DAE / IIA - 2018

MATH - 123 APPLIED MATHEMATICS - I

PAPER 'B' PART - A (OBJECTIVE)

Time : 30Minutes Marks : 15

Q.1: *Encircle the correct answer.*

1. Sum of $-3+5i$ and $4-7i$ is:

[a] $1-2i$ [b] $-1-2i$
[c] $1-12i$ [d] $-7+12i$

2. If $z=a+bi$ then $z+\overline{z}$ is equal to:

[a] $2a$ [b] $2b$
[c] 0 [d] $2a+2bi$

5. $i(1-2i)$ is equal to:

[a] $1+2i$ [b] $2i+i^2$
[c] 3 [d] $3i$

4. The Fractions $\frac{2x+5}{x^2+5x+6}$ is known as:

[a] Proper [b] Improper
[c] Neither proper nor improper
[d] None of these

5. The equivalent partial fractions of $\frac{x+11}{(x+1)(x-3)^2}$ is:

[a] $\frac{A}{x+1}+\frac{B}{(x-3)^2}$ [b] $\frac{A}{x+1}+\frac{B}{x-3}$

[c] $\frac{A}{x+1}+\frac{B}{x-3}+\frac{C}{(x-3)^2}$

[d] $\frac{A}{x+1}+\frac{Bx+C}{(x-3)^2}$

6. Conversion of 9 to binary system is:

[a] $(1001)_2$ [b] $(101)_2$
[c] $(11)_2$ [d] None of these

7. ~~In Boolean Algebra~~ $\overline{X+Y}$ ~~is equal to:~~

[a] $\overline{X}+\overline{Y}$ [b] $\overline{X}.\overline{Y}$
[c] XY [d] $X+Y$

8. Symbol  is used for:

[a] NOT gate [b] NOR gate
[c] OR gate [d] NAND gate

9. $y=2$ is a line parallel to:

[a] $x-\text{axis}$ [b] $y-\text{axis}$
[c] $y=x$ [d] $x=3$

10. Equation of the line is slope intercept form is:

[a] $\frac{x}{y}+\frac{y}{b}=1$ [b] $y=mx+c$

[c] $y-y_1=m(x-x_1)$

[d] None of these

11. Distance between (4,3) and (7,5) is:

[a] 25 [b] $\sqrt{13}$
[c] 5 [d] None of these

12. ~~Ratio formula for y – coordinate is:~~

[a] $\frac{x_1r_2+x_2r_1}{r_1+r_2}$ [b] $\frac{y_1r_2+y_2r_1}{r_1+r_2}$

[c] $\frac{x-y}{2}$ [d] None of these

13. Given three points are collinear if their slopes are:

[a] Equal [b] Unequal
[c] $m_1m_2=-1$ [d] None of these

14. Straight line from center to the circumference is:

[a] Circle [b] Radius
[c] Diameter [d] None of these

15. Radius of the circle $(x-1)^2+(y-2)^2=16$ is:

[a] 2 [b] 1
[c] 4 [d] None of these

Answer Key

1	a	2	a	3	a	4	a	5	c
6	a	7	b	8	a	9	a	10	b
11	b	12	b	13	a	14	b	15	c

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DAE / IIA - 2018

MATH-123 APPLIED MATHEMATICS-I

PAPER 'B' PART-B (SUBJECTIVE)

Time: 2:30 Hrs Marks: 60

Section-I

Q.1. Write short answers to any Eighteen (18) questions.

1. Write the conjugate and modulus of $-\frac{2}{3}-\frac{4}{9}i$.

Sol. Let $z=-\frac{2}{3}-\frac{4}{9}i$

Conjugate $=\bar{z}=\overline{-\frac{2}{3}-\frac{4}{9}i}=\boxed{-\frac{2}{3}+\frac{4}{9}i}$

As, $a=-\frac{2}{3}$ & $b=-\frac{4}{9}$

Modulus $=|z|=\sqrt{a^2+b^2}$

$|z|=\sqrt{\left(-\frac{2}{3}\right)^2+\left(-\frac{4}{9}\right)^2}=\sqrt{\frac{4}{9}+\frac{16}{81}}$

$|z|=\sqrt{\frac{36+16}{81}}=\sqrt{\frac{52}{81}}=\boxed{\frac{\sqrt{52}}{9}}$

2. Prove that if $Z=\overline{Z}$ then $\overline{Z}$ is real.

Sol. Let $Z=a+bi\rightarrow(i)$

then $\overline{Z}=a-bi\rightarrow(ii)$

Given that: $Z=\overline{Z}$

$\Rightarrow a+bi=a-bi$ {By using eq.(i) & eq.(ii)}

$\Rightarrow a+bi-a+bi=0$

$\Rightarrow 2bi=0 \Rightarrow b=0 \quad \because 2i\neq 0$

Put $b=0$ in eq.(ii)

$\overline{Z}=a-0i=a\in\mathbb{R}$

Hence $\overline{Z}$ is real. **Proved.**

3. Factorize $2x^2+5y^2$

Sol. $2x^2+5y^2=2x^2-5y^2i^2$

$=\left(\sqrt{2}x\right)^2-\left(\sqrt{5}yi\right)^2$

$=\boxed{\left(\sqrt{2}x-\sqrt{5}yi\right)\left(\sqrt{2}x+\sqrt{5}yi\right)}$

4. ~~Express the complex number $3-\sqrt{3}i$ in polar (trigonometric) form.~~

Sol. Let, $z=3-\sqrt{3}i$

Here: $a=3$ & $b=-\sqrt{3}$

$r=|z|=\sqrt{a^2+b^2}$

$r=\sqrt{(3)^2+\left(-\sqrt{3}\right)^2}$

$r=\sqrt{9+3}=\sqrt{12}$

$r=\sqrt{4\times3}=2\sqrt{3}$

$\theta=\tan^{-1}\left(\frac{b}{a}\right)$

$\theta=\tan^{-1}\left(\frac{-\sqrt{3}}{3}\right)$

$\theta=-30^\circ$

$z=r\,\text{cis}\theta=2\sqrt{3}\,\text{cis}(-30^\circ)$

$z=\boxed{2\sqrt{3}\left(\cos30^\circ-i\sin30^\circ\right)}$

5. ~~Express $|z|=3$ and $\arg z=-\frac{\pi}{2}$ in the form $x+yi$.~~

Sol. $z=r\text{ cis }\theta=3\text{cis}\left(-\frac{\pi}{2}\right)=3\text{cis}(-90^\circ)$

$z=3\left[\cos(-90^\circ)+i\sin(-90^\circ)\right]$

$z=3\left[0+i(-1)\right]=3[-i]=\boxed{-3i}$

6. Define improper fraction and give one example.

Sol. A fraction in which the degree of the numerator is greater then or equal to the degree of denominator is called improper fraction.

Example: $\frac{x^2+1}{(x+1)(x-1)}$

7. **Resolve $\frac{1}{x^2-1}$ into partial fractions .**

Sol. $\frac{1}{x^2-1}=\frac{1}{(x)^2-(1)^2}=\frac{1}{(x-1)(x+1)}$

$\frac{1}{(x-1)(x+1)}=\frac{A}{x-1}+\frac{B}{x+1}\rightarrow(i)$

$1=A(x+1)+B(x-1)\rightarrow(ii)$

Put $x=1$ in eq.(ii)

$1=A(1+1)+B(1-1)$

$1=A(2)+B(0)$

$1=2A+0 \Rightarrow \boxed{A=\frac{1}{2}}$

Put $x=-1$ in eq.(ii)

$1=A(-1+1)+B(-1-1)$

$1=A(0)+B(-2)$

$1=0-2B \Rightarrow \boxed{B=-\frac{1}{2}}$

Put values of A, & B in eq. (i),

we get: $\boxed{\frac{1}{2(x-1)}-\frac{1}{2(x+1)}}$

8. **Write an identity equation of $\frac{2x+5}{x^2+5x+6}$**

Sol. $\frac{2x+5}{x^2+5x+6}$

$=\frac{2x+5}{(x+2)(x+3)}$

$=\boxed{\frac{A}{x+2}+\frac{B}{x+3}}$

x^2+5x+6

$=x^2+3x+2x+6$

$=x(x+3)+2(x+3)$

$=(x+3)(x+2)$

9. **Form of partial fractions $\frac{1}{(x+1)^2(x-2)}$ is ________**

Sol. $\frac{1}{(x+1)^2(x-2)}=\boxed{\frac{A}{(x+1)}+\frac{B}{(x+1)^2}+\frac{C}{(x-2)}}$

10. **Define "Binary Numbers".**

Sol. The Binary number system is a number system of base equal to 2.

11. **Convert octal number $(107)_8$ to binary number.**

Sol.

$2\,|\,1$ → $0\text{-}1$; $2\,|\,0$ → $0\text{-}0$; $2\,|\,7$ → $2\,|\,3\text{-}1$ → $1\text{-}1$

001 000 111

$(107)_8=\boxed{(001000111)_2}$

12. **Prove $X+XZ=X$ by Boolean Algebra rules.**

Sol. $L.H.S.=X+XZ$

$=X(1+Z)$

$=X(1)$ $\because 1+Z=1$

$=X=R.H.S.$ **Proved.**

13. **Prove $X(\overline{X}+Y)=XY$ by Boolean Algebra rules.**

Sol. $L.H.S.=X(\overline{X}+Y)$

$=X\overline{X}+XY$

$=0+XY$ $\because X\overline{X}=0$

$=XY=R.H.S.$ **Proved.**

14. **Prepare a truth table for $X(X+Y)=X$**

Sol. $X(X+Y)=X$

L.H.S. X	Y	X+Y	R.H.S. X(X+Y)
0	0	0	0
0	1	1	0
1	0	1	1
1	1	1	1

15. Find the coordinates of the mid-point of the segment $P_1(3, 7)$, $P_2(-2, 3)$.

Sol. $\text{Mid-point} = \left(\dfrac{x_1+x_2}{2}, \dfrac{y_1+y_2}{2}\right)$

$$= \left(\frac{3+(-2)}{2}, \frac{7+3}{2}\right) = \boxed{\left(\frac{1}{2}, 5\right)}$$

16. For the triangle whose vertices are A(0, 1), B(7, 2) and C(3, 8). Find the length of the median from C to AB.

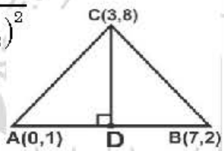
Sol. Let D be midpoint of $\overline{AB}$:

$$D = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$D = \left(\frac{0+7}{2}, \frac{1+2}{2}\right) \Rightarrow D = \left(\frac{7}{2}, \frac{3}{2}\right)$$

Distance between $|\overline{CD}| =$

$$= \sqrt{(x_1-x_2)^2 + (y_1-y_2)^2}$$



$$= \sqrt{\left(3-\frac{7}{2}\right)^2 + \left(8-\frac{3}{2}\right)^2}$$

$$= \sqrt{\left(\frac{6-7}{2}\right)^2 + \left(\frac{16-3}{2}\right)^2}$$

$$= \sqrt{\left(-\frac{1}{2}\right)^2 + \left(\frac{13}{2}\right)^2} = \sqrt{\frac{1}{4} + \frac{169}{4}}$$

$$= \sqrt{\frac{1+169}{4}} = \sqrt{\frac{170}{4}} = \boxed{\sqrt{\frac{85}{2}}}$$

17. ~~Find the angle between the lines having slopes -3 and 2.~~

Sol. Let, $m_1 = -3$ and $m_2 = 2$

$$\theta = \tan^{-1}\left(\frac{m_2-m_1}{1+m_2m_1}\right) = \tan^{-1}\left(\frac{2-(-3)}{1+(2)(-3)}\right)$$

$$\theta = \tan^{-1}\left(\frac{2+3}{1-6}\right) = \tan^{-1}\left(\frac{5}{-5}\right) = \boxed{135^\circ}$$

18. ~~Find the equation of a line through the point (3, -2) with slope $m = \frac{3}{4}$.~~

Sol. Equation of line in point - slope form :

$$y - y_1 = m(x - x_1)$$

$$y + 2 = \frac{3}{4}(x-3)$$

$$4y + 8 = 3x - 9 \Rightarrow 4y + 8 - 3x + 9 = 0$$

$$-3x + 4y + 17 = 0 \Rightarrow \boxed{3x - 4y - 17 = 0}$$

19. ~~Find the equation of a line whose perpendicular distance from the origin is 2 and inclination of the perpendicular is 225°.~~

Sol. Here P = 2 and $\theta = 225^\circ$

Normal form of equation of line is

$$x\cos\theta + y\sin\theta = P$$

$$x\cos 225^\circ + y\sin 225^\circ = 2$$

$$x\left(-\frac{1}{\sqrt{2}}\right) + y\left(-\frac{1}{\sqrt{2}}\right) = 2$$

Multipling each term by $\sqrt{2}$ we get :

$$-x - y = 2\sqrt{2} \Rightarrow \boxed{x + y + 2\sqrt{2} = 0}$$

20. ~~Find the equation of the line passing through the point (1, -2) making an angle of 135° with the X-axis.~~

Sol. Let, $\theta = 135^\circ$

$\text{Slope} = m = \tan\theta$

$m = \tan 135^\circ = -1$

Equation of line in point - slope form :

$$y - y_1 = m(x - x_1)$$

$$y - (-2) = -1(x-1)$$

$$y + 2 = -x + 1$$

$$y + 2 + x - 1 = 0 \Rightarrow \boxed{x + y + 1 = 0}$$

21. Find points of intersection of the lines : $x+2y-3=0$, $2x-3y+8=0$

Sol. Let $\begin{cases} \ell_1 : x+2y-3=0 \rightarrow (i) \\ \ell_2 : 2x-3y+8=0 \rightarrow (ii) \end{cases}$

Multiplying eq. (i) by 2 & subtracting eq.(ii) from it:

$$\begin{array}{r} 2x+4y-6=0 \\ \pm 2x \mp 3y \pm 8=0 \\ \hline 7y-14=0 \end{array}$$

$7y=14 \Rightarrow y=\frac{14}{7}=2$

Put $y=2$ in eq.(i), we have :

$x+2(2)-3=0$

$x+4-3=0$

$x+1=0 \Rightarrow x=-1$

Point of intersection is $\boxed{(-1,2)}$

22. Show that the points (1, 9), (-2, 3) and (-5, -3) are collinear.

Sol. $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 9 & 1 \\ -2 & 3 & 1 \\ -5 & -3 & 1 \end{vmatrix}$

$=1\begin{vmatrix} 3 & 1 \\ -3 & 1 \end{vmatrix} - 9\begin{vmatrix} -2 & 1 \\ -5 & 1 \end{vmatrix} + 1\begin{vmatrix} -2 & 3 \\ -5 & -3 \end{vmatrix}$

$=1(3-(-3))-9(-2-(-5))+1(6-(-15))$

$=1(3+3)-9(-2+5)+1(6+15)$

$=1(6)-9(3)+1(21)=6-27+21=0$

Hence given points are collinear. **Proved.**

23. Show that the lines passing through the points (0, -7), (8, -5) and (5, 7), (8, -5) are perpendicular.

Sol.

$\ell_1 : (0,-7)$ & $(8,-5)$	$\ell_2 : (5,7)$ & $(8,-5)$
Slope of $\ell_1 = m_1$	Slope of $\ell_2 = m_2$
$m_1 = \frac{y_2-y_1}{x_2-x_1}$	$m_2 = \frac{y_2-y_1}{x_2-x_1}$
$m_1 = \frac{-5+7}{8-0}$	$m_2 = \frac{-5-7}{8-5}$
$m_1 = \frac{2}{8} = \frac{1}{4}$	$m_2 = -\frac{12}{3} = -4$

As, $m_1 m_2 = \left(\frac{1}{4}\right)(-4) = -1$

Hence both lines ℓ_1 & ℓ_2 are $\boxed{\text{perpendicular}}$. **Proved.**

24. Write the equation of circle with, center at (h, k) and radius 'r'.

Sol. $\boxed{(x-h)^2+(y-k)^2=r^2}$

25. Find center and radius of the circle $x^2+y^2+9x-7y-33=0$

Sol. Comparing with general equation of circle.

$x^2+y^2+9x-7y-33=0$

$2g=9$	$2f=-7$	$c=-33$
$g=\frac{9}{2}$	$f=-\frac{7}{2}$	

Center $=(-g,-f)=$

Center $=\left(-\frac{9}{2},-\left(-\frac{7}{2}\right)\right)=\boxed{\left(-\frac{9}{2},\frac{7}{2}\right)}$

Radius $= r = \sqrt{g^2+f^2-c}$

$r=\sqrt{\left(\frac{9}{2}\right)^2+\left(-\frac{7}{2}\right)^2-(-33)}$

$r=\sqrt{\frac{81}{4}+\frac{49}{4}+33}$

$r=\sqrt{\frac{81+49+132}{4}}=\sqrt{\frac{262}{4}}=\boxed{\sqrt{\frac{131}{2}}}$

26. Reduce the equation of the circle $x^2+y^2-4x+6y-12=0$ into standard form.

Sol. As given equation:

$$x^2+y^2-4x+6y-12=0$$

$$x^2-4x+y^2+6y=12$$

Adding the square of one half of the coefficient of x & y on both sides:

$$x^2-4x+(2)^2+y^2+6y+(3)^2=12+(2)^2+(3)^2$$

$$(x-2)^2+(y+3)^2=12+4+9$$

$$(x-2)^2+(y+3)^2=25$$

$$\boxed{(x-2)^2+(y+3)^2=(5)^2}$$

27. Find the equation of the circle which touches both the axes of 4th quadrant and has a radius of 5 units.

Sol. As circle touches both the axes of 4^{th} – quad. & Radius $= r = 5$

So, centre $= (h, k) = (5, -5)$

Standard form of eq. of circle:

$$(x-h)^2+(y-k)^2=r^2$$

Put $h=5, k=-5$ & $r=5$

$$(x-5)^2+(y+5)^2=r^2$$

$$(x)^2-2(x)(5)+(5)^2+(y)^2+2(y)(5)+(5)^2=25$$

$$x^2-10x+25+y^2+10y+25-25=0$$

$$\boxed{x^2+y^2-10x+10y+25=0}$$

Section - II

Note : Attemp any three (3) questions $3\times8=24$

Q.2.(a) Extract the square root of $-3+4i$.

Sol. See Q.9(i) of Ex# 8.1 (Page # 310)

(b) ~~**Write complex number of**~~ **$4cis240°$** ~~**in the form**~~ **$a+bi$.**

Sol. See Q.3(i) of Ex# 8.2 (Page # 318)

Q.3.(a) Resolve $\dfrac{1}{(1-x)(1-2x)(1-3x)}$ into partial fractions.

Sol. See Q.7 of Ex# 9.1 (Page # 351)

(b) Resolve $\dfrac{1}{x^3-1}$ into partial fractions.

Sol. See Q.9 of Ex# 9.3 (Page # 377)

Q.4.(a) Convert 18×24 to binary form and then perform binary multiplication.

Sol. See Q.7 of Ex# 10 (Page # 410)

(b) Minimize the expression

$$X=\overline{A}B\overline{C}+A\overline{B}\overline{C}+\overline{A}\overline{B}\overline{C}+\overline{A}\overline{B}\overline{C}$$

Sol. See example 05[b] of chapter 11

Q.5.(a) The point (x, y) is on the x-axis and is 6 units away from the point $(1, 4)$. Find x and y.

Sol. See Q.5 of Ex# 12.1 (Page # 450)

(b) ~~**Find the points trisecting the join of**~~ **$A(-1, 4)$ and $B(6, 2)$.**

Sol. See Q.9 of Ex# 12.2 (Page # 461)

Q.6. Find the equation of a circle passing through the points $(0, 1)$, $(3, -3)$ and $(3, -1)$.

Sol. See Q.3[b] of Ex# 13 (Page # 525)

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