DAE / IIA - 2018

MATH - 113 APPLIED MATHEMATICS - I

PAPER 'A' PART - A (OBJECTIVE)

Time : 30 Minutes Marks : 15

Q.1: *Encircle the correct answer.*

1. To make $x^2 - 5x$ a complete square we should add:

[a] 25 [b] $\frac{25}{4}$ [c] $\frac{25}{9}$ [d] $\frac{25}{16}$

2. If ± 3 are the roots of the equation, then the equation is:

[a] $x^2 - 3 = 0$ [b] $x^2 - 9 = 0$
[c] $x^2 + 3 = 0$ [d] $x^2 + 9 = 0$

3. The nth term of an A.P whose first term is 'a' and common difference is 'd' is:

[a] $2a + (n+1)d$ [b] $n + (n+1)d$
[c] $a + (n-1)d$ [d] $a + (d-1)n$

4. The nth term of a G.P, a, ar, ar², ... Is:

[a] ar^2 [b] ar^{n+1} [c] $\frac{1}{a}r^{n-1}$ [d] ar^{n-1}

5. The sum of a terms of a geometric series $a + ar + ar^2 + \ldots; |r| < 1$ is:

[a] $\frac{ar^{n-1}}{r-1}$ [b] $\frac{a(1-r^n)}{1-r}$
[c] $\frac{ar^{n+1}}{1-r}$ [d] $\frac{a(r^n-1)}{1-r}$

6. The second last term in the expansion of $(a + b)^7$ is:

[a] $7a^6b$ [b] $7ab^5$
[c] $7b^7$ [d] 15

7. The value of $\binom{2n}{n}$ is:

[a] $\frac{2n}{n!n!}$ [b] $\frac{(2n)!}{n!n!}$
[c] $\frac{(2n)!}{n!}$ [d] $\frac{(2n)!}{n(n-1!)}$

8. Partial fraction of $\frac{2x+3}{(x-2)(x+5)}$ is:

[a] $\frac{2}{x-2}+\frac{1}{x+5}$ [b] $\frac{3}{x-2}+\frac{1}{x+5}$
[c] $\frac{2}{x-2}+\frac{3}{x+5}$ [d] $\frac{1}{x-2}+\frac{1}{x+5}$

9. The degree measure of one radian is approximately equal to:

[a] 57.3º [b] 57.2º
[c] 57.1º [d] 57.0º

10. An angle subtended at the center of a circle by an arc equal to the radius of the circle is called:

[a] Right angle [b] Degree
[c] Radian [d] Acute angle

11. $\sec^2\theta + \operatorname{cosec}^2\theta$ is equal to:

[a] $\sec^2\theta\operatorname{cosec}^2\theta$ [b] $\sin\theta\cos\theta$
[c] $2\sec^2\theta$ [d] $2\operatorname{cosec}^2\theta$

12. $\cos\left(\frac{\pi}{2}+\theta\right)$ is equal to:

[a] $\cos\theta$ [b] $-\cos\theta$
[c] $\sin\theta$ [d] $-\sin\theta$

13. $\cos A - \cos B$ is equal to:

[a] $2\cos\frac{A+B}{2}\cos\frac{A-B}{2}$
[b] $-2\sin\frac{A+B}{2}\sin\frac{A-B}{2}$
[c] $2\sin\frac{A+B}{2}\cos\frac{A-B}{2}$
[d] $2\cos\frac{A+B}{2}\sin\frac{A-B}{2}$

14. When angle of elevation is viewed by an observer, the object is:

[a] above [b] below
[c] at same level [d] none of these

15. In a right triangle if one angle is 45º, then the other will be:

[a] 45° [b] 50° [c] 60° [d] 75°

Answer Key

1	b	2	b	3	c	4	d	5	b
6	b	7	b	8	d	9	a	10	c
11	a	12	d	13	b	14	a	15	a

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DAE / IIA - 2018

MATH - 113 APPLIED MATHEMATICS - I

PAPER 'A' PART - B (SUBJECTIVE)

Time: 2:30 Hrs   Marks: 60

Section - I

Q.1. Write short answers to any Eighteen (18) questions.

1. Solve the Quadratic equation $6x^2 - 5x = 4$ by factorization.

Sol. $6x^2 - 5x = 4 \Rightarrow 6x^2 - 5x - 4 = 0$

$6x^2 - 8x + 3x - 4 = 0$

$2x(3x - 4) + 1(3x - 4) = 0$

$(3x - 4)(2x + 1) = 0$

Either | **OR**

$3x - 4 = 0$ | $2x + 1 = 0$

$3x = 4 \Rightarrow x = \frac{4}{3}$ | $2x = -1 \Rightarrow x = -\frac{1}{2}$

$S.S. = \left\{-\frac{1}{2}, \frac{4}{3}\right\}$

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2. Discuss the nature of the roots of the equation $2x^2 - 7x + 3 = 0$

Sol. Here: $a = 2,\ b = -7,\ c = 3$

Disc. $= b^2 - 4ac$

$= (-7)^2 - 4(2)(3) = 49 - 24 = 25$

$\therefore$ The roots are **Rational unequal and Real.**

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3. ~~If α, β are the roots of the equation~~ $x^2 - 4x + 2 = 0$ ~~find the equation whose roots are:~~ $-\alpha,\ -\beta$

Sol. As α, β are the roots of the given equation. $x^2 - 4x + 2 = 0$

Here: $a = 1,\ b = -4,\ c = 2$

Sum of Roots | Products of Roots

$\alpha + \beta = -\frac{b}{a}$ | $\alpha\beta = \frac{c}{a}$

$= -\left(-\frac{4}{1}\right) = 4$ | $= \frac{2}{1} = 2$

As, $-\alpha, -\beta$ are the roots of require equation.

$S = -\alpha + (-\beta)$ | $P = (-\alpha)(-\beta)$

$S = -\alpha - \beta$ | $P = \alpha\beta = 2$

$S = -(\alpha + \beta) = -4$

$x^2 - Sx + P = 0$

$x^2 - (-4)x + 2 = 0 \Rightarrow \boxed{x^2 + 4x + 2 = 0}$

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4. Define finite Sequence.

Sol. A sequence is called finite sequence, if it has finite terms.

Example: 2, 4, 6, 8, . . . ,50

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5. Find the 7th term of A.P., in which the first term is 7 and the common difference is -3.

Sol. Here: $a_7 = ?\ ,\ a_1 = 7\ \&\ d = -3$

$a_7 = a + 6d \Rightarrow a_7 = 7 + 6(-3)$

$a_7 = 7 - 18 \Rightarrow \boxed{a_7 = -11}$

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6. ~~Find the sum of the series 3 + 11 + 19 + ... to 16 terms.~~

Sol. Here: $a_1 = 3,\ d = 11 - 3 = 8\ \&\ n = 16$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{16} = \frac{16}{2}[2(3) + (16-1)(8)]$

$S_{16} = 8[6 + 120] = 8(126) = \boxed{1008}$

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7. Write down the geometric sequence in which the 1st term is 2 and second term is -6 and n = 5.

Sol. Here: $a_1 = 2,\ a_2 = -6\ \&\ n = 5.$

$r = \frac{a_2}{a_1} = \frac{-6}{2} = -3$

$a_3 = ar^2 = 2(-3)^2 = 2(9) = \boxed{18}$

$a_4 = ar^3 = 2(-3)^3 = 2(-27) = \boxed{-54}$

$a_5 = ar^4 = 2(-3)^4 = 2(81) = \boxed{162}$

Hence $\boxed{2, -6, 18, -54, 162, \ldots}$ is required G.P.

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8. Find the G.M. between $\frac{4}{3}$ and 243.

Sol. Here: $a = 4/3$, $b = 243$

$$G = \pm\sqrt{ab} = \pm\sqrt{\frac{4}{3}\times 243}$$

$$G = \pm\sqrt{324} \Rightarrow \boxed{G = \pm 18}$$

9. ~~Find the sum of infinite geometric series in which~~ $a = 128$ & $r = -1/2$.

Sol. $$S_\infty = \frac{a}{1-r} = \frac{128}{1-\left(-\frac{1}{2}\right)} = \frac{128}{1+\frac{1}{2}}$$

$$S_\infty = \frac{128}{\frac{2+1}{2}} = \frac{128}{3/2} = 128\times\frac{2}{3} = \boxed{\frac{256}{3}}$$

10. **Expand by Bi-nomial theorem** $\left(\frac{x}{2}-\frac{2}{y}\right)^4$

Sol. $\left(\frac{x}{2}-\frac{2}{y}\right)^4$

$$= \binom{4}{0}\left(\frac{x}{2}\right)^4\left(\frac{2}{y}\right)^0 - \binom{4}{1}\left(\frac{x}{2}\right)^3\left(\frac{2}{y}\right)^1 + \binom{4}{2}\left(\frac{x}{2}\right)^2\left(\frac{2}{y}\right)^2 - \binom{4}{3}\left(\frac{x}{2}\right)^1\left(\frac{2}{y}\right)^3 + \binom{4}{4}\left(\frac{x}{2}\right)^0\left(\frac{2}{y}\right)^4$$

$$= (1)\left(\frac{x^4}{16}\right)(1) - 4\left(\frac{x^3}{8}\right)\left(\frac{2}{y}\right) + 6\left(\frac{x^2}{4}\right)\left(\frac{4}{y^2}\right) - 4\left(\frac{x}{2}\right)\left(\frac{8}{y^3}\right) + (1)(1)\left(\frac{16}{y^4}\right)$$

$$= \boxed{\frac{x^4}{16} - \frac{x^3}{y} + 6\frac{x^2}{y^2} - 16\frac{x}{y^3} + \frac{16}{y^4}}$$

11. **Find the 7th term in the expansion of** $\left(x-\frac{1}{x}\right)^9$

Sol. Here: $a = x,\ b = -1/x$, $n = 9$ & $r = 6$

Using general term formula:

$$T_{r+1} = \binom{n}{r}a^{n-r}b^r \Rightarrow T_{6+1} = \binom{9}{6}(x)^{9-6}\left(-\frac{1}{x}\right)^6$$

$$T_7 = 84x^3\left(\frac{1}{x^6}\right) \Rightarrow \boxed{T_7 = \frac{84}{x^3}}$$

12. ~~Expand~~ $\frac{1}{\sqrt{1+x}}$ ~~to three terms.~~

Sol. $$\frac{1}{\sqrt{1+x}} = (1+x)^{-1/2}$$

$$= 1 + \left(-\frac{1}{2}\right)(x) + \frac{\left(-\frac{1}{2}\right)\left(-\frac{1}{2}-1\right)}{2!}(x)^2 + \ldots$$

$$= 1 - \frac{x}{2} + \frac{1}{2}\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)x^2 + \ldots = \boxed{1 - \frac{x}{2} + \frac{3}{8}x^2 + \ldots}$$

13. **Which will be the middle term/terms in the expansion of** $\left(x+\frac{3}{x}\right)^{15}$.

Sol. As $n = 15$ (Odd), so

$$\text{Middle terms} = \left(\frac{n+1}{2}\right)^{th} + \left(\frac{n+3}{2}\right)^{th} \text{ terms.}$$

$$\text{Middle terms} = \left(\frac{15+1}{2}\right)^{th} + \left(\frac{15+3}{2}\right)^{th} \text{ terms.}$$

$$\text{Middle terms} = 8^{th} + 9^{th} \text{ terms.}$$

$\boxed{\text{Hence } T_8 \text{ \& } T_9 \text{ are two middle terms.}}$

14. **Resolve** $\frac{1}{x^2-x}$ **into partial fractions.**

Sol. $$\frac{1}{x^2-x} = \frac{1}{x(x-1)}$$

$$\frac{1}{x(x-1)} = \frac{A}{x} + \frac{B}{x-1} \rightarrow (i)$$

$$1 = A(x-1) + Bx \rightarrow (ii)$$

Put $x = 0$ in eq.(ii)

$$1 = A(0-1) + B(0)$$

$$1 = -A \Rightarrow \boxed{A = -1}$$

Put $x = 1$ in eq.(ii)

$$1 = A(1-1) + B(1)$$

$$1 = A(0) + B \Rightarrow \boxed{B = 1}$$

Put values of A, & B

in eq. (i), we get: $\boxed{-\frac{1}{x} + \frac{1}{x-1}}$

15. Form of partial fractions of $\frac{1}{(x^3-1)(x^2+1)}$ is ______.

Sol.

$$\frac{1}{(x^3-1)(x^2+1)}=\frac{1}{(x-1)(x^2+x+1)(x^2+1)}$$

$$=\boxed{\frac{A}{(x-1)}+\frac{Bx+C}{(x^2+x+1)}+\frac{Dx+E}{(x^2+1)}}$$

16. ~~Convert~~ $\frac{2\pi}{3}$ ~~radians into degree measure.~~

Sol. $\frac{2\pi}{3}\text{rad}=\frac{2\pi}{3}\times\frac{180}{\pi}=\boxed{120^\circ}$

17. Find 'x' if

$$\tan^2 45^\circ-\cos^2 60^\circ=x\sin 45^\circ\cos 45^\circ\tan 60^\circ$$

Sol. $\tan^2 45^\circ-\cos^2 60^\circ=x\sin 45^\circ\cos 45^\circ\tan 60^\circ$

$$(1)^2-\left(\frac{1}{2}\right)^2=x\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)\left(\sqrt{3}\right)$$

$$1-\frac{1}{4}=x\left(\frac{2\sqrt{3}}{4}\right)\Rightarrow\frac{4-1}{4}\times\frac{4}{2\sqrt{3}}=x$$

$$\frac{3}{2\sqrt{3}}=x\quad\Rightarrow\quad\boxed{x=\frac{\sqrt{3}}{2}}$$

18. Prove that:

$$\cos 30^\circ\cos 60^\circ-\sin 30^\circ\sin 60^\circ=0$$

Sol. L.H.S. $=\cos 30^\circ\cos 60^\circ-\sin 30^\circ\sin 60^\circ$

$$=\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right)-\left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$$

$=\frac{\sqrt{3}}{4}-\frac{\sqrt{3}}{4}=0=$ R.H.S. **Proved.**

19. Prove that:

$$1-2\sin^2\theta=2\cos^2\theta-1$$

Sol. L.H.S. $=1-2\sin^2\theta$

$$=1-2\left(1-\cos^2\theta\right)\quad\because\left\{\sin^2\theta=1-\cos^2\theta\right\}$$

$$=1-2+2\cos^2\theta$$

$=2\cos^2\theta-1=$ R.H.S. **Proved.**

20. Prove that: $\cos\left(\frac{\pi}{2}-\theta\right)=\sin\theta$

Sol. L.H.S. $=\cos\left(\frac{\pi}{2}-\theta\right)$

$$=\cos(90^\circ-\theta)\quad\because\left\{\frac{\pi}{2}\times\frac{180^\circ}{\pi}=90^\circ\right\}$$

$$=\cos 90^\circ\cos\theta+\sin 90^\circ\sin\theta$$

$$=(0)\cos\theta+(1)\sin\theta\quad\because\left\{\begin{array}{c}\text{Using calculator}\\ \cos 90^\circ=0\ \&\ \sin 90^\circ=1\end{array}\right\}$$

$=0+\sin\theta=\sin\theta=$ R.H.S. **Proved.**

21. Show that:

$$\sin\left(\theta+\frac{\pi}{6}\right)+\cos\left(\theta+\frac{\pi}{3}\right)=\cos\theta$$

Sol. L.H.S. $=\sin\left(\theta+\frac{\pi}{6}\right)+\cos\left(\theta+\frac{\pi}{3}\right)$

$$=\sin(\theta+30^\circ)+\cos(\theta+60^\circ)$$

$$=\sin\theta\cos 30^\circ+\cos\theta\sin 30^\circ+\cos\theta\cos 60^\circ-\sin\theta\sin 60^\circ$$

$$=\sin\theta\left(\frac{\sqrt{3}}{2}\right)+\cos\theta\left(\frac{1}{2}\right)+\cos\theta\left(\frac{1}{2}\right)-\sin\theta\left(\frac{\sqrt{3}}{2}\right)$$

$=\frac{\cos\theta}{2}+\frac{\cos\theta}{2}=2.\frac{\cos\theta}{2}=\cos\theta=$ R.H.S. **Proved.**

22. Express $\cos(a+b)\cos(a-b)-\sin(a+b)\sin(a-b)$ as single term.

Sol.

$$\cos(a+b)\cos(a-b)-\sin(a+b)\sin(a-b)$$

$$=\cos(a+b+a-b)$$

$$=\boxed{\cos 2a}$$

23. ~~Prove that:~~ $\cos\alpha=\cos^2\frac{\alpha}{2}-\sin^2\frac{\alpha}{2}$

Sol. L.H.S. $=\cos\alpha=\cos\left(\frac{\alpha}{2}+\frac{\alpha}{2}\right)$

$$=\cos\frac{\alpha}{2}\cos\frac{\alpha}{2}-\sin\frac{\alpha}{2}\sin\frac{\alpha}{2}$$

$=\cos^2\frac{\alpha}{2}-\sin^2\frac{\alpha}{2}=$ R.H.S. **Proved.**

24. ~~Given that, γ = 90º, α = 35º, a = 5, find angle β.~~

Sol. We know that in any triangle:

$$\alpha+\beta+\gamma=180^\circ$$

$$\beta=180^\circ-\alpha-\gamma$$

$$\beta=180^\circ-35^\circ-90^\circ \Rightarrow \boxed{\beta=55^\circ}$$

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25. **Define angle of elevation.**

Sol. **Angle of Elevation:**

If the line of sight is upward from the horizontal, the angle is called angle of Elevation.

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26. **In any triangle ABC in which a =5, c = 6, α = 45º, find γ.**

Sol. By using law of sines:

$$\frac{a}{\sin\alpha}=\frac{b}{\sin\beta}=\frac{c}{\sin\gamma}$$

$$\text{we take}: \frac{a}{\sin\alpha}=\frac{c}{\sin\gamma}$$

$$5\sin\gamma=6\sin 45^\circ$$

$$\sin\gamma=\frac{6\sin 45^\circ}{5}=\boxed{0.8485}$$

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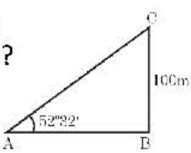
27. **Find the distance of man from the foot of tower 100m high if the angle of elevation of its top as observed by the man is $52^\circ 30'$.**

Sol. Let, A be the position of man and B be the foot of tower BC Height of tower = BC = 100m.

in right ΔABC, AB = ?



$$\tan 52^\circ 32'=\frac{\overline{BC}}{\overline{AB}}$$

$$\overline{AB}=\frac{\overline{BC}}{\tan 52^\circ 32'}=\frac{100}{1.3048}=\boxed{76.64\text{m}}$$

Section - II

Note : Attemp any three (3) questions $3\times 8=24$

Q.2.(a) Solve the equation.

$$\frac{a}{ax-1}+\frac{b}{bx-1}=a+b \text{ by}$$

factorization.

Sol. See Q.1(ix) of Ex # 1.1 (Page # 10)

(b) ~~The roots of the equation px2 + qx + q = 0 are α and β, prove that :~~

$$\sqrt{\frac{\alpha}{\beta}}+\sqrt{\frac{\beta}{\alpha}}+\sqrt{\frac{q}{p}}=0$$

Sol. See Q.6 of Ex # 1.3 (Page # 46)

Q.3.(a) ~~If S_1, S_2, S_3 be sums to n, 2n, 3n terms of an arithmetic progression, Show that $S_3 = 3(S_2 - S_1)$~~

Sol. See Q.9 of Ex # 2.3 (Page # 95)

(b) **The A.M of two positive integral numbers exceeds their (positive) G.M by 2 and their sum is 20. Find the numbers.**

Sol. See Q.8 of Ex # 2.5 (Page # 115)

Q.4.(a) Find the middle term in the expansion of $\left(3x^2+\frac{1}{2x}\right)^{10}$.

Sol. See Q.6(i) of Ex # 3.1 (Page # 150)

(b) Resolve $\frac{x^2}{(x-1)^3(x+2)}$ into partial fractions.

Sol. See Q.9 of Ex # 4.2 (Page # 201)

Q.5.(a) ~~A railway train is traveling on a curve of half a kilometer radius at the rate of 20 km per hour, Through what angle had it turned in 10 seconds.~~

Sol. See example # 06 of Ch # 05

(b) If m = tanθ + sinθ and n = tanθ - sinθ than prove that $m^2 - n^2 = 4\sqrt{mn}$

Sol. See Q.22 of Ex # 5.3 (Page # 262)

Q.6.(a) Prove that:

$$\tan 3\theta=\frac{3\tan\theta-\tan^3\theta}{1-3\tan^2\theta}$$

Sol. See Triple angles proof (Page # 294)

(b) **On walking 300 meters towards a tower in a horizontal line through its base, the measure of the angle of elevation of this top changes from 30º to 60º. Find the height of the tower.**

Sol. See Q.4 of Ex # 7.2 (Page # 336)

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