DAE / IIA - 2019

MATH - 123 APPLIED MATHEMATICS - I

PAPER 'B' PART - A(OBJECTIVE)

Time : 30 Minutes Marks : 15

Q.1: *Encircle the correct answer.*

1. Modulus of $3+4i$ is:

[a] 47 [b] 16
[c] 5 [d] 3

2. If $z=a+bi$ then $z+\bar{z}$ is equal to:

[a] 2a [b] 2b
[c] 0 [d] $2a+2bi$

3. $i(2-i)$ is equal to:

[a] $1+2i$ [b] $2i+i^2$
[c] 3 [d] $3i$

4. The number of partial fractions of $\dfrac{x^3-3x^2+1}{(x-1)(x+1)(x^2-1)}$ are:

[a] 2 [b] 3
[c] 4 [d] 5

5. The fraction $\dfrac{(x-1)(x-2)(x-3)}{(x-4)(x-5)(x-6)}$ is called:

[a] Proper [b] Improper
[c] Neither proper nor improper
[d] None of these

6. Numbers of digit in a binary system are:

[a] 2 [b] 7
[c] 10 [d] 8

7. If the switch is on it is represented by:

[a] 0 [b] 1
[c] OR [d] NOT

8. $X(\overline{X}+Y)$ is equal to:

[a] $X.Y$ [b] $X.\overline{X}$
[c] $X+\overline{X}Y$ [d] $X+Y$

9. ~~In Boolean algebra~~ $\overline{X+Y}$ ~~is equal to:~~

[a] $\overline{X}+\overline{Y}$ [b] $\overline{X}.\overline{Y}$
[c] XY [d] $X+Y$

10. Equation of the line in slope-intercept form is:

[a] $\dfrac{x}{a}+\dfrac{y}{b}=1$
[b] $y-y_1=m(x-x_1)$
[c] $y=mx+c$ [d] None of these

11. Distance between (4, 3) and (7, 5) is:

[a] 25 [b] $\sqrt{13}$
[c] 5 [d] None of these

12. When two lines are perpendicular then:

[a] $m_1=m_2$ [b] $m_1=-m_2$
[c] $m_1m_2=-1$ [d] None of these

13. Point $(-4,-5)$ lies in the quadrant:

[a] 1^{st} [b] 2^{nd}
[c] 3^{rd} [d] 4^{th}

14. Radius of the circle $x^2+y^2=1$ is:

[a] 1 [b] (0 , 0)
[c] 2 [d] None of these

15. Center of the circle $(x-1)^2+(y-2)^2=16$ is:

[a] (1 , 2) [b] (2 , 1)
[c] (4 , 0) [d] None of these

Answer Key

1	c	2	a	3	a	4	c	5	b
6	a	7	b	8	a	9	b	10	c
11	b	12	c	13	c	14	a	15	a

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DAE / IIA - 2019

MATH - 123 APPLIED MATHEMATICS - I

PAPER 'B' PART - B(SUBJECTIVE)

Time: 2:30 Hrs Marks: 60

Section - I

Q.1. Write short answers to any Eighteen (18) questions.

1. **Write the conjugate and Modulus of $(-2+i)$.**

Sol. Let $z=-2+i$

Conjugate $=\bar{z}=\overline{-2+i}=\boxed{-2-i}$

As, $a=-2$, $b=1$

Modulus $=|z|=\sqrt{a^2+b^2}$

$|z|=\sqrt{(-2)^2+(1)^2}=\sqrt{4+1}=\boxed{\sqrt{5}}$

2. **Simplify the complex number** $\dfrac{-9+4i}{8-3i}$

Sol. $\dfrac{-9+4i}{8-3i}=\dfrac{-9+4i}{8-3i}\times\dfrac{8+3i}{8+3i}$

$=\dfrac{-72-27i+32i+12i^2}{(8)^2-(3i)^2}$

$=\dfrac{-72+5i-12}{64+9}$

$=\dfrac{-84+5i}{73}=\boxed{-\dfrac{84}{73}+\dfrac{5}{73}i}$

3. **Show that** $\dfrac{1+2i}{2-i}$

Sol. L.H.S. $=\dfrac{1+2i}{2-i}$

$=\dfrac{\sqrt{(1)^2+(2)^2}}{\sqrt{(2)^2+(-1)^2}}=\dfrac{\sqrt{1+4}}{\sqrt{4+1}}$

$=\dfrac{\sqrt{5}}{\sqrt{5}}=1=$ R.H.S. **Proved.**

4. **Find the multiplicative inverse of $(-3, 4)$.**

Sol. Let $z=(-3,4)=-3+4i$

Multiplicative Inverse of $Z=\dfrac{1}{Z}=\dfrac{1}{-3+4i}$

$=\dfrac{1}{-3+4i}\times\dfrac{-3-4i}{-3-4i}=\dfrac{-3-4i}{(-3)^2-(4i)^2}$

$=\dfrac{-3-4i}{9+16}=\dfrac{-3-4i}{25}=\boxed{-\dfrac{3}{25}-\dfrac{4}{25}i}$

5. **Factorize $(36a^2+100b^2)$.**

Sol. $36a^2+100b^2=36a^2-100b^2i^2$

$=(6a)^2-(10bi)^2=\boxed{(6a-10bi)(6a+10bi)}$

6. **Resolve into partial fractions.** $\dfrac{7x+25}{(x+3)(x+4)}$.

Sol. $\dfrac{7x+25}{(x+3)(x+4)}=\dfrac{A}{x+3}+\dfrac{B}{x+4}\rightarrow(i)$

$7x+25=A(x+4)+B(x+3)\rightarrow(ii)$

Put $x=-3$ in eq.(ii)

$7(-3)+25=A(-3+4)+B(-3+3)$

$-21+25=A(1)+B(0)$

$4=A+0\Rightarrow\boxed{A=4}$

Put $x=-4$ in eq.(ii)

$7(-4)+25=A(-4+4)+B(-4+3)$

$-28+25=A(0)+B(-1)$

$-3=0-B\Rightarrow\boxed{B=3}$

Put values of A, & B in eq. (i),

we get: $\boxed{\dfrac{4}{x+3}+\dfrac{3}{x+4}}$

7. **Resolve $\frac{1}{x^2-1}$ into partial fractions .**

Sol. $\frac{1}{x^2-1}=\frac{1}{(x)^2-(1)^2}=\frac{1}{(x-1)(x+1)}$

$$\frac{1}{(x-1)(x+1)}=\frac{A}{x-1}+\frac{B}{x+1}\rightarrow(i)$$

$$1=A(x+1)+B(x-1)\rightarrow(ii)$$

Put $x=1$ in eq.(ii)

$$1=A(1+1)+B(1-1)$$

$$1=A(2)+B(0)$$

$$1=2A+0 \Rightarrow \boxed{A=1/2}$$

Put $x=-1$ in eq.(ii)

$$1=A(-1+1)+B(-1-1)$$

$$1=A(0)+B(-2)$$

$$1=0-2B \Rightarrow \boxed{B=-1/2}$$

Put values of A, & B in eq. (i),

we get: $\boxed{\frac{1}{2(x-1)}-\frac{1}{2(x+1)}}$

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8. **Write in the form of partial fractions $\frac{x^5}{x^4-1}$.**

Sol. $\frac{x^5}{x^4-1}\rightarrow(i)$ {**Improper Fraction**}

$$\begin{array}{r} x \\ x^4-1\overline{)\,x^5} \\ \underline{-x^5\mp x} \\ x \end{array}$$

x^4-1

$=(x^2)^2-(1)^2$

$=(x^2-1)(x^2+1)$

$=(x-1)(x+1)(x^2+1)$

So, eq.(i) becomes:

$$=x+\frac{x}{(x-1)(x+1)(x^2+1)}$$

$$=\boxed{x+\frac{A}{x-1}+\frac{B}{x+1}+\frac{Cx+D}{x^2+1}}$$

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9. **Write in the form of partial fractions $\frac{1}{(x+2)^2(x-1)}$.**

Sol.

$$\frac{1}{(x+2)^2(x-1)}=\boxed{\frac{A}{(x+2)}+\frac{B}{(x+2)^2}+\frac{C}{(x-1)}}$$

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10. **Define octal numbers.**

Sol. The Octal number system is a number system of base equal to 8.

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11. **Add the binary numbers**

$$(1101)_2+(1011_2)$$

Sol.

$$\begin{array}{r} 1101 \\ \underline{+1011} \\ 11000 \end{array}$$

$$\boxed{(11000)_2}$$

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12. **Define (i) OR Gate (ii) AND Gate.**

ANS.

OR Gate: The OR gate is an electronic circuit that gives a high output (1) if one or more of its inputs are 1.

AND Gate: The AND gate is an electronic circuit that gives a high output (1) when all of its inputs are 1.

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13. **Prove by Boolean algebra rules:**

$$X+\overline{X}Y=X+Y$$

Sol. $\text{L.H.S.}=X+\overline{X}Y$

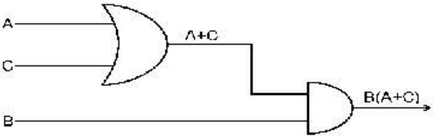
$=(X+\overline{X})(X+Y)$ {By Dual of Distributive law}

$=1(X+Y)\quad \because X+\overline{X}=1$

$=X+Y=\text{R.H.S.}$ **Proved.**

14. **Construct a logic Diagram for expression $\mathbf{B(A+C)}$.**

Sol.



15. **Write distance formula between two points $(\mathbf{x_1, y_1})$ and $(\mathbf{x_2, y_2})$.**

Sol. $\text{Distance} = |AB| = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

16. **Find the slope of a line which is perpendicular to the line joining $\mathbf{P_1(2, 4)}$, $\mathbf{P_2(-2, 1)}$.**

Sol. Slope of line joining given point:

$$= m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1-4}{-2-2} = \frac{-3}{-4} = \frac{3}{4}$$

Slope of require line $= m_2 = ?$

As, both lines are perpendicular

So, $m_1 m_2 = -1$

$$\Rightarrow \left(\frac{3}{4}\right) m_2 = -1$$

$$\Rightarrow \quad m_2 = -1 \times \frac{4}{3} \quad \Rightarrow \quad \boxed{m_2 = -\frac{4}{3}}$$

17. **~~Find the equation of line having x-intercept -2 and y-intercept 3.~~**

Sol. $\text{Let, } x - \text{intercept} = a = -2$

$\& \quad y - \text{intercept} = b = 3$

Equation of line in intercept

form: $\frac{x}{a} + \frac{y}{b} = 1$

$$\Rightarrow \quad \frac{x}{-2} + \frac{y}{3} = 1$$

$$\Rightarrow \quad \frac{-3x + 2y}{6} = 1$$

$$\Rightarrow \quad -3x + 2y = 6$$

$$\Rightarrow \quad -3x + 2y - 6 = 0$$

$$\Rightarrow \quad \boxed{3x - 2y + 6 = 0}$$

18. **~~Find the equation of a line whose perpendicular distance from the origin is 2 and inclination of the perpendicular is 225°.~~**

Sol. Here $P = 2$ and $\theta = 225°$

Normal form of equation of line is

$$x\cos\theta + y\sin\theta = P$$

$$x\cos 225° + y\sin 225° = 2$$

$$x\left(-\frac{1}{\sqrt{2}}\right) + y\left(-\frac{1}{\sqrt{2}}\right) = 2$$

$$-\frac{x}{\sqrt{2}} - \frac{y}{\sqrt{2}} = 2$$

$$-x - y = 2\sqrt{2} \Rightarrow \boxed{x + y + 2\sqrt{2} = 0}$$

19. **Show that the points (1, 9), (-2, 3) and (-5, -3) are collinear.**

Sol.
$$\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 9 & 1 \\ -2 & 3 & 1 \\ -5 & -3 & 1 \end{vmatrix}$$

$$= 1\begin{vmatrix} 3 & 1 \\ -3 & 1 \end{vmatrix} - 9\begin{vmatrix} -2 & 1 \\ -5 & 1 \end{vmatrix} + 1\begin{vmatrix} -2 & 3 \\ -5 & -3 \end{vmatrix}$$

$$= 1(3 - (-3)) - 9(-2 - (-5)) + 1(6 - (-15))$$

$$= 1(3+3) - 9(-2+5) + 1(6+15)$$

$$= 1(6) - 9(3) + 1(21) = 6 - 27 + 21 = 0$$

Hence given points are collinear. **Proved.**

20. **~~Find the distance from the point $(-2, 1)$ to the line $3x + 4y - 12 = 0$~~**

Sol.

Distance between point & line

$$D=\frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}$$

$$D=\frac{|3(-2)+4(1)-12|}{\sqrt{(3)^2+(4)^2}}$$

$$D=\frac{|-6+4-12|}{\sqrt{9+16}}$$

$$D=\frac{|-14|}{\sqrt{25}}=\boxed{\frac{14}{5}}$$

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21. **Find the co-ordinates of the mid-point of the segment A(3, 7), B(-2, 3).**

Sol. $\text{Mid}-\text{point}=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$

$$=\left(\frac{3+(-2)}{2},\frac{7+3}{2}\right)$$

$$=\left(\frac{3-2}{2},\frac{10}{2}\right)$$

$$=\boxed{\left(\frac{1}{2},\ 5\right)}$$

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22. **Reduce the equation $3x+4y-2=0$ into intercept form.**

Sol. $3x+4y-2=0$

$$3x+4y=2$$

Dividing both sides by 2, we have:

$$\frac{3x}{2}+\frac{4y}{2}=\frac{2}{2}$$

$$\frac{x}{2/3}+\frac{y}{2/4}=1$$

$$\boxed{\frac{x}{2/3}+\frac{y}{1/2}=1}$$

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23. **Define a circle.**

Sol. A circle is the set of all points in a plane that are equally distance from a fixed point.

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24. **Write the general form of the equation of a circle.**

Sol. $x^2+y^2+2gx+2fy+c=0$

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25. **Find center and radius of the circle $x^2+y^2+9x-7y-33=0$**

Sol. Comparing with general equation of circle.

$$x^2+y^2+9x-7y-33=0$$

$$2g=9,\quad g=\frac{9}{2}\qquad\left|\qquad 2f=-7,\quad f=-\frac{7}{2}\qquad\right|\qquad c=-33$$

Center $=(-g,-f)=$

Center $=\left(-\frac{9}{2},-\left(-\frac{7}{2}\right)\right)=\boxed{\left(-\frac{9}{2},\frac{7}{2}\right)}$

$$\text{Radius}=r=\sqrt{g^2+f^2-c}$$

$$r=\sqrt{\left(\frac{9}{2}\right)^2+\left(-\frac{7}{2}\right)^2-(-33)}$$

$$r=\sqrt{\frac{81}{4}+\frac{49}{4}+33}$$

$$r=\sqrt{\frac{81+49+132}{4}}$$

$$r=\sqrt{\frac{262}{4}}$$

$$r=\boxed{\sqrt{\frac{131}{2}}}$$

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26. **Find the equation of a circle with center at (-1, 3) and tangent to x-axis.**

Sol. $\text{Here : Centre} = (h,k) = (-1,3)$

$\& \quad \text{Radius} = r = 3$

Standard form of eq. of circle :

$$(x-h)^2+(y-k)^2=r^2$$

$\text{Put } h=-1, \quad k=3 \quad \& \quad r=3$

$$(x+1)^2+(y-3)^2=(3)^2$$

$$(x)^2+2(x)(1)+(1)^2+(y)^2-2(y)(3)+(3)^2=9$$

$$x^2+2x+1+y^2-6y+9-9=0$$

$$\boxed{x^2+y^2+2x-6y+1=0}$$

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27. **Reduce the equation of the circle $x^2+y^2-4x+6y-12=0$ into standard form.**

Sol. As given equation :

$$x^2+y^2-4x+6y-12=0$$

$$x^2-4x+y^2+6y=12$$

Adding the square of one half of the coefficient of x & y on both sides :

$$x^2-4x+(2)^2+y^2+6y+(3)^2=12+(2)^2+(3)^2$$

$$(x-2)^2+(y+3)^2=12+4+9$$

$$(x-2)^2+(y+3)^2=25$$

$$\boxed{(x-2)^2+(y+3)^2=(5)^2}$$

Section - II

Note : Attemp any three (3) questions $3\times 8 = 24$

Q.2.(a) **Simplify $\left(-1+\sqrt{3}i\right)^3$.**

Sol. See Q.4(vii) of Ex# 8.1 (Page # 305)

(b) **Find the multiplicative inverse of $(-3,\ 4)$.**

Sol. See Q.8(i) of Ex# 8.1 (Page # 309)

Q.3. **Resolve $\dfrac{1}{x^4(x+1)}$ into partial fractions.**

Sol. See example 07 of Chapter 09

Q.4.(a) (i) **Convert $(35)_8$ into decimal number.**

Sol. See example 08[a] of Chapter 10

(ii) **Convert $(245)_{10}$ to its octal equivalent.**

Sol. See example 09[a] of Chapter 10

(b) **Prepare a truth table for the Boolean expression $XYZ+\overline{X}.\overline{Y}.\overline{Z}$**

Sol. See Q.1(i) of Ex # 11 (Page # 425)

Q.5. **Reduce the equation $3x+4y=10$ to .**

(a) **Slope-intercept form.**

(b) **Intercept form.**

(c) **Normal form.**

Sol. See example 23 of Chapter 12

Q.6. **(a)** **Find the equation of the circle having $(-2,\ 5)$ and $(3,\ 4)$ as the end point of its diameter.**

Sol. See Q.9[a] of Ex # 13 (Page # 541)

(b) **Find the center and radius of the circle $x^2+y^2-6x+6y=0.$**

Sol. See Q.2[a] of Ex # 13 (Page # 522)

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