DAE / IIA - 2017

MATH - 123 APPLIED MATHEMATICS - I

PAPER 'A' PART - A (OBJECTIVE)

Time : 30 Minutes   Marks : 15

Q.1: *Encircle the correct answer.*

1. $15°$ is equal to:

[a] $\frac{\pi}{6}$ rad [b] $\frac{\pi}{3}$ rad

[c] $\frac{\pi}{12}$ rad [d] $\frac{\pi}{15}$ rad

2. $\sin(90° - \theta)$ is equal to:

[a] $-\sin\theta$ [b] $\sin\theta$

[c] $-\cos\theta$ [d] $\cos\theta$

3. $\sin(\alpha+\beta)$ is equal to:

[a] $\sin\alpha\cos\beta + \cos\alpha\sin\beta$

[b] $\cos\alpha\cos\beta - \sin\alpha\sin\beta$

[c] $\sin\alpha\cos\beta - \cos\alpha\sin\beta$

[d] $\cos\alpha\cos\beta + \sin\alpha\sin\beta$

4. If in a triangle ABC, a = 1, b = 2, and C = 60º then c is:

[a] $\sqrt{3}$ [b] 2 [c] 1 [d] 3

5. Unit vector of $\underline{i}+\underline{j}+\underline{k}$ is:

[a] $\underline{i}+\underline{j}+\underline{k}$ [b] $\frac{1}{3}(\underline{i}+\underline{j}+\underline{k})$

[c] $\frac{1}{\sqrt{3}}(\underline{i}+\underline{j}+\underline{k})$

[d] $\frac{1}{2}(\underline{i}+\underline{j}+\underline{k})$

6. The number of terms in the expansion $(a+b)^{13}$ are:

[a] 12 [b] 13

[c] 14 [d] 15

7. ~~One radian is equal to:~~

[a] $90°$ [b] $\left(\frac{90}{\pi}\right)^{\circ}$

[c] $180°$ [d] $\left(\frac{180}{\pi}\right)^{\circ}$

8. The terminal side of angle θ lies in 4^{th} quadrant, sign of sin θ will be:

[a] Positive [b] Negative

[c] Both +ve and –ve

[d] None of these

9. If $\vec{a}$ and $\vec{b}$ are collinear vectors then:

[a] $\bar{a}\times\bar{b}=0$ [b] $\bar{a}.\bar{b}=0$

[c] $\bar{a}-\bar{b}=0$ [d] $\bar{a}+\bar{b}=0$

10. The cross product of two vector $\vec{a}$ and $\vec{b}$ are:

[a] $ab\sin\theta$ [b] $ab\cos\theta$

[c] $ab\sin\theta\hat{n}$ [d] $ab\cos\theta\hat{n}$

11. The standard form of a quadratic equation is:

[a] $ax^2+bx=0$ [b] $ax^2=0$

[c] $ax^2+bx+c=0$

[d] $ax^2+c=0$

12. If discriminant of an equation is zero, then the roots will be:

[a] Imaginary [b] Real

[c] Equal [d] Irrational

13. ~~The product of roots of~~ $2x^2-3x-5=0$ ~~is:~~

[a] $-\frac{5}{2}$ [b] $\frac{5}{2}$ [c] $\frac{2}{5}$ [d] $-\frac{2}{5}$

14. The value of is $\binom{n}{n}$ equal to:

[a] 0 [b] 1 [c] n [d] $-n$

15. Third term of $(x+y)^4$ is:

[a] $4x^2y^2$ [b] $4x^3y$

[c] $6x^2y^2$ [d] $6x^3y$

Answer Key

1	c	2	d	3	a	4	a	5	c
6	c	7	d	8	b	9	a	10	c
11	c	12	c	13	a	14	b	15	a

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DAE / IIA - 2017

MATH - 123 APPLIED MATHEMATICS - I

PAPER 'A' PART - B (SUBJECTIVE)

Time: 2:30 Hrs Marks: 60

Section - I

Q.1. Write short answers to any Eighteen (18) questions.

1. Solve the equation $x^2 - 3x = 2x - 6$ by factorization.

Sol. $x^2 - 3x = 2x - 6$

$x^2 - 3x - 2x + 6 = 0$

$x(x-3) - 2(x-3) = 0$

$(x-3)(x-2) = 0$

Either | OR
$x - 3 = 0$ | $x - 2 = 0$
$x = 3$ | $x = 2$

$S.S. = \{2, 3\}$

2. Solve the quadratic equation:

$x^2 + 7x + 12 = 0$

Sol. $x^2 + 7x + 12 = 0$

$x^2 + 4x + 3x + 12 = 0$

$x(x+4) + 3(x+4) = 0$

$(x+4)(x+3) = 0$

Either | OR
$x + 4 = 0$ | $x + 3 = 0$
$x = -4$ | $x = -3$

$S.S. = \{-3, -4\}$

3. ~~Find the value of 'k', given that the sum of roots of the equation~~ $3x^2 + kx + 5 = 0$ ~~will be equal to the product of its roots.~~

Sol. Here: $a = 3,\ b = k,\ c = 5$

As, Sum of roots = Product of roots

$$\frac{-b}{a} = \frac{c}{a}$$

$$\frac{-k}{3} = \frac{5}{3}$$

$$-k = 5 \Rightarrow \boxed{k = -5}$$

4. ~~From the quadratic equation whose roots are~~ $3+i$ & $3-i$

Sol. $S = 3 + i + 3 - i$ | $P = (3+i)(3-i)$

$S = 6$ | $P = (3)^2 - (i)^2$

$P = 9 - (-1)$

$P = 9 + 1 = 10$

$x^2 - Sx + P = 0$

$$\boxed{x^2 - 6x + 10 = 0}$$

5. ~~Find the sum and product of the roots of~~ $x^2 - 9 = 0$.

Sol. Here: $a = 1,\ b = 0,\ c = -9$

Sum of the Roots $= S = -\frac{b}{a} = -\frac{0}{1} = \boxed{0}$

Product of the Roots $= P = \frac{c}{a} = \frac{-9}{1} = \boxed{-9}$

6. Expand $\left(x + \frac{1}{x}\right)^4$ by Binomial theorem.

Sol. By using binomial theorem.

$$= \binom{4}{0}(x)^4\left(\frac{1}{x}\right)^0 + \binom{4}{1}(x)^3\left(\frac{1}{x}\right)^1 + \binom{4}{2}(x)^2\left(\frac{1}{x}\right)^2$$

$$+ \binom{4}{3}(x)^1\left(\frac{1}{x}\right)^3 + \binom{4}{4}(x)^0\left(\frac{1}{x}\right)^4$$

$$= (1)(x^4)(1) + (4)(x^3)\left(\frac{1}{x}\right) + (6)(x^2)\left(\frac{1}{x^2}\right)$$

$$+ (4)(x)\left(\frac{1}{x^3}\right) + (1)(1)\left(\frac{1}{x^4}\right)$$

$$= \boxed{x^4 + 4x^2 + 6 + \frac{4}{x^2} + \frac{1}{x^4}}$$

7. Find the 5th term in the expansion of $\left(2x - \frac{x^2}{4}\right)^7$

Sol. Here : $a = 2x$, $b = -\frac{x^2}{4}$, $n = 7$ & $r = 4$

$$T_{r+1} = \binom{n}{r} a^{n-r} b^r$$

$$T_{4+1} = \binom{7}{4} (2x)^{7-4} \left(-\frac{x^2}{4}\right)^4$$

$$T_5 = (35)(8x^3)\left(\frac{x^8}{256}\right) \Rightarrow \boxed{T_5 = \frac{35}{32} x^{11}}$$

8. ~~**Expand**~~ $(1+x)^{-3}$ ~~**up to three terms.**~~

Sol. $(1+x)^{-3}$

Put $b = x$ & $n = -3$ in Binomial series Formula, $(1+b)^n = 1 + nb + \frac{n(n-1)}{2!}b^2 + \ldots$ we have :

$$= 1 + (-3)(x) + \frac{-3(-3-1)}{2!}(x)^2 + \ldots$$

$$= 1 - 3x + \frac{-3(-4)}{2}x^2 + \ldots$$

$$= \boxed{1 - 3x + 6x^2 + \cdots}$$

9. ~~**Compute**~~ $(1.02)^4$ ~~**to two decimal places by use of Binomial formula.**~~

Sol. $(1.02)^4 = (1 + 0.02)^4$

$$= \binom{4}{0}(1)^4(0.02)^0 + \binom{4}{1}(1)^3(0.02)^1 + \binom{4}{2}(1)^2(0.02)^2 + \ldots$$

$$= (1)(1)(1) + 4(1)(0.02) + 6(1)(0.0004) + \ldots$$

$$= 1 + 0.08 + 0.0024 + \ldots = 1.0824 = \boxed{1.08}$$

10. **Which term is the middle term/terms in the Binomial expansion of $(a+b)^n$.**

(i) When 'n' is even. (ii) When 'n' is odd.

Sol. (i) **When n is even**

Then, Middle term = $\left(\frac{n+2}{2}\right)^{th}$ term

Sol. (ii) **When n is odd**

Then, there are two middle terms:

Middle term = $\left(\frac{n+1}{2}\right)^{th} + \left(\frac{n+3}{2}\right)^{th}$ terms.

11. ~~**Find the missing elements ℓ, r, θ, when ℓ = 8.4m, θ = 2.8 rad.**~~

Sol. We know that : $\ell = r\theta$

$$\Rightarrow r = \frac{\ell}{\theta} = \frac{8.4}{2.8} = \boxed{3m}$$

12. **Evaluate:**

$$\mathbf{\cos 30^\circ \cos 60^\circ - \sin 30^\circ \sin 60^\circ}$$

Sol. $\cos 30^\circ \cos 60^\circ - \sin 30^\circ \sin 60^\circ$

$$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$$

$$= \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4} = \boxed{0}$$

13. **Prove that:**

$$\mathbf{1 - 2\sin^2\theta = 2\cos^2\theta - 1}$$

Sol. $\text{L.H.S.} = 1 - 2\sin^2\theta$

$$= 1 - 2(1 - \cos^2\theta)$$

$$= 1 - 2 + 2\cos^2\theta$$

$= 2\cos^2\theta - 1 = \text{R.H.S.}$ **Proved.**

14. **Show that:**

$$\mathbf{\sin(\alpha+\beta) + \sin(\alpha-\beta) = 2\sin\alpha\cos\beta}$$

Sol. $\text{L.H.S.} = \sin(\alpha+\beta) + \sin(\alpha-\beta)$

$$= \sin\alpha\cos\beta + \cancel{\cos\alpha\sin\beta} + \sin\alpha\cos\beta - \cancel{\cos\alpha\sin\beta}$$

$= 2\sin\alpha\cos\beta = \text{R.H.S.}$ **Proved.**

15. **Find the value of $\sin 105^\circ$, without using calculator.**

Sol. $\sin 105^\circ = \sin(45^\circ + 60^\circ)$

$$= \sin 45^\circ \cos 60^\circ + \cos 45^\circ \sin 60^\circ$$

$$= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right) + \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right)$$

$$= \frac{1}{2\sqrt{2}} + \frac{\sqrt{3}}{2\sqrt{2}} = \boxed{\frac{1+\sqrt{3}}{2\sqrt{2}}}$$

16. ~~Express~~ $\sin 5\theta - \sin\theta$ ~~as product.~~

Sol. $\sin 5\theta - \sin\theta$

$$= 2\cos\left(\frac{5\theta+\theta}{2}\right)\sin\left(\frac{5\theta-\theta}{2}\right)$$

$$= 2\cos\left(\frac{6\theta}{2}\right)\sin\left(\frac{4\theta}{2}\right)$$

$$= \boxed{2\cos 3\theta \sin 2\theta}$$

17. **Prove that:**

$$\cos^4\theta - \sin^4\theta = \frac{1}{\sec 2\theta}$$

Sol. $\text{L.H.S.} = \cos^4\theta - \sin^4\theta$

$$= \left(\cos^2\theta\right)^2 - \left(\sin^2\theta\right)^2$$

$$= \left(\cos^2\theta - \sin^2\theta\right)\left(\cos^2\theta + \sin^2\theta\right)$$

$$= \left(\cos^2\theta - \sin^2\theta\right)(1)$$

$$= \cos^2\theta - \sin^2\theta$$

$$= \cos 2\theta = \frac{1}{\sec 2\theta} = \text{R.H.S.}$$ **Proved.**

18. **Prove that:** $\sin(180^\circ - \theta) = \sin\theta$

Sol. $\text{L.H.S.} = \sin(180^\circ - \theta)$

$$= \sin 180^\circ \cos\theta - \cos 180^\circ \sin\theta$$

$$= (0)\cos\theta - (-1)\sin\theta$$

$$= 0 + \sin\theta$$

$$= \sin\theta = \text{R.H.S.}$$ **Proved.**

19. **In any triangle ABC if $a = 3$, $b = 7$, $\beta = 85^\circ$ find α.**

Sol. By using law of sines:

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$$

$$\text{We take: } \frac{a}{\sin\alpha} = \frac{b}{\sin\beta}$$

$$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b}$$

$$\sin\alpha = \frac{a\sin\beta}{b}$$

$$\sin\alpha = \frac{3\sin 85^\circ}{7}$$

$$\sin\alpha = 0.4269$$

$$\alpha = \sin^{-1}(0.4269)$$

$$\boxed{\alpha = 25^\circ 16'}$$

20. **Write the law of Cosines.**

Sol. In any triangle ABC, with usual notations.

i. $a^2 = b^2 + c^2 - 2bc\cos\alpha$

ii. $b^2 = c^2 + a^2 - 2ca\cos\beta$

iii. $c^2 = a^2 + b^2 - 2ab\cos\gamma$

21. **In any triangle ABC if b = 5, c = 8, α= 60º, Find 'a'.**

Sol. By using law of cosines:

$$a^2 = b^2 + c^2 - 2bc\cos\alpha$$

$$a^2 = (5)^2 + (8)^2 - 2(5)(8)\cos 60^\circ$$

$$a^2 = 25 + 64 - 40$$

$$a^2 = 49$$

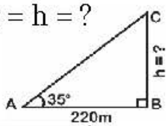
$$\sqrt{a^2} = \sqrt{49}$$

$$\boxed{a = 7}$$

22. ~~The shadow of a building is 220 meters when the measure of the angle of elevation of the sun is 35º. Find the height of the building.~~

Sol. Let height of building $= h = ?$

$$\tan 35^\circ = \frac{h}{220}$$



$$h = 220\tan 35^\circ = \boxed{154.05\,\text{m}}$$

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23. **Show that the vectors**

$$\mathbf{4i - 6j + 9k \;\;\&\;\; -6i + 9j - \frac{27}{2}k}$$

are parallel.

Sol.

Let, $\vec{a} = 4i - 6j + 9k$ & $\vec{b} = -6i + 9j - \frac{27}{2}k$

$$\vec{a}\times\vec{b} = \begin{vmatrix} i & j & k \\ 4 & -6 & 9 \\ -6 & 9 & -27/2 \end{vmatrix}$$

$$= i\begin{vmatrix} -6 & 9 \\ 9 & -27/2 \end{vmatrix} - j\begin{vmatrix} 4 & 9 \\ -6 & -27/2 \end{vmatrix} + k\begin{vmatrix} 4 & -6 \\ -6 & 9 \end{vmatrix}$$

$$= i\left(-6\left(-\frac{27}{2}\right) - 81\right) - j\left(4\left(-\frac{27}{2}\right) - (-54)\right) + k(36 - 36)$$

$$= i(81 - 81) - j(-54 + 54) + k(36 - 36)$$

$$= i(0) - j(0) + k(0) = \boxed{0}$$

Hence $\vec{a}$ & $\vec{b}$ are parallel.

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24. **Find $\vec{a}.\vec{b}$ if**

$$\mathbf{\vec{a} = 2i + 3j + 4k \;\;\&\;\; \vec{b} = i - j + k}$$

Sol. $\vec{a}.\vec{b} = (2i + 3j + 4k).(i - j + k)$

$$= (2)(1) + (3)(-1) + (4)(1)$$

$$= 2 - 3 + 4 = \boxed{3}$$

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25. **For what value of λ, the vectors $\mathbf{2i - j + 2k \;\&\; 3i + 2\lambda j}$ are perpendicular.**

Sol. Let, $\vec{a} = 2i - j + 2k$ & $\vec{b} = 3i + 2\lambda j$

As given vectors are perpendicular.

So, $\vec{a}.\vec{b} = 0$

$\Rightarrow (2i - j + 2k).(3i + 2\lambda j) = 0$

$\Rightarrow (2)(3) + (-1)(2\lambda) + (2)(0) = 0$

$\Rightarrow 6 - 2\lambda + 0 = 0$

$\Rightarrow -2\lambda = -6$

$\Rightarrow \lambda = \frac{-6}{-2}$

$\Rightarrow \boxed{\lambda = 3}$

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26. **Given the vectors $\mathbf{\vec{a} = 3i + j - k}$ and $\mathbf{\vec{b} = 2i + j - k}$, find magnitude of $\mathbf{3\vec{a} - \vec{b}}$.**

Sol. $3\vec{a} - \vec{b} = 3(3i + j - k) - (2i + j - k)$

$3\vec{a} - \vec{b} = 9i + 3j - 3k - 2i - j + k$

$3\vec{a} - \vec{b} = 7i + 2j - 2k$

$|3\vec{a} - \vec{b}| = \sqrt{(7)^2 + (2)^2 + (-2)^2}$

$= \sqrt{49 + 4 + 4} = \boxed{\sqrt{57}}$

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27. **Given vectors $\mathbf{\vec{a} = 3i - 2j + 4k}$, $\mathbf{\vec{b} = 2i + j + 3k}$. Find the magnitude and Direction Cosines a $\mathbf{\vec{a} - \vec{b}}$.**

Sol. Let, $\vec{v} = \vec{a} - \vec{b}$

$\vec{v} = (3i - 2j + 4k) - (2i + j + 3k)$

$\vec{v} = 3i - 2j + 4k - 2i - j - 3k$

$\vec{v} = i - 3j + k$

Magnitude $= |\vec{v}|$

$= \sqrt{(1)^2 + (-3)^2 + (1)^2}$

$= \sqrt{1 + 9 + 1} = \boxed{\sqrt{11}}$

Direction cosine:

$$\boxed{\frac{1}{\sqrt{11}},\; -\frac{3}{\sqrt{11}},\; \frac{1}{\sqrt{11}}}$$

Section - II

Note : Attemp any three (3) questions $3 \times 8 = 24$

Q.2.(a) Solve the equation

$$\frac{4}{x-1}-\frac{5}{x+2}=\frac{3}{x}$$ by Factorization.

Sol. See Q.1(v) of Ex # 1.1 (Page # 07)

(b) Show that the roots of the equation $(mx+c)^2 = 4x$ will be equal if $c=\frac{a}{m}$

Sol. See Q.3(ii) of Ex # 1.2 (Page # 33)

Q.3. Find the middle term in the expansion of $\left(3x^2+\frac{1}{2x}\right)^{10}$.

Sol. See Q.6(i) of Ex # 2.1 (Page # 85)

Q.4.(a) If $\cos\theta = -\frac{\sqrt{3}}{2}$ and the terminal side of the angle lies in the third quadrant. Find the remaining trigonometric ratios of θ.

Sol. See Q.1 of Ex # 3.2 (Page # 121)

(b) Prove that $\frac{\cot^2\theta-1}{\cot^2\theta+1}=2\cos^2\theta-1$

Sol. See Q.11 of Ex # 3.3 (Page # 135)

Q.5.(a) Show that :

$$\frac{\sin 75^\circ - \sin 15^\circ}{\cos 75^\circ + \cos 15^\circ}=\frac{1}{\sqrt{3}}$$

Sol. See Q.13(i) of Ex # 4.3 (Page # 190)

(b) ~~Find the right triangle ABC in which~~ $\gamma = 90^\circ,\ a = 250,\ \alpha = 42^\circ 25'$

Sol. See Q.1 of Ex # 5.1 (Page # 208)

Q.6.(a) Find the cosine of the angle between the vector $\vec{a}=2i-8j+3k$, $\vec{b}=4j+3k$

Sol. See Q.3(i) of Ex # 6.2 (Page # 256)

(b) Find the vector whose magnitude is 5 and which is in the direction of vector $4\underline{i}-3\underline{j}+\underline{k}$.

Sol. See Q.2 of Ex # 6.1 (Page # 247)

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