DAE / IIA - 2018

MATH - 113 APPLIED MATHEMATICS - I

PAPER 'B' PART - A (OBJECTIVE)

Time: 30 Minutes Marks: 15

Q.1: *Encircle the correct answer.*

1. If $\hat{i}$, $\hat{j}$ and $\hat{k}$ are orthogonal unit vector, then $\hat{j}\times\hat{i} = ?$

[a] $\hat{k}$ **[b]** $-\hat{k}$ **[c]** 1 **[d]** -1

2. $|\vec{a}\times\vec{b}|$ is area of the figure called:

[a] Triangle [b] Rectangle
[c] Parallelogram [d] Sector

3. If A(5, 2, -3), B(6, 1, 4), $\overline{AB}$

[a] $6i+j+4k$ **[b]** $5i+2j-3k$
[c] $i-j+7k$ **[d]** $5i+2j-k$

4. The order of the matrix $\begin{bmatrix}2\\3\\4\end{bmatrix}$ is:

[a] 2×1 **[b]** 2×2
[c] 3×1 **[d]** 1×3

5. ~~If two rows of a determinant are identical then its value is:~~

[a] 1 [b] 0 [c] -1 [d] None

6. Matrix $\left[c_{ij}\right]_{m\times n}$ is a rectangular if:

[a] $i\neq j$ **[b]** $i=j$
[c] $m=n$ **[d]** $m-n\neq 0$

7. Area of equilateral triangle with side 'x' is;

[a] $\frac{\sqrt{3}}{4}x^2$ **[b]** $\frac{\sqrt{3}}{2}x^2$
[c] $\frac{2}{\sqrt{3}}x$ **[d]** None of these

8. Area of parallelogram having 'a' and 'b' as adjacent sides and θ is the included angles is:

[a] $ab\cos\theta$ **[b]** $\frac{1}{2}ab\sin\theta$
[c] $ab\sin\theta$ **[d]** $a\sin\theta$

9. If a is the side of polygon of n side, then its area is:

[a] $\frac{na^2}{4}\cot\frac{180°}{n}$ **[b]** $\frac{na^2}{3}\frac{180°}{n}$
[c] $\frac{na^2}{2}\sin\frac{360°}{n}$ **[d]** None of these

10. Area of a sector of 60º in a circle of radius 6cm is:

[a] 6π [b] $6\pi^2$ [c] 36π [d] 3π

11. S, stands for, in calculating area of irregular figures.

[a] breadth of strip
[b] height of strip
[c] volume of strip [d] None of these

12. The cube is a right prism with

[a] square base [b] rectangular base
[c] Triangular base
[d] None of these

13. Total surface area of cylinder of radius r and height h is:

[a] $2\pi(h+r)$ [b] $\pi(h+r)$
[c] $2\pi r(h+r)$ [d] $\pi r(h+r)$

14. A solid figure whose base is a place polygon and sides are triangles that meet in a common vertex is known as:

[a] pyramid [b] cube
[c] frustum of a pyramid
[d] none of these

15. The surface area of a sphere of radius 'r' is:

[a] $4\pi r^3$ **[b]** $4\pi r^2$
[c] πr^2 **[d]** $\frac{4}{3}\pi r^3$

Answer Key

1	b	2	c	3	c	4	c	5	b
6	d	7	a	8	c	9	a	10	c
11	a	12	a	13	c	14	a	15	b

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DAE / IIA - 2018

MATH - 113 APPLIED MATHEMATICS - I

PAPER 'B' PART - B (SUBJECTIVE)

Time: 2:30 Hrs Marks: 60

Section - I

Q.1. Write short answers to any Eighteen (18) questions.

1. If $\cos\alpha,\ \cos\beta,\ \cos\gamma$ are direction cosines of a vector $\vec{r} = xi + yj + zk$, then show that $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

Sol. As, $\vec{r} = xi + yj + zk$

$\Rightarrow \quad |\vec{r}| = \sqrt{x^2 + y^2 + z^2}$

Direction cosines of $\vec{r}$ are:

$$\cos\alpha = \frac{x}{\sqrt{x^2+y^2+z^2}} \rightarrow (i)$$

$$\cos\beta = \frac{y}{\sqrt{x^2+y^2+z^2}} \rightarrow (ii)$$

$$\cos\gamma = \frac{z}{\sqrt{x^2+y^2+z^2}} \rightarrow (iii)$$

Adding eq.(i), eq.(ii) & eq.(iii)

L.H.S. $= \cos^2\alpha + \cos^2\beta + \cos^2\gamma$

$$= \left(\frac{x}{\sqrt{x^2+y^2+z^2}}\right)^2 + \left(\frac{y}{\sqrt{x^2+y^2+z^2}}\right)^2 + \left(\frac{z}{\sqrt{x^2+y^2+z^2}}\right)^2$$

$$= \frac{x^2}{x^2+y^2+z^2} + \frac{y^2}{x^2+y^2+z^2} + \frac{z^2}{x^2+y^2+z^2}$$

$$= \frac{x^2+y^2+z^2}{x^2+y^2+z^2} = 1 = \text{R.H.S.}$$ **Proved.**

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2. **Define scalar product of two vectors.**

Sol. The scalar product of two vectors $\vec{a}$ & $\vec{b}$ is denoted by $\vec{a} \cdot \vec{b}$ and defined as $\vec{a}.\vec{b} = |\vec{a}||\vec{b}|\cos\theta$.

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3. **For what value of λ, the vectors $2i - j + 2k$ & $3i + 2\lambda j$ are perpendicular.**

Sol. Let, $\vec{a} = 2i - j + 2k$ & $\vec{b} = 3i + 2\lambda j$

As given vectors are perpendicular.

So, $\quad \vec{a}.\vec{b} = 0$

$\Rightarrow \quad (2i - j + 2k).(3i + 2\lambda j) = 0$

$\Rightarrow \quad (2)(3) + (-1)(2\lambda) + (2)(0) = 0$

$\Rightarrow \quad 6 - 2\lambda + 0 = 0$

$\Rightarrow \quad -2\lambda = -6$

$\Rightarrow \quad \lambda = \frac{-6}{-2} \quad \Rightarrow \quad \boxed{\lambda = 3}$

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4. **If $\vec{a} = 3i - 2j + 5k$ and $\vec{b} = -2i - j + k$. Find $2\vec{a} - 3\vec{b}$ and also its unit vectors.**

Sol. $2\vec{a} - 3\vec{b}$

$= 2(3\hat{i} - 2\hat{j} + 5\hat{k}) - 3(-2\hat{i} - \hat{j} + \hat{k})$

$= 6\hat{i} - 4\hat{j} + 10\hat{k} + 6\hat{i} + 3\hat{j} - 3\hat{k}$

$= \boxed{12\hat{i} - \hat{j} + 7\hat{k}}$

$|2\vec{a} - 3\vec{b}| = \sqrt{(12)^2 + (-1)^2 + (7)^2}$

$= \sqrt{144 + 1 + 49} = \sqrt{194}$

Unit vector of $2\vec{a} - 3\vec{b} = \frac{2\vec{a} - 3\vec{b}}{|2\vec{a} - 3\vec{b}|}$

$= \boxed{\frac{12\hat{i} - \hat{j} + 7\hat{k}}{\sqrt{194}}}$

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5. **If $\vec{a} = 2i + 3j + 4k$ & $\vec{b} = i - j + k$ Find $|\vec{a} \times \vec{b}|$**

Sol. $\vec{a} \times \vec{b} = \begin{vmatrix} i & j & k \\ 2 & 3 & 4 \\ 1 & -1 & 1 \end{vmatrix}$

$$= i\begin{vmatrix} 3 & 4 \\ -1 & 1 \end{vmatrix} - j\begin{vmatrix} 2 & 4 \\ 1 & 1 \end{vmatrix} + k\begin{vmatrix} 2 & 3 \\ 1 & -1 \end{vmatrix}$$

$$= i(3+4) - j(2-4) + k(-2-3)$$

$$= i(7) - j(-2) + k(-5)$$

$$= 7i + 2j - 5k$$

$$|\bar{a} \times \bar{b}| = \sqrt{49+4+25} = \boxed{\sqrt{78}}$$

6. **Define rectangular matrix.**

Sol. A matrix in which no. of rows and no. of columns are not equal is called rectangular matrix.

7. ~~**Without expansion, show that :**~~

$$\begin{vmatrix} \mathbf{1} & \mathbf{2} & \mathbf{3} \\ \mathbf{4} & \mathbf{5} & \mathbf{6} \\ \mathbf{7} & \mathbf{8} & \mathbf{9} \end{vmatrix} = \mathbf{0}$$

Sol. $\text{L.H.S.} = \begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix}$

$$= \begin{vmatrix} 1+3 & 2 & 3 \\ 4+6 & 5 & 6 \\ 7+9 & 8 & 9 \end{vmatrix} \quad \text{By } C_1 + C_3$$

$$= \begin{vmatrix} 4 & 2 & 3 \\ 10 & 5 & 6 \\ 16 & 8 & 9 \end{vmatrix}$$

$$= 2\begin{vmatrix} 2 & 2 & 3 \\ 5 & 5 & 6 \\ 8 & 8 & 9 \end{vmatrix} \quad \text{Taking 2 common from } C_1$$

$$= 2(0) \qquad \because C_1 = C_2$$

$$= 0 = \text{R.H.S.}$$ **Proved.**

8. **Find 'k' if** $\mathbf{A} = \begin{bmatrix} \mathbf{4} & \mathbf{k} & \mathbf{3} \\ \mathbf{7} & \mathbf{3} & \mathbf{6} \\ \mathbf{2} & \mathbf{3} & \mathbf{1} \end{bmatrix}$ **is a singular matrix.**

Sol. As, A is singular so $|A| = 0$

$$\begin{vmatrix} 4 & k & 3 \\ 7 & 3 & 6 \\ 2 & 3 & 1 \end{vmatrix} = 0$$

$$4\begin{vmatrix} 3 & 6 \\ 3 & 1 \end{vmatrix} - k\begin{vmatrix} 7 & 6 \\ 2 & 1 \end{vmatrix} + 3\begin{vmatrix} 7 & 3 \\ 2 & 3 \end{vmatrix} = 0$$

$$4(3-18) - k(7-12) + 3(21-6) = 0$$

$$4(-15) - k(-5) + 3(15) = 0$$

$$-60 + 5k + 45 = 0$$

$$5k - 15 = 0$$

$$5k = 15 \Rightarrow k = \frac{15}{5} \Rightarrow \boxed{k = 3}$$

9. ~~**Without expansion, verify that:**~~

$$\begin{vmatrix} \boldsymbol{\alpha} & \boldsymbol{\beta+\gamma} & \mathbf{1} \\ \boldsymbol{\beta} & \boldsymbol{\gamma+\alpha} & \mathbf{1} \\ \boldsymbol{\gamma} & \boldsymbol{\alpha+\beta} & \mathbf{1} \end{vmatrix} = \mathbf{0}$$

Sol. $\text{L.H.S.} = \begin{vmatrix} \alpha & \beta+\gamma & 1 \\ \beta & \gamma+\alpha & 1 \\ \gamma & \alpha+\beta & 1 \end{vmatrix}$

$$= \begin{vmatrix} \alpha+\beta+\gamma & \beta+\gamma & 1 \\ \alpha+\beta+\gamma & \gamma+\alpha & 1 \\ \alpha+\beta+\gamma & \alpha+\beta & 1 \end{vmatrix} \quad \text{By } C_1 + C_2$$

$$= (\alpha+\beta+\gamma)\begin{vmatrix} 1 & \beta+\gamma & 1 \\ 1 & \gamma+\alpha & 1 \\ 1 & \alpha+\beta & 1 \end{vmatrix} \quad \text{By taking } (\alpha+\beta+\gamma) \text{ common from } C_1$$

$$= (\alpha+\beta+\gamma)(0) \qquad \because C_1 \equiv C_3$$

$$= 0 = \text{R.H.S.}$$ **Proved.**

10. **If** $\mathbf{A} = \begin{bmatrix} \mathbf{1} & \mathbf{2} \\ \mathbf{3} & \mathbf{4} \end{bmatrix}$, $\mathbf{B} = \begin{bmatrix} \mathbf{2} & \mathbf{3} \\ \mathbf{4} & \mathbf{5} \end{bmatrix}$**, Then find A + B**

Sol. $A + B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} = \boxed{\begin{bmatrix} 3 & 5 \\ 7 & 9 \end{bmatrix}}$

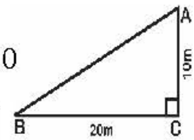
11. **Find the area of right triangle if base and altitude are 20m and 10m respectively.**

Sol. Given: base = 20m & Altitude = 10m

$$\text{Area} = \frac{1}{2} \times \text{base} \times \text{Altitude}$$

$$\text{Area} = \frac{1}{2} \times 20 \times 10$$

$$\text{Area} = \boxed{100\ \text{m}^2}$$



12. What is the side of the equilateral triangle whose area is $9\sqrt{3}$ sq.cm.

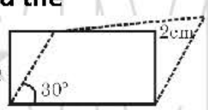
Sol. Let 'a' be length of each side of an equilateral triangle.

As, Area of equilateral triangle $= 9\sqrt{3}$ sq.cm

$$\Rightarrow \frac{\sqrt{3}}{4}a^2 = 9\sqrt{3} \quad \Rightarrow \quad a^2 = 9\sqrt{3}\left(\frac{4}{\sqrt{3}}\right)$$

$$\Rightarrow a^2 = 36 \Rightarrow \sqrt{a^2} = \sqrt{36} \Rightarrow \boxed{a = 6\,\text{cm}}$$

13. A wire rectangle of original size 2cm by 3 cm is passed to form a parallelogram. The included angle is reduced to 30º. Find the reduction in area.



Sol. A_1 = Area of Rectangle

$A_1 = \ell \times w = 2 \times 3 = 6$ sq.cm

A_2 = Area of Parallelogram

$A_2 = \ell w \sin\theta = (2)(3)\sin 30° = \boxed{3\text{sq.cm}}$

Reduction in Area $= A_R = A_1 - A_2$

$A_R = 6 - 3 = \boxed{3\,\text{sq.cm}}$

14. The sides of a cyclic quadrilateral are 75, 55, 140 and 40m, find its area.

Sol. Let, $a = 75\text{m}, b = 55\text{m}, c = 140\text{m}, d = 40\text{m}$

$$s = \frac{a+b+c+d}{2} = \frac{75+55+140+40}{2} = 155\text{m}$$

$$\text{Area} = \sqrt{(s-a)(s-b)(s-c)(s-d)}$$

$$= \sqrt{(155-75)(155-55)(155-140)(155-40)}$$

$$= \sqrt{80(100)(15)(115)} = \sqrt{13800000} = \boxed{3714.8\,\text{sq.m}}$$

15. Define circumscribed polygon.

Sol. If a polygon is drawn outside a circle, so that every side of the polygon touches the circle, then this polygon is called circumscribed polygon.

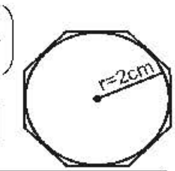
16. A regular octagon circumscribes a circle of 2cm radius. Find the area of the octagon.

Sol. Here n = 8, r = 2cm & A = ?

$$\text{Area} = nr^2 \tan\left(\frac{180°}{n}\right)$$

$$A = 8(2)^2 \tan\left(\frac{180°}{8}\right)$$

$$A = 32\tan 22.5° \Rightarrow \boxed{A = 13.25\,\text{cm}^2}$$



17. Define diameter of circle.

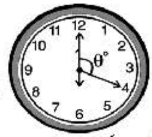
Sol. A chord passing through the center of the circle is called a diameter.

18. The minute hand of a clock is 12cm long. Find the area which is described on the clock face between 6 A.M to 6.20 A.M.

Sol. Here: radius = r = 12cm

$$\theta = \frac{360°}{12} \times 4 = 120°$$

$$\theta = 120 \times \frac{\pi}{180} = \frac{2\pi}{3}\text{rad}$$



$$\text{Area of sector} = \frac{1}{2}r^2\theta = \frac{1}{2}(12)^2\left(\frac{2\pi}{3}\right)$$

$$\text{Area} = \frac{144\pi}{3} = \boxed{150.7\text{cm}^2}$$

19. The inside measurement of a room are 8.5m, 6.4 and 4.5m height. How many men should sleep in the room, if each man is allowed 13.6 cu. m of air?

Sol. Let, ℓ = Length of room = 8.5m

b = Breadth of room = 6.4m

h = Height of room = 4.5m

Volume of room $= \ell bh$

$= 8.5 \times 6.4 \times 4.5 = 244.80\text{m}^3$

Air allowed for one men = $13.6\,\text{m}^3$

No. of men that can sleep in room

$= \frac{244.80}{13.6} = \boxed{18\,\text{men}}$

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20. An open rectangular tank of length 16cm, width 9cm and height 13cm contains. Water up to a height of 8cm. calculate:

[a] volume in liters

[b] total surface area of the tank.

Sol. Here: ℓ = 16cm, b = 9cm,

$h =$ Heigth of tank $= 13\text{cm}$ &

$h_1 =$ Heigth of water $= 8\text{cm}$

[a] $\text{Volume} = \ell b h_1 = 16 \times 9 \times 8$

$V = 1152\text{cm}^3 = \boxed{1.152 \text{ liters}}$

[b] L.S.A. = Perimeter of base × Height

$\text{L.S.A.} = 2(\ell + b) \times h$

$\text{L.S.A.} = 2(16 + 9) \times 13$

$\text{L.S.A.} = 50 \times 13 = \boxed{650\text{ cm}^2}$

Area of base of Tank $= \ell \times w$

$= 9 \times 16 = 144\text{ cm}^2$

T.S.A. = Area of base + L.S.A.

$\text{T.S.A.} = 144 + 650 = \boxed{794\text{ cm}^2}$

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21. Write the formula for Area of irregular figure by Trapezoidal Rule.

Sol. Area

$= S\left[\frac{\text{sum of first \& last ordinates}}{2} + (\text{sum of remaining ordinates})\right]$

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22. Define Hollow Circular Cylinder.

Sol. The space between two concentric cylinders is called Hollow Circular Cylinder.

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23. Write formula of volume of elliptic cylinder and total area.

Sol. **(i)** $\text{Volume} = \pi abh \text{ cu.unit}$

(ii) $\text{T.S.A.} = \pi(a + b)h + 2\pi ab \text{ sq.unit}$

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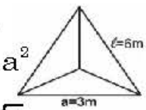
24. Find the whole surface of a pyramid whose base is an equilateral triangle of side 3m and its slant height is 6m.

Sol. Here: T.S.A=? a = 3m, ℓ = 6m

Whole surface area of the pyramid = Lateral surface area + Area of base

$\text{T.S.A.} = \frac{1}{2}(3a)\ell + \frac{\sqrt{3}}{4}a^2$



$\text{T.S.A.} = \frac{1}{2}(3 \times 3)6 + \frac{\sqrt{3}}{4}(3)^2$

$\text{T.S.A.} = 27 + 3.897 = \boxed{30.897\text{m}^2}$

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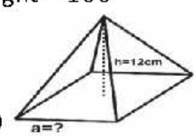
25. The height of pyramid with square base is 12cm, and its volume is 100cu.cm. Find length of side of square base.

Sol. Here: $h = 12\text{cm}, V = 100\text{cm}^3$ & $a = ?$

As, Volume of Pyramid $= 100\text{cm}^3$

$\frac{1}{3} \times$ Area of base × Height $= 100$

$\frac{1}{3} \times a^2 \times 12 = 100$



$4a^2 = 100 \Rightarrow a^2 = \frac{100}{4}$

$a^2 = 25 \Rightarrow \boxed{a = 5\text{cm}}$

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26. A right triangle of which the sides are 3cm and 4cm length is made to turn around the longer side. Find the volume of the cone thus formed.

Sol. Here: r = 3cm, h = 4cm & V=?

$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(3)^2(4) = \boxed{37.70\text{cm}^3}$

27. **Write the formula for surface area of segment area of segment of a sphere.**

Sol. Surface area of segment of sphere $= \boxed{2\pi rh \text{ sq.unit}}$

Section - II

Note : Attemp any three (3) questions $3 \times 8 = 24$

Q.2.(a) **Find the length of the sides of a triangle, whose vertices are A (2, 4, -1), B(4, 5, 1), C(3, 6, -3) and show that the triangle is right angled.**

Sol. See Q.5 of Ex # 8.1 (Page # 372)

(b) **Find sine of the angle and the unit vector perpendicular to $\vec{a} = i + j + k$ and $\vec{b} = 2i + 3j - k$**

Sol. See Q.19(i) of Ex # 8.2 (Page # 388)

Q.3.(a) **Show that:**

$$\begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ +\sin\theta & 0 & \cos\theta \end{bmatrix} \begin{bmatrix} \cos\theta & 0 & +\sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Sol. See Q.8(ii) of Ex # 9.1 (Page # 412)

(b) **Find the inverse, if it exists, of the matrix** $A = \begin{bmatrix} 0 & -2 & -3 \\ 1 & 3 & 3 \\ -1 & -2 & -2 \end{bmatrix}$

Sol. See Q.4(ii) of Ex # 9.3 (Page # 439)

Q.4.(a) **The sides of a triangular lawn are proportional to the numbers 5, 12 and 13. The cost of fencing it at the rate of Rs.2 per meter is Rs. 120. Find the sides. Also find the cost of turfing the lawn at 25 paisa per square meter.**

Sol. See Q.6 of Ex # 10 (Page # 465)

(b) **The inner diameter of a circular building is 54m and the base of the will occupies a space of 352sq.m. Find the thickness of the wall.**

Sol. See Q.8 of Ex # 13 (Page # 499)

Q.5.(a) **Find the area of an irregular plane figure whose ordinates are 20, 23, 28, 32, 34, 37 and 40m respectively and the width of each trip is 7 meter.**

Sol. See Q.4 of Ex # 14 (Page # 508)

(b) **Find the cost of painting the outside of a rectangular box whose length is 64cm, breadth is 45cm and height is 51cm, at the rate of 37 paisa per sq.m.**

Sol. See Q.2 of Ex # 15 (Page # 518)

Q.6.(a) **Find the cost of the canvas for 50 conical tents, the height of each being 45cm and the diameter of the base 36cm at the rate of Rs.9.40sq.m.**

Sol. See Q.4 of Ex # 18[A] (Page # 566)

(b) ~~The core for a cast iron piece has the shape of a spherical segment of two bases. The diameters of the upper and lower bases are 2ft. and 6ft. respectively, and the distance between the bases is 2ft. If the average weight of a cu.ft. of crore is 100 Lbs. Find the weight of the core.~~

Sol. See Q.6 of Ex # 19[B] (Page # 596)

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