k=\frac{-7}{-18} \Rightarrow \boxed{k=\frac{7}{18}}$$

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4. **Define a sequence.**

Sol. A set of numbers arranged in order by some fixed rule is called a sequence. For examples:

(i) $2, 4, 6, \ldots$ **(ii)** $3, 9, 27, \ldots$

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5. **Find the 7th term of an A.P. $\mathbf{1, 4, 7, \ldots}$**

Sol. Here : $a_1=1$ & $d=4-1=3$

$a_7=a_1+6d$

$a_7=1+6(3)$

$a_7=1+18 \Rightarrow \boxed{a_7=19}$

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6. ~~**Find the sum of the series 3 + 11 + 19 + ... to 16 terms.**~~

Sol. Here: $a_1=3,\ d=11-3=8\ \&\ n=16$

$$S_n=\frac{n}{2}[2a+(n-1)d]$$

$$S_{16}=\frac{16}{2}[2(3)+(16-1)(8)]$$

$$S_{16}=8[6+120]=8(126)=\boxed{1008}$$

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7. **Find the A.M between $\mathbf{\sqrt{5}-4}$ and $\mathbf{\sqrt{5}+4}$**

Sol. Let $a=\sqrt{5}-4$ and $b=\sqrt{5}+4$

$$\text{A.M.}=A=\frac{a+b}{2}$$

$$A=\frac{\sqrt{5}-4+\sqrt{5}+4}{2}=\frac{2\sqrt{5}}{2}=\boxed{\sqrt{5}}$$

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8. ~~**Sum to 5 term the series**~~ $\mathbf{1+3+9+\ \ldots}$

Sol. Here: $a=1,\ r=\frac{3}{1}=3\ \&\ n=5$

$$S_n = \frac{a\left(r^n - 1\right)}{r-1} = \frac{1\left((3)^5 - 1\right)}{3-1}$$

$$S_5 = \frac{243-1}{2} = \frac{242}{2} \Rightarrow \boxed{S_5 = 121}$$

9. ~~**Find the sum of infinite geometric series in which**~~

$$a = 128 \text{ and } r = -\tfrac{1}{2}.$$

Sol. $$S_\infty = \frac{a}{1-r} = \frac{128}{1-\left(-\frac{1}{2}\right)} = \frac{128}{1+\frac{1}{2}}$$

$$S_\infty = \frac{128}{\frac{2+1}{2}} = \frac{128}{3/2} = 128 \times \frac{2}{3} = \boxed{\frac{256}{3}}$$

10. **Expand the expression** $\left(\frac{x}{y}+\frac{y}{x}\right)^4$

Sol. $$\left(\frac{x}{y}+\frac{y}{x}\right)^4$$

$$= \binom{4}{0}\left(\frac{x}{y}\right)^4\left(\frac{y}{x}\right)^0 + \binom{4}{1}\left(\frac{x}{y}\right)^3\left(\frac{y}{x}\right)^1 + \binom{4}{2}\left(\frac{x}{y}\right)^2\left(\frac{y}{x}\right)^2$$

$$+\binom{4}{3}\left(\frac{x}{y}\right)^1\left(\frac{y}{x}\right)^3 + \binom{4}{4}\left(\frac{x}{y}\right)^0\left(\frac{y}{x}\right)^4$$

$$= (1)\left(\frac{x^4}{y^4}\right)(1) + 4\left(\frac{x^3}{y^3}\right)\left(\frac{y}{x}\right) + 6\left(\frac{x^2}{y^2}\right)\left(\frac{y^2}{x^2}\right)$$

$$+4\left(\frac{x}{y}\right)\left(\frac{y^3}{x^3}\right) + 1(1)\left(\frac{y^4}{x^4}\right)$$

$$= \boxed{\frac{x^4}{y^4} + 4\frac{x^2}{y^2} + 6 + 4\frac{y^2}{x^2} + \frac{y^4}{x^4}}$$

11. **Calculate** $(1.04)^5$ **by binomial theorem up to two decimal places.**

Sol. $$(1.04)^5 = (1+0.04)^5$$

$$= \binom{5}{0}(1)^5(0.04)^0 + \binom{5}{1}(1)^4(0.04)^1 + \binom{5}{2}(1)^3(0.04)^2 + \dots$$

$$= (1)(1)(1) + 5(1)(0.04) + 10(1)(0.0016) + \dots$$

$$= 1 + 0.2 + 0.016 + \dots = 1.2160 = \boxed{1.22}$$

12. **Find the 5th term in the expansion of** $\left(2x - \frac{x^2}{4}\right)^7$

Sol. Here : $a = 2x$, $b = -\frac{x^2}{4}$, $n = 7$ & $r = 4$

$$T_{r+1} = \binom{n}{r}a^{n-r}b^r \Rightarrow T_{4+1} = \binom{7}{4}(2x)^{7-4}\left(-\frac{x^2}{4}\right)^4$$

$$T_5 = (35)\left(8x^3\right)\left(\frac{x^8}{256}\right) \Rightarrow \boxed{T_5 = \frac{35}{32}x^{11}}$$

13. **Define proper fraction and give example.**

Sol. A fraction in which the degree of the numerator is less than the degree of the denominator is called proper fraction.

Example: $\dfrac{2x}{(x-2)(x+5)}$

14. **Resolve** $\dfrac{1}{x^2 - x}$ **into partial fractions.**

Sol. $$\frac{1}{x^2 - x} = \frac{1}{x(x-1)}$$

$$\frac{1}{x(x-1)} = \frac{A}{x} + \frac{B}{x-1} \rightarrow (i)$$

$$1 = A(x-1) + Bx \rightarrow (ii)$$

Put $x = 0$ in eq. (ii)

$$1 = A(0-1) + B(0)$$

$$1 = -A \quad \Rightarrow \quad \boxed{A = -1}$$

Put $x = 1$ in eq. (ii)

$$1 = A(1-1) + B(1)$$

$$1 = A(0) + B \quad \Rightarrow \quad \boxed{B = 1}$$

Put values of A, & B

in eq. (i), we get: $\boxed{-\frac{1}{x} + \frac{1}{x-1}}$

15. **Write in the form of partial fractions $\dfrac{x^5}{x^4-1}$.**

Sol. $\dfrac{x^5}{x^4-1} \rightarrow (i) \left\{\begin{matrix}\text{Improper}\\ \text{Fraction}\end{matrix}\right\}$

So, eq.(i) becomes:

$$= x + \frac{x}{(x-1)(x+1)(x^2+1)}$$

$$= \boxed{x + \frac{A}{x-1} + \frac{B}{x+1} + \frac{Cx+D}{x^2+1}}$$

$$\begin{array}{r} x \\ x^4-1\overline{)\;x^5\quad\;\;} \\ \underline{_-x^5 \mp x} \\ x \end{array}$$

$$x^4-1 = (x^2)^2 - (1)^2$$
$$= (x^2-1)(x^2+1)$$
$$= (x-1)(x+1)(x^2+1)$$

16. ~~**Convert**~~ $12°40'$ ~~**into radian measure.**~~

Sol. $12°40' = \left(12 + \dfrac{40}{60}\right)^{\circ}$

$$= (12 + 0.666)^{\circ} = 12.6666^{\circ}$$

$$= 12.6666^{\circ} \times \frac{\pi}{180} = \boxed{0.22\,\text{rad}}$$

17. ~~**What is the length of an arc of a circle of radius 5 cm whose central angle is 140º.**~~

Sol. Here : $\ell = ?,\ r = 5\text{cm},\ \theta = 140^{\circ}$

$$\theta = 140^{\circ} = 140 \times \frac{\pi}{180} = 2.44\text{rad.}$$

By using formula : $\ell = r\theta$

$$\ell = r\theta = (5)(2.44) = \boxed{12.22\text{cm}}$$

18. **If $\sin\theta = \dfrac{5}{6}$ and θ is an obtuse angle, find $\cos\theta$.**

Sol. As, we know that : $\cos^2\theta = 1 - \sin^2\theta$

$$\cos^2\theta = 1 - \left(\frac{5}{6}\right)^2 = 1 - \frac{25}{36} = \frac{36-25}{36} = \frac{11}{36}$$

$$\sqrt{\cos^2\theta} = \pm\sqrt{\frac{11}{36}} \Rightarrow \boxed{\cos\theta = -\frac{\sqrt{11}}{6}} \because \left\{\begin{matrix}\theta \text{ is an}\\ \text{obtuse}\end{matrix}\right\}$$

19. **Prove that:**

$$2\sin 45° + \frac{1}{2}\operatorname{cosec} 45° = \frac{3}{\sqrt{2}}$$

Sol. $\text{L.H.S} = 2\sin 45° + \dfrac{1}{2}\operatorname{cosec} 45°$

$$= 2.\frac{1}{\sqrt{2}} + \frac{\sqrt{2}}{2} = \frac{4+2}{2\sqrt{2}} = \frac{6}{2\sqrt{2}} = \frac{3}{\sqrt{2}} = \text{R.H.S.}\ \textbf{Proved.}$$

20. **Prove that:**

$$1 - 2\sin^2\theta = 2\cos^2\theta - 1$$

Sol. $\text{L.H.S.} = 1 - 2\sin^2\theta$

$$= 1 - 2(1 - \cos^2\theta) \quad \because \left\{\begin{matrix}\sin^2\theta\\ =1-\cos^2\theta\end{matrix}\right\}$$

$$= 1 - 2 + 2\cos^2\theta$$

$$= 2\cos^2\theta - 1 = \text{R.H.S.}\ \textbf{Proved.}$$

21. **Prove that: $\cos\left(\dfrac{\pi}{2} - \theta\right) = \sin\theta$**

Sol. $\text{L.H.S.} = \cos\left(\dfrac{\pi}{2} - \theta\right)$

$$= \cos(90° - \theta) \because \left\{\frac{\pi}{2} \times \frac{180°}{\pi} = 90°\right\}$$

$$= \cos 90° \cos\theta + \sin 90° \sin\theta$$

$$= (0)\cos\theta + (1)\sin\theta \because \left\{\begin{matrix}\text{Using calculator}\\ \textbf{cos90°=0 \& sin90°=1}\end{matrix}\right\}$$

$$= 0 + \sin\theta = \sin\theta = \text{R.H.S.}\quad \textbf{Proved.}$$

22. **Show that:**

$$\cos(\alpha+\beta) - \cos(\alpha-\beta) = -2\sin\alpha\sin\beta$$

Sol. $\text{L.H.S.} = \cos(\alpha+\beta) - \cos(\alpha-\beta)$

$$= [\cos\alpha\cos\beta - \sin\alpha\sin\beta] - [\cos\alpha\cos\beta + \sin\alpha\sin\beta]$$

$$= \cancel{\cos\alpha\cos\beta} - \sin\alpha\sin\beta - \cancel{\cos\alpha\cos\beta} - \sin\alpha\sin\beta$$

$$= -2\sin\alpha\sin\beta = \text{R.H.S.}\quad \textbf{Proved.}$$

23. **Prove that:**

$$\tan(45° + \theta)\tan(45° - \theta) = 1$$

Sol. $\text{L.H.S.} = \tan(45° + \theta)\tan(45° - \theta)$

$$= \frac{\tan 45° + \tan\theta}{1 - \tan 45° \tan\theta} \times \frac{\tan 45° - \tan\theta}{1 + \tan 45° \tan\theta}$$

$$= \frac{1 + \tan\theta}{1 - (1)\tan\theta} \times \frac{1 - \tan\theta}{1 + (1)\tan\theta} \because \left\{\begin{matrix}\text{Using calculator}\\ \tan 45°=1\end{matrix}\right\}$$

$$= \frac{1 + \tan\theta}{1 - \tan\theta} \times \frac{1 - \tan\theta}{1 + \tan\theta} = 1 = \text{R.H.S.}\ \textbf{Proved.}$$

24. ~~**Express the sum**~~ $\cos 12\theta - \cos 4\theta$ ~~**as product.**~~

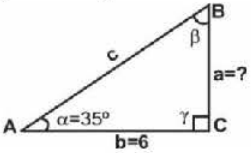
Sol. $\cos 12\theta - \cos 4\theta$

$$= -2\sin\left(\frac{12\theta + 4\theta}{2}\right)\sin\left(\frac{12\theta - 4\theta}{2}\right)$$

$$= -2\sin\left(\frac{16\theta}{2}\right)\sin\left(\frac{8\theta}{2}\right) = \boxed{-2\sin 8\theta \sin 4\theta}$$

25. ~~In right triangle ABC, b = 6, α = 35º, γ = 90º, Find side 'a'.~~

Sol. We know that, from figure:



$$\tan\alpha = \frac{a}{b}$$

$$\tan 35^\circ = \frac{a}{6}$$

$$6\tan 35^\circ = a \Rightarrow \boxed{a = 4.2}$$

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26. **The sides of a triangle are 16, 20 and 33 meters respectively. Find its greatest angle.**

Sol. Let a = 16, b = 20, c = 33

As side 'c' is greatest so, we will find angle 'γ'.

$$\text{Using law of cosines}: \cos\gamma = \frac{a^2 + b^2 - c^2}{2ab}$$

$$\cos\gamma = \frac{(16)^2 + (20)^2 - (33)^2}{2(16)(20)} = -0.6765$$

$$\gamma = \cos^{-1}(-0.6765) \Rightarrow \boxed{\gamma = 132^\circ 34'}$$

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27. **In any triangle ABC in which a =5, c = 6, α = 45º, find γ.**

Sol. From law of sines

$$\text{we take}: \frac{a}{\sin\alpha} = \frac{c}{\sin\gamma}$$

$$5\sin\gamma = 6\sin 45^\circ$$

$$\sin\gamma = \frac{6\sin 45^\circ}{5} \Rightarrow \boxed{\sin\gamma = 0.8485}$$

Section - II

Note : Attemp any three (3) questions $3 \times 8 = 24$

Q.2.(a) Solve the given equation by factorization $\frac{1}{a+b+x} = \frac{1}{a} + \frac{1}{b} + \frac{1}{x}$

Sol. See Q.1(vi) of Ex # 1.1 (Page # 8)

(b) ~~The roots of the equation px2 + qx + q = 0 are α, β prove that~~ :

$$\sqrt{\frac{\alpha}{\beta}} + \sqrt{\frac{\beta}{\alpha}} + \sqrt{\frac{q}{p}} = 0$$

Sol. See Q.6 of Ex # 1.3 (Page # 46)

Q.3.(a) What term in the arithmetic progression 4,1,-2,..... is -77.

Sol. See Q.3(i) of Ex # 2.1 (Page # 77)

(b) ~~A rubber ball is dropped from a height of 4.8cm. It continuously rebounds, each time rebounding ¾ of the distance of the proceeding fall. How much distance has it traveled when it strike the ground for the sixth time.~~

Sol. See Q.5 of Ex # 2.6 (Page # 122)

Q.4(a) Find the term independent of x in the expansion of $\left(2x^2 + \frac{1}{x}\right)^9$.

Sol. See Q.10(ii) of Ex # 3.1 (Page # 162)

(b) Resolve into partial fractions

$$\frac{3x + 7}{(x^2 + x + 1)(x^2 - 4)}$$

Sol. See Q.6 of Ex # 4.3 (Page # 209)

Q.5.(a) Prove that:

$$\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = \sec\theta\operatorname{cosec}\theta + 1$$

Sol. See Q.12 of Ex # 5.3 (Page # 257)

(b) If $\sin\alpha = \frac{4}{5}$ and $\sin\beta = \frac{12}{13}$, both α & β are in 1st quadrant, find $\sin(\alpha - \beta)$.

Sol. See Q.7(i) of Ex # 6.1 (Page # 283)

Q.6.(a) ~~Express the sum sin 3θ + sin 5θ + sin 7θ + sin 9θ as a product.~~

Sol. See Q.3 of Ex # 6.3 (Page # 307)

(b) ~~Form a light-house, angles of depression of two ships on opposite of the light-house are observed to beº and 45º. If the height of the light-house be 300m, find the distance between the chips if the line joining them passing through foot of light house.~~

Sol. See Q.8 of Ex # 7.2 (Page # 338)

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