DAE/IIA-2019/05 SECOND YEAR

(Common with Bio Medical, Computer, Food Computer Information, Electrical, Electronics, Food Processing & Preservation, Instrument, Critical Health Care and Telecommunication Technologies.)

<u>MATH-233 APPLIED MATHEMATICS – II</u> <u>PAPER 'A' (Subjective)</u>

Time: 2:30 Hours

SECTION - I

earn Math

Marks: 60

 $18 \times 2 = 36$

 $\mathbf{Q.1:}$ Write short answer to any Eighteen (18) of the questions: -

1. If
$$f(x) = \ell n(x)$$
, then prove that: $f(pq) = f(p) + f(q)$.

2. If
$$f(x) = ln(x)$$
, then prove that: $f\left(\frac{p}{q}\right) = f(p) - f(q)$.

- 3. If $f(x) = \sin x + \cos x$, show that: $f(x + \pi) = -f(x)$.
- **4.** Show that the function $f(x) = x^4 7x^2 + 7$ is an even function of x.
- **5.** Find the derivative of $(a + x)\sqrt{a x}$ w.r.t. 'x'.

6. If
$$y = \frac{1}{(x-3)(x+2)}$$
, find $\frac{dy}{dx}$.

7. Differentiate
$$\frac{x}{x^2+1}$$
 w.r.t. 'x'.

8. If
$$y = \frac{1+x}{1-x}$$
, find $\frac{d}{d}$

9. If
$$y = \frac{x^2 + 1}{x - 1}$$
, find $\frac{dy}{dx}$ at $x = 2$

10. If
$$y = u^n \& u = (3x^3 - 7x^2 + x + 1)$$
 find $\frac{dy}{dx}$

- **11.** Find the derivative of $x^2 \sec 4x$.
- **12.** Differentiate $\sin^{-1}\sqrt{x}$ w.r.t. 'x'.

13. Differentiate
$$\sin^{-1}\left(\frac{x}{3}\right)$$
 w.r.t. 'x'.

- **14.** Differentiate $x^2 \cot^{-1} x$ w.r.t. 'x'.
- **15.** Differentiate $l \mathbf{n} \sqrt{\mathbf{x}}$ w.r.t. 'x'.
- **16.** Differentiate $\sin(\ell n \tan x)$.
- **17.** Differentiate $x \ell n 3x$ w.r.t. 'x'.
- **18.** Find slope of tangent to $x = a \cos \theta$, $y = b \sin \theta$ at $\theta = \frac{\pi}{4}$.
- **19.** Find the slope of tangent to the curve $y = x^3 3x + 2$ at (0, 2).
- **20.** For the curve $x = t^2 1$, $y = t^2 t$, the tangent is parallel to x-axis, find the value of t.

21. If
$$x = a\cos\theta$$
, $y = a\sin\theta$, find $\frac{d^2y}{dx^2}$.

- **22.** If mode = 15, Median = 12 find mean.
- **23.** Define arithmetic mean.
- **24.** The arithmetic mean of 7 values is 6 find the sum of values.
- **25.** If two coins are tossed find the probability that only one head.
- **26.** Tickets numbered 1 to 20 are mixed up and then a ticket is drawn at random. What is the probability that ticket drawn has a number multiple of 3 or 5.
- **27.** If a die is rolled once, what is the probability of getting a 4.

SECTION - II

2

Note: Attempt any three (03) questions.

Q.2. (a) Show that
$$x \cdot \frac{a^x + 1}{a^x - 1}$$
 is an Even function of x.

(b) Evaluate:
$$\lim_{\theta \to 0} \frac{1 - \cos p\theta}{1 - \cos q\theta}$$
.

- **Q.3.** (a) Differentiate $x^{\frac{2}{3}}$ by ab-initio method (or first principle).
 - (b) Differentiate $x\sqrt{\frac{a+x}{a-x}}$ w.r.t. 'x'.

Q.4 (a) If
$$\sin y = x \sin(a + y)$$
, prove that : $\frac{dy}{dx} = \frac{\sin^2(a + y)}{\sin a}$

- (b) Find the derivative of $ln\left(\frac{x^2+x+1}{x^2-x+1}\right)$
- **Q.5.** (a) Use differentials to find the approximate value of $\sqrt{65}$.
 - (b) Find the maximum and minimum (extreme) values of the function $\frac{x^3}{3} \frac{3x^2}{2} + 2x + 5$.
- **Q.6. (a)** Find the mean for the following distribution showing marks obtained by 50 students in English.

| Marks | Frequency |
|---------|-----------|
| 20 - 24 | 1 |
| 25 - 29 | 4 |
| 30 - 34 | 8 |
| 35 – 39 | 11 |
| 40 - 44 | 15 |
| 45 - 49 | 9 |
| 50 - 54 | 2 |

(b) Calculate the S.D. from the Mean for the following data, 2, 6, 9, 12, 8, 13, 5, 6, 23, 16.