DAE / IIA - 2018

MATH-123 APPLIED MATHEMATICS - I

PAPER 'A' PART - A(OBJECTIVE)

Time : 30 Minutes Marks : 15

Q.1: *Encircle the correct answer.*

1. To make $49x^2 + 5x$ a complete square we must add:

[a] $\left(\frac{5}{14}\right)^2$ [b] $\left(\frac{14}{5}\right)^2$

[c] $\left(\frac{5}{7}\right)^2$ [d] $\left(\frac{7}{5}\right)^2$

2. ~~If ± 3 are the roots of the equation, then the equation is:~~

[a] $x^2 - 3 = 0$ [b] $x^2 - 9 = 0$

[c] $x^2 + 3 = 0$ [d] $x^2 + 9 = 0$

3. ~~Product of roots of~~ $ax^2 + bx - c = 0$ ~~is:~~

[a] $\frac{c}{a}$ [b] $-\frac{c}{a}$ [c] $\frac{a}{b}$ [d] $-\frac{a}{b}$

4. In the expansion of $(a+b)^n$ the general term is:

[a] $\binom{n}{r}a^r b^r$ [b] $\binom{n}{r}a^{n-r}b^r$

[c] $\binom{n}{r-1}a^{n-r+1}b^{r-1}$ [d] $\binom{n}{r}a^{n-r-1}b^{r-1}$

5. In the expansion of $(a+b)^n$ the sum of exponents of a and b in any term is:

[a] n [b] $n-1$

[c] $n+1$ [d] None of these

6. The middle term of $\left(\frac{x}{y} - \frac{y}{x}\right)^4$ is:

[a] 6 [b] 8 [c] $\frac{4x^2}{y^2}$ [d] $\frac{4x}{y}$

7. ~~The degree measure of one radian is approximately equal to:~~

[a] 37.3º [b] 57.2º

[c] 57.1º [d] 57.0º

8. ~~π rad is equal to:~~

[a] $360°$ [b] $270°$

[c] $180°$ [d] $90°$

9. $\cos\left(\frac{\pi}{2} + 0\right)$ is equal to:

[a] $\cos\theta$ [b] $\sin\theta$

[c] $-\cos\theta$ [d] $-\sin\theta$

10. sin (A + B) – sin (A – B) is equal to:

[a] 2sin A cos B [b] 2cos A cos B

[c] -2sin A sin B [d] 2cos A sin B

11. If in a triangle ABC, b = 2, c = 2, A = 60º, then a is:

[a] 2 [b] 3 [c] 4 [d] 5

12. ~~In a right triangle of one angle is 45º, then the other will be:~~

[a] 45º [b] 50º

[c] 60º [d] 75º

13. If θ in the angle between the vector $\vec{a}$ and $\vec{b}$ then cos θ is:

[a] $\vec{a}.\vec{b}$ [b] $\frac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|}$

[c] $\frac{\vec{a}.\vec{b}}{|\vec{a}|}$ [d] $\frac{\vec{a}.\vec{b}}{|\vec{b}|}$

14. If $\vec{a} \times \vec{b} = 0$ then $\vec{a}$ and $\vec{b}$ is:

[a] Non-parallel [b] Parallel

[c] Perpendicular [d] None of these

15. ~~If E(cosθ + jsinθ) then the exponential form is:~~

[a] $Ee^{j\theta}$ [b] $Ee^{-j\theta}$

[c] $e^{j\theta}$ [d] $e^{-j\theta}$

Answer Key

1	a	2	b	3	a	4	b	5	a
6	b	7	a	8	c	9	d	10	d
11	a	12	a	13	b	14	b	15	a

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DAE / IIA - 2018

MATH - 123 APPLIED MATHEMATICS - I

PAPER 'A' PART - B (SUBJECTIVE)

Time: 2:30 Hrs Marks: 60

Section - I

Q.1. Write short answers to any Eighteen (18) questions.

1. Solve the quadratic equation $3x^2 + 5x = 2$ by factorization.

Sol. $3x^2 + 5x = 2$

$3x^2 + 5x - 2 = 0$

$3x^2 + 6x - x - 2 = 0$

$3x(x+2) - 1(x+2) = 0$

$(x+2)(3x-1) = 0$

Either OR

$x + 2 = 0$ | $3x - 1 = 0$

$x = -2$ | $3x = 1 \Rightarrow x = \frac{1}{3}$

$$S.S. = \left\{-2, \frac{1}{3}\right\}$$

2. Solve the equation $x^2 - 2x - 899 = 0$ by completing the square.

Sol. $x^2 - 2x - 889 = 0$

$x^2 - 2x = 889$

Adding the square of one half of the coefficient of x i.e., $(1)^2$ on both sides

$x^2 - 2x + (1)^2 = 889 + (1)^2$

$(x-1)^2 = 889 + 1$

$(x-1)^2 = 900$

$\sqrt{(x-1)^2} = \pm\sqrt{900}$

$x - 1 = \pm 30$

$x = \pm 30 + 1$

Either OR

$x = 30 + 1$ | $x = -30 + 1$

$x = 31$ | $x = -29$

$$S.S. = \{-29, 30\}$$

3. Discuss the nature of the roots of the equation $9x^2 + 6x + 1 = 0$

Sol. Here: $a = 9,\ b = 6,\ c = 1$

$\text{Disc.} = b^2 - 4ac$

$= (6)^2 - 4(9)(1) = 36 - 36 = 0$

$\therefore$ The roots are **Equal and Real.**

4. ~~Find the value of k if the sum of the roots of the equation $(2k-1)x^2 + (4k-1)x + (k+3) = 0$ is 5/2.~~

Sol. Here: $a = 2k - 1,\ b = 4k - 1,\ c = k + 3$

As, sum of roots $= \frac{5}{2}$

$-\frac{b}{a} = \frac{5}{2}$

$\Rightarrow \frac{-(4k-1)}{2k-1} = \frac{5}{2}$

$-2(4k-1) = 5(2k-1)$

$-8k + 2 = 10k - 5$

$-8k - 10k = -5 - 2$

$-18k = -7 \Rightarrow k = \frac{-7}{-18} \Rightarrow \boxed{k = \frac{7}{18}}$

5. ~~For what value of k, the sum of roots of equation~~ $3x^2 + kx + 5 = 0$ ~~may be equal to the product of roots.~~

Sol. Here: $a = 3,\ b = k,\ c = 5$

As, Sum of roots = Product of roots

$\frac{-b}{a} = \frac{c}{a}$

$\frac{-k}{3} = \frac{5}{3}$

$-k = 5 \Rightarrow \boxed{k = -5}$

6. **Expand $\left(\frac{x}{y}+\frac{y}{x}\right)^4$ by Binomial theorem.**

Sol. $\left(\frac{x}{y}+\frac{y}{x}\right)^4$

$$=\binom{4}{0}\left(\frac{x}{y}\right)^4\left(\frac{y}{x}\right)^0+\binom{4}{1}\left(\frac{x}{y}\right)^3\left(\frac{y}{x}\right)^1+\binom{4}{2}\left(\frac{x}{y}\right)^2\left(\frac{y}{x}\right)^2$$

$$+\binom{4}{3}\left(\frac{x}{y}\right)^1\left(\frac{y}{x}\right)^3+\binom{4}{4}\left(\frac{x}{y}\right)^0\left(\frac{y}{x}\right)^4$$

$$=(1)\left(\frac{x^4}{y^4}\right)(1)+4\left(\frac{x^3}{y^3}\right)\left(\frac{y}{x}\right)+6\left(\frac{x^2}{y^2}\right)\left(\frac{y^2}{x^2}\right)$$

$$+4\left(\frac{x}{y}\right)\left(\frac{y^3}{x^3}\right)+1(1)\left(\frac{y^4}{x^4}\right)$$

$$=\boxed{\frac{x^4}{y^4}+4\frac{x^2}{y^2}+6+4\frac{y^2}{x^2}+\frac{y^4}{x^4}}$$

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7. **Compute $(1.02)^{10}$ by Binomial theorem up to two decimal places.**

Sol. $(1.02)^{10}=(1+0.02)^{10}$

$$=\binom{10}{0}(1)^{10}(0.02)^0+\binom{10}{1}(1)^9(0.02)^1$$

$$+\binom{10}{2}(1)^8(0.02)^2+\ldots$$

$$=(1)(1)(1)+10(1)(0.02)$$

$$+45(1)(0.0004)+\ldots$$

$$=1+0.2+0.018+\ldots$$

$$=1.2180=\boxed{1.22}$$

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8. **Find the 6th term in the expansion of $(x+3y)^{10}$**

Sol. Here: $a=x,\ b=3y,\ n=10\ \&\ r=5$

$$T_{r+1}=\binom{n}{r}a^{n-r}b^r$$

$$T_{5+1}=\binom{10}{5}(x)^{10-5}(3y)^5$$

$$T_6=(252)(x^5)(243y^5)$$

$$\boxed{T_6=61236x^5y^5}$$

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9. ~~**Expand $\frac{1}{\sqrt{1+x}}$ to three terms.**~~

Sol. $\frac{1}{\sqrt{1+x}}=(1+x)^{-1/2}$

$$=1+\left(-\frac{1}{2}\right)(x)+\frac{\left(-\frac{1}{2}\right)\left(-\frac{1}{2}-1\right)}{2!}(x)^2+\ldots$$

$$=1-\frac{x}{2}+\frac{1}{2}\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)x^2+\ldots$$

$$=\boxed{1-\frac{x}{2}+\frac{3}{8}x^2+\ldots}$$

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10. ~~**Using the Binomial series, calculate $\sqrt{80}$ to the nearest hundredth.**~~

Sol. $\sqrt{80}=\sqrt{81-1}=\sqrt{81\left(1-\frac{1}{81}\right)}$

$$=9\left(1-\frac{1}{81}\right)^{1/2}=9\left[1-\left(\frac{1}{2}\right)\left(\frac{1}{81}\right)+\ldots\right]$$

$$=9\left[1-\frac{1}{162}+\ldots\right]=9[1-0.0062+\ldots]$$

$$=9[0.9938]=\boxed{8.94}$$

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11. ~~**Convert $\frac{2\pi}{3}$ radius into degree measure.**~~

Sol. $\frac{2\pi}{3}\text{ rad}=\frac{2\pi}{3}\times\frac{180}{\pi}\text{ rad}=\boxed{120^\circ}$

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12. **Prove that:**

$$2\sin 45^\circ+\frac{1}{2}\operatorname{cosec} 45^\circ=\frac{3}{\sqrt{2}}$$

Sol. $\text{L.H.S} = 2\sin 45° + \frac{1}{2}\operatorname{cosec}45°$

$$= 2\left(\frac{\sqrt{2}}{2}\right) + \frac{1}{2}\left(\frac{2}{\sqrt{2}}\right) = \sqrt{2} + \frac{1}{\sqrt{2}}$$

$$= \frac{(\sqrt{2})^2 + 1}{\sqrt{2}} = \frac{2+1}{\sqrt{2}}$$

$$= \frac{3}{\sqrt{2}} = \text{R.H.S.}$$ **Proved.**

13. **Prove that:**

$$\cos 30° \cos 60° - \sin 30° \sin 60° = 0$$

Sol. $\text{L.H.S.} = \cos 30° \cos 60° - \sin 30° \sin 60°$

$$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$$

$$= \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4}$$

$$= 0 = \text{R.H.S.}$$ **Proved.**

14. **Prove that:**

$$\frac{1}{1+\sin\theta} + \frac{1}{1-\sin\theta} = 2\sec^2\theta$$

Sol. $\text{L.H.S.} = \frac{1}{1+\sin\theta} + \frac{1}{1-\sin\theta}$

$$= \frac{1(1-\sin\theta) + 1(1+\sin\theta)}{(1+\sin\theta)(1-\sin\theta)}$$

$$= \frac{1-\sin\theta+1+\sin\theta}{(1)^2 - (\sin\theta)^2}$$

$$= \frac{2}{1-\sin^2\theta} = \frac{2}{\cos^2\theta}$$

$$= 2\sec^2\theta = \text{R.H.S.}$$ **Proved.**

15. **Prove that:** $\cos\left(\frac{\pi}{2} - \beta\right) = \sin\beta$

Sol. $\text{L.H.S.} = \cos\left(\frac{\pi}{2} - \beta\right)$

$$= \cos(90° - \beta) \because \left\{\frac{\pi}{2} \times \frac{180°}{\pi} = 90°\right\}$$

$$= \cos 90° \cos\beta + \sin 90° \sin\beta$$

$$= (0)\cos\beta + (1)\sin\beta \because \left\{\begin{array}{c}\text{Using calculator} \\ \cos 90° = 0 \ \& \ \sin 90° = 1\end{array}\right\}$$

$$= 0 + \sin\beta = \sin\beta = \text{R.H.S.}$$ **Proved.**

16. **Show that:**

$$\cos(\alpha+\beta) - \cos(\alpha-\beta) = -2\sin\alpha\sin\beta$$

Sol. $\text{L.H.S.} = \cos(\alpha+\beta) - \cos(\alpha-\beta)$

$$= [\cos\alpha\cos\beta - \sin\alpha\sin\beta] - [\cos\alpha\cos\beta + \sin\alpha\sin\beta]$$

$$= \cancel{\cos\alpha\cos\beta} - \sin\alpha\sin\beta - \cancel{\cos\alpha\cos\beta} - \sin\alpha\sin\beta$$

$$= -2\sin\alpha\sin\beta = \text{R.H.S.}$$ **Proved.**

17. ~~**Prove that:** $\sin\alpha = 2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}$~~

Sol. $\text{L.H.S.} = \sin\alpha = \sin\left(\frac{2\alpha}{2}\right)$

$$= \sin\left(\frac{\alpha+\alpha}{2}\right) = \sin\left(\frac{\alpha}{2} + \frac{\alpha}{2}\right)$$

$$= \sin\frac{\alpha}{2}\cos\frac{\alpha}{2} + \cos\frac{\alpha}{2}\sin\frac{\alpha}{2}$$

$$= 2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2} = \text{R.H.S.}$$ **Proved.**

18. **Find $\cos\theta$ if $\sin\theta = \frac{7}{25}$ and angle θ is an acute angle.**

Sol. $\sin\theta = {}^{7}\!/_{25}$

$$\cos^2\theta = 1 - \sin^2\theta = 1 - \left(\frac{7}{25}\right)^2 = 1 - \frac{49}{625} = \frac{625-49}{625} = \frac{576}{625}$$

$$\cos\theta = \pm\frac{24}{25}$$

As θ is acute angle, so $\boxed{\cos\theta = \frac{24}{25}}$

19. **Define the law of Sine.**

Sol. In any triangle ABC, with usual notations.

$$\boxed{\frac{a}{\sin\alpha}=\frac{b}{\sin\beta}=\frac{c}{\sin\gamma}}$$

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20. **In any triangle ABC, if a = 20, c = 32 and γ = 70º find angle α.**

Sol. By using law of sines:

$$\frac{a}{\sin\alpha}=\frac{b}{\sin\beta}=\frac{c}{\sin\gamma}$$

$$\text{we take: } \frac{a}{\sin\alpha}=\frac{c}{\sin\gamma}$$

$$\frac{\sin\alpha}{a}=\frac{\sin\gamma}{c}$$

$$\sin\alpha=\frac{a\sin\gamma}{c}$$

$$\sin\alpha=\frac{20\sin 70^\circ}{32}$$

$$\sin\alpha=0.5873$$

$$\alpha=\sin^{-1}(0.5873)$$

$$\boxed{\alpha=35^\circ 57' 58''}$$

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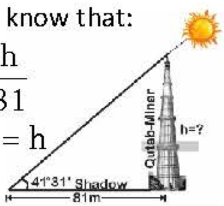
21. ~~**The shadow of Qutab Minar is 81m long when the measure of the angle of elevation of the sun is 41º31′. Find the height of the Qutab Minar.**~~

Sol. From figure, we know that:

$$\tan 41^\circ 31'=\frac{h}{81}$$

$$81\tan 41^\circ 31'=h$$

$$\boxed{h=71.70\text{m}}$$



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22. **In any triangle ABC if b = 25, c = 37, A = 65º, Find a.**

Sol. By using law of cosines:

$$a^2=b^2+c^2-2bc\cos 52^\circ$$

$$a^2=(25)^2+(37)^2-2(25)(37)\cos 65^\circ$$

$$a^2=625+1369-781.84$$

$$a^2=1212.16$$

$$\sqrt{a^2}=\sqrt{1212.16}\Rightarrow \boxed{a=34.82}$$

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23. **Find 'α' so that $\left|\alpha i+(\alpha+1)j+2k\right|=3$**

Sol. $\left|\alpha\hat{i}+(\alpha+1)\hat{j}+2\hat{k}\right|=3$

$$\sqrt{(\alpha)^2+(\alpha+1)^2+(2)^2}=3$$

$$\sqrt{\alpha^2+\alpha^2+2\alpha+1+4}=3$$

$$\sqrt{2\alpha^2+2\alpha+5}=3$$

Squaring both sides, we get:

$$2\alpha^2+2\alpha+5=9$$

$$2\alpha^2+2\alpha+5-9=0$$

$$2\alpha^2+2\alpha-4=0$$

$$2\alpha^2+4\alpha-2\alpha-4=0\ \{\text{By Factorization}\}$$

$$2\alpha(\alpha+2)-2(\alpha+2)=0$$

$$(\alpha+2)(2\alpha-2)=0$$

Either	OR
$\alpha+2=0$	$2\alpha-2=0$
$\boxed{\alpha=-2}$	$2\alpha=2\Rightarrow\boxed{\alpha=1}$

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24. **Define Vector product.**

Sol. The vector product of two vectors $\vec{a}$ & $\vec{b}$ is denoted by $\vec{a}\times\vec{b}$ and is defined as $\vec{a}\times\vec{b}=|\vec{a}||\vec{b}|\sin\theta\,\hat{n}$.

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25. ~~**Under what condition does the relation**~~ **$\vec{a}.\vec{b}=|\vec{a}||\vec{b}|$** ~~**hold for two vectors**~~ $\vec{a}$ & $\vec{b}$.

Sol. As $\vec{a}.\vec{b}=|\vec{a}||\vec{b}|$

$\Rightarrow |\vec{a}||\vec{b}|\cos\theta = |\vec{a}||\vec{b}| \because \left\{\begin{matrix}\text{By Definition}\\ \vec{a}.\vec{b}=|\vec{a}||\vec{b}|\cos\theta\end{matrix}\right\}$

$\Rightarrow \cos\theta = \frac{|\vec{a}||\vec{b}|}{|\vec{a}||\vec{b}|} \Rightarrow \cos\theta = 1$

$\Rightarrow \theta = \cos^{-1}(1) \Rightarrow \boxed{\theta = 0°}$

26. ~~**Express $\sqrt{2}\ \angle 45°$ in Rectangular form i.e., a + jb**~~

Sol. $\sqrt{2}\ \angle 45° = \sqrt{2}(\cos 45° + j\sin 45°)$

$= \sqrt{2}\left(\frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}\right) = \sqrt{2}\left(\frac{1+j}{\sqrt{2}}\right) = \boxed{1+j}$

27. ~~**Find the product of**~~

$\mathbf{Z_1 = 2\angle 15°,\ Z_2 = -1\angle 30°}$

Sol. $Z_1Z_2 = (2\angle 15°)(-1\angle 30°)$

$= (2)(-1)\angle(15° + 30°) = \boxed{-2\angle 45°}$

Section - II

Note: Attemp any three (3) questions $3 \times 8 = 24$

Q.2.(a) Solve the equation

$x^2 + (m - n)x - 2(m - n)^2 = 0$

by using quadratic formula.

Sol. See Q.3(v) of Ex # 1.1 (Page # 22)

(b) ~~The roots of the equation $px^2 + qx + q = 0$ are α and β~~

~~Prove that~~ $\sqrt{\frac{\alpha}{\beta}} + \sqrt{\frac{\beta}{\alpha}} + \sqrt{\frac{q}{p}} = 0$

Sol. See Q.6 of Ex # 1.3 (Page # 46)

Q.3.(a) Find the term involving q^8 in the expansion of $\left(\frac{p^2}{2} + 6q^2\right)^{12}$

Sol. See Q.7(v) of Ex # 2.1 (Page # 90)

(b) Find the coefficient of x^5 in the expansion of $\frac{(1+x)^2}{(1-x)^2}$

Sol. See Q.3(i) of Ex # 2.2 (Page # 103)

Q.4. **(a)** If $\sin\theta = \frac{2}{3}$, and the terminal side of the angle lies in the second quadrant, find the remaining trigonometric ratios of θ.

Sol. See Q.3 of Ex # 3.2 (Page # 122)

(b) Prove that

$$\frac{1+\tan^2\theta}{1+\cos^2\theta} = \frac{(1-\tan\theta)^2}{(1-\cot\theta)^2}$$

Sol. See Q.14 of Ex # 3.3 (Page # 136)

Q.5. **(a)** Prove that

$$\sin^4\theta = \frac{3}{8} - \frac{1}{2}\cos 2\theta + \frac{1}{8}\cos 4\theta$$

Sol. See Q.19 of Ex # 4.2 (Page # 179)

(b) ~~A television antenna is on the roof of a building. From a point on the ground 36m from the building, the angle of elevation of the top and the bottom of the antenna are 51° and 42° respectively. How tall is the antenna?~~

Sol. See Q.11 of Ex # 5.2 (Page # 218)

Q.6.(a) Using cross product, find the area of triangle whose vertices are (0, 0, 0), (1, 1, 1), (0, 0, 3).

Sol. See Q.21(i) of Ex # 6.2 (Page # 266)

(b) ~~Simplify $(\sqrt{3} + j)^7$ and express the result in a + jb form.~~

Sol. See Example Q.9(a) of Ch # 07

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