DAE / IIA - 2017

MATH - 113 APPLIED MATHEMATICS - I

PAPER 'B' PART - A (OBJECTIVE)

Time: 30 Minutes Marks: 15

Q.1: *Encircle the correct answer.*

1. If a = 2cm, b = 3cm and c = 5cm are sides of triangle, then perimeter of triangle is:
[a] 8cm [b] 6cm [c] 10cm [d] 30cm

2. If a = 4cm, b = 2cm are adjacent sides of triangle and $\theta = 30°$ is the included angle then area is:
[a] 2 sq.cm [b] 4 sq.cm
[c] 8 sq.cm [d] 12 sq.cm

3. Area of parallelogram having 'a' and 'b' as adjacent sides and θ is the included angles is:
[a] $ab\cos\theta$ [b] $\frac{1}{2}ab\sin\theta$
[c] $ab\sin\theta$ [d] $a\sin\theta$

4. Area of parallelogram having base 2cm and height 5cm is:
[a] 20 sq.cm [b] 30 sq.cm
[c] 15 sq.cm [d] 10 sq.cm

5. If R and r denote radii of the outer and inner circles, then area of annulus (ring) is:
[a] $\pi(R^2 - r^2)$ [b] $\frac{\pi}{2}(R^2 - r^2)$
[c] $\pi(R^2 + r^2)$ [d] $(R^2 - r^2)$

6. Total surface area of the cube of side 'a' is:
[a] a^2 [b] $3a^2$ [c] $6a^2$ [d] $4a^2$

7. Volume of hollow cylinder if R and r are external and internal radii respectively is:
[a] $\pi(R - r)$ [b] $2\pi(R^2 - r^2)$
[c] $\pi(R^2 - r^2)$ [d] $\pi(R - r)h$

8. Lateral surface are of regular pyramid if perimeter of base is P and slant height 'ℓ' is;
[a] $P\ell$ [b] $\frac{1}{3}P\ell$ [c] $\frac{1}{2}P\ell$ [d] $\frac{1}{6}P\ell$

9. The curved surface of a right circular cone of radius 3cm and slant height is 6cm.
[a] 54π [b] 9π [c] 18π [d] 6π

10. The unit vector of $\mathbf{i} - 2\mathbf{j} - 2\mathbf{k}$ is:
[a] $i - 2j - 2k$ [b] $\frac{1}{3}(i - 2j - 2k)$
[c] $\frac{1}{\sqrt{3}}(i - 2j - 2k)$ [d] $\frac{1}{2}(i - 2j - 2k)$

11. If $\vec{a} = \mathbf{i} + \mathbf{j} + \mathbf{k}$ and $\vec{b} = -\mathbf{i} - \mathbf{j} - m\mathbf{k}$ are perpendicular then 'm' will be equal to:
[a] 1 [b] -2 [c] ± 1 [d] ± 3

12. $|\vec{a} \times \vec{b}|$ is a
[a] Vector quant. [b] Scalar quant.
[c] Unity [d] None of these

13. ~~If two rows of a determinant are identical then its value is:~~
[a] 1 [b] 0 [c] -1 [d] None

14. The value of 'm' for which matrix $\begin{bmatrix} 2 & 3 \\ 6 & m \end{bmatrix}$ is singular.
[a] 6 [b] 3 [c] 8 [d] 9

15. If A and B are symmetric, then $(AB)^t =$
[a] BA [b] A^tB^t
[c] B^tA^t [d] Both a and c

Answer Key

1	c	2	b	3	c	4	d	5	a
6	c	7	c	8	c	9	c	10	b
11	b	12	b	13	b	14	d	15	c

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DAE / IIA - 2017

MATH - 113 APPLIED MATHEMATICS - I

PAPER 'B' PART - B (SUBJECTIVE)

Time: 2:30 Hrs Marks: 60

Section - I

Q.1. Write short answers to any Eighteen (18) questions.

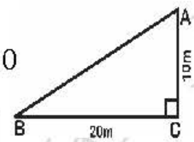
1. Find the area of right triangle if base and altitude are 20m and 10m respectively.

Sol. Given: base = 20m & Altitude = 10m

$$\text{Area} = \frac{1}{2} \times \text{base} \times \text{Altitude}$$

$$\text{Area} = \frac{1}{2} \times 20 \times 10$$

$$\text{Area} = \boxed{100\ m^2}$$



2. Find the area of triangle with sides 5, 4 and 3 meters respectively.

Sol. Let, $a = 5m,\ b = 4m,\ c = 3m$

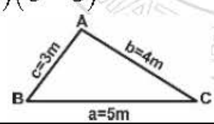
$$s = \frac{a+b+c}{2} = \frac{5+4+3}{2} = \frac{12}{2} = 6m$$

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$A = \sqrt{6(6-5)(6-4)(6-3)}$$

$$A = \sqrt{6(1)(2)(3)}$$

$$A = \sqrt{36} = \boxed{6\ sq.m}$$



3. Find the base of a parallelogram whose area is 256sq.cm and height 32cm.

Sol. Here: base = ? & height = 32cm

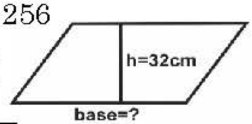
$$\text{Area of parallelogram} = 256$$

$$\text{Base} \times \text{Height} = 256$$

$$\text{Base} \times 32 = 256$$

$$\text{Base} = \frac{256}{32}$$

$$\boxed{\text{Base} = 8\,cm}$$



4. Define a rhombus.

Sol. A quadrilateral having all sides are equal with unequal diagonals.

5. Write the formula of area of a regular polygon of 'n' sides when the radius of inscribed circle 'r' is given.

Sol. $\text{Area} = \boxed{nr^2 \tan\left(\frac{180^\circ}{n}\right)\ \text{sq.unit}}$

6. The perimeter of a regular hexagon is 12cm, find its area.

Sol. Perimeter of hexagon = 12 cm

$$6a = 12$$

$$\Rightarrow \quad a = 12/6 = 2cm$$

$$\text{Area} = \frac{na^2}{4}\cot\left(\frac{180^\circ}{n}\right)$$

$$A = \frac{6(2)^2}{4}\cot\left(\frac{180^\circ}{6}\right)$$

$$A = 6\cot 30^\circ$$

$$A = \frac{6}{\tan 30^\circ}$$

$$A = \boxed{10.39\,sq.cm}$$

7. Find the radius of a circle the area of which is 9.3129 sq.cm.

Sol. As, Area of circle = 9.3129 sq.cm

$$\pi r^2 = 9.3129$$

$$r^2 = \frac{9.3129}{\pi}$$

$$r^2 = 2.96 \Rightarrow \boxed{r = 1.72\,cm}$$

8. Define area of the Annulus (Ring).

Sol. Let, Radius of inner circle $= r$

and, Radius of outer circle $= R$

$$\text{Area of Annulus (Ring)} = \pi\left(R^2 - r^2\right)\ \text{sq.unit}$$

9. Find the area of cross-section of river along a line where the depths at equal interval of 10m are noted 0, 7, 11, 15, 0 meters respectively.

Sol.

Sr.#	1	2	3	4	5
Ordinate	0	7	11	15	0

$S = 10m$

$A = 0 + 0 = 0$

$D = 11$

$E = 7 + 15 = 22$

By using Simpson's Rule:

$$\text{Area} = \frac{S}{3}[A + 2D + 4E]$$

$$\text{Area} = \frac{10}{3}\left[0 + 2(11) + 4(22)\right]$$

$$\text{Area} = \frac{10}{3}\left[22 + 88\right]$$

$$\text{Area} = \frac{10}{3}\left[110\right]$$

$$\text{Area} = \boxed{366.67\,m^2}$$

10. The base of a right prism is an equilateral triangle with a side of 4cm and its height is 25cm, find its volume.

Sol. Here: $a = 4cm,\ h = 25cm\ \&\ V = ?$

Area of base (equilateral triangle)

$$= \frac{\sqrt{3}}{4}a^2 = \frac{\sqrt{3}}{4}(4)^2 = 6.92cm^2$$

Volume = Area of base × height

$$V = 6.92 \times 25 = \boxed{173.2\,cm^3}$$

11. The inside measurement of a room are 8.5m, 6.4 and 4.5m height. How many men should sleep in the room, if each man is allowed 13.6 cu. m of air?

Sol. Let, ℓ = Length of room $= 8.5m$

b = Breadth of room $= 6.4m$

h = Height of room $= 4.5m$

Volume of room $= \ell bh$

$= 8.5 \times 6.4 \times 4.5 = 244.80m^3$

Air allowed for one men = $13.6\,m^3$

No. of men that can sleep in room

$$= \frac{244.80}{13.6} = \boxed{18\,\text{men}}$$

12. The diameter of the base of a right circular cylinder is 14cm and its height is 10cm. Find the volume of cylinder.

Sol. Here: $d = 14cm\ \&\ h = 10cm$

As, $d = 14cm \Rightarrow r = \frac{d}{2} = \frac{14}{2} = 7cm$

Volume $= \pi r^2 h$

$V = \pi(7)^2 10 = \boxed{1539.38\,cm^3}$

13. Find the diameter of the cylinder if its volume is 704cm³ and height is 14cm.

Sol. Here: $d = ?,\ V = 704cm^3\ \&\ h = 14cm$

As, Volume of Cylinder $= 704cm^3$

$\Rightarrow \quad \pi r^2 h = 704$

$\Rightarrow \quad r^2 = \dfrac{704}{\pi h}$

$\Rightarrow \quad r^2 = \dfrac{704}{\pi(14)}$

$\Rightarrow \quad r^2 = 16$

$\Rightarrow \quad r = 4\,cm$

Diameter $= d = 2r = 2(4) = \boxed{8\,cm}$

14. Define pyramid.

Sol. A pyramid is a solid, whose base is a plane polygon and sides being triangles that meet in a common vertex.

15. **Find the volume of a pyramid with a square base of side 10cm and height 15cm.**

Sol. Here : V = ? , a = 10cm & h = 15cm

$$\text{Area of base (square)} = a^2$$

$$= (10)^2 = 100\text{cm}^2$$

$$\text{Volume} = \frac{1}{3}\times \text{Area of base} \times \text{height}$$

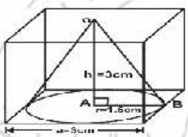
$$V = \frac{1}{3}\times 100\times 15 = \boxed{500\text{cm}^3}$$

16. **Find the volume of the largest cone that can be cut out of a cube whose edge is 3cm.**

Sol. Let 'a' = edge of the cube = 3cm

Then h = 3cm &

$$r = \frac{a}{2} = \frac{3}{2} = 1.5\text{cm}$$



$$\text{Volume} = \frac{1}{3}\pi r^2 h$$

$$V = \frac{1}{3}\pi(1.5)^2(3) = \boxed{7.069\,\text{cm}^3}$$

17. **How many square meter of copper will be required to cover a hemi-spherical dome of 30m diameter.**

Sol. Surface area of hemi - sphere dome

$$= \frac{1}{2}\pi d^2 = \frac{1}{2}\pi(30)^2 = \boxed{1413.72\text{m}^2}$$

18. **Given the vectors: $\vec{a} = 3i - 2j + k$, $\vec{b} = 2i - 4j - 3k$, $\vec{c} = -i + 2j + 2k$ Find $\vec{a} + \vec{b} + \vec{c}$**

Sol. $\vec{a} + \vec{b} + \vec{c}$

$$= 3i - 2j + k + 2i - 4j - 3k - i + 2j + 2k$$

$$= \boxed{4i - 4j + 0k}$$

19. **Given the vectors $\vec{a} = 3i + j - k$ and $\vec{b} = 2i + j - k$ = 2i + j − k, find magnitude of $3\vec{a} - \vec{b}$.**

Sol. $3\vec{a} - \vec{b} = 3(3i + j - k) - (2i + j - k)$

$$3\vec{a} - \vec{b} = 9i + 3j - 3k - 2i - j + k$$

$$3\vec{a} - \vec{b} = 7i + 2j - 2k$$

$$|3\vec{a} - \vec{b}|$$

$$= \sqrt{(7)^2 + (2)^2 + (-2)^2}$$

$$= \sqrt{49 + 4 + 4}$$

$$= \boxed{\sqrt{57}}$$

20. **Find $\vec{a}.\vec{b}$ if $\vec{a} = i + 2j + 2k$ & $\vec{b} = 3i - 2j - 2k$**

Sol. $\vec{a}.\vec{b} = (i + 2j + 2k)\bullet(3i - 2j - 4k)$

$$= (1)(3) + (2)(-2) + (2)(-4)$$

$$= 3 - 4 - 8$$

$$= \boxed{-9}$$

21. **For what value of λ, the vectors $2i - j + 2k$ & $3i + 2\lambda j$ are perpendicular.**

Sol. Let, $\vec{a} = 2i - j + 2k$ & $\vec{b} = 3i + 2\lambda j$

As given vectors are perpendicular.

So, $\vec{a}.\vec{b} = 0$

$\Rightarrow$ $(2i - j + 2k).(3i + 2\lambda j) = 0$

$\Rightarrow$ $(2)(3) + (-1)(2\lambda) + (2)(0) = 0$

$\Rightarrow$ $6 - 2\lambda + 0 = 0$

$\Rightarrow$ $-2\lambda = -6$

$\Rightarrow$ $\lambda = \dfrac{-6}{-2}$

$\Rightarrow$ $\boxed{\lambda = 3}$

22. **Find scalar x and y such that $x(i + 2j) + y(3i + 4j) = 7i + 9j$**

Sol. $x(i + 2j) + y(3i + 4j) = 7i + 9j$

$$xi + 2xj + 3yi + 4yj = 7i + 9j$$
$$(x+3y)i + (2x+4y)j = 7i + 9j$$

Comparing coefficients of i & j, we have:

$$x + 3y = 7 \rightarrow (i) \mid 2x + 4y = 9 \rightarrow (ii)$$

Multiplying eq. (i) by 2:

$$2x + 6y = 14 \rightarrow (iii)$$

Subtracting eq. (iii) & eq. (ii)

$$\begin{array}{r} 2x + 6y = 14 \\ -2x \pm 4y = -9 \\ \hline 2y = 5 \end{array}$$

$$\Rightarrow \boxed{y = 5/2}$$

Put $y = 5/2$ in eq. (i)

$$x + 3\left(\frac{5}{2}\right) = 7$$
$$x = 7 - \frac{15}{2}$$
$$x = \frac{14-15}{2}$$
$$x = \boxed{-\frac{1}{2}}$$

23. Define scalar matrix.

Sol. A diagonal matrix in which all diagonal elements are same is called scalar matrix.

24. Find x and y if

$$\begin{bmatrix} x+3 & 1 \\ -3 & 3y-4 \end{bmatrix} = \begin{bmatrix} y & 1 \\ -3 & 2x \end{bmatrix}$$

Sol. $$\begin{bmatrix} x+3 & 1 \\ -3 & 3y-4 \end{bmatrix} = \begin{bmatrix} y & 1 \\ -3 & 2x \end{bmatrix}$$

Comparing corresponding elements of both matrices:

$$x + 3 = y \text{ and } 3y - 4 = 2x$$
$$x - y = -3 \rightarrow (i) \mid 2x - 3y = -4 \rightarrow (ii)$$

Multipling eq. (i) by 2 and subtracting eq. (ii)

$$\begin{array}{r} 2x - 2y = -6 \\ -2x \mp 3y = \mp 4 \\ \hline y = -2 \end{array}$$

$$\boxed{y = -2}$$

Put $y = -2$ in eq. (i)

$$x - (-2) = -3$$
$$x = -3 - 2$$
$$\boxed{x = -5}$$

25. ~~**Without expansion, verify that:**~~

$$\begin{vmatrix} \alpha & \beta+\gamma & 1 \\ \beta & \gamma+\alpha & 1 \\ \gamma & \alpha+\beta & 1 \end{vmatrix} = 0$$

Sol. $$\text{L.H.S.} = \begin{vmatrix} \alpha & \beta+\gamma & 1 \\ \beta & \gamma+\alpha & 1 \\ \gamma & \alpha+\beta & 1 \end{vmatrix}$$

$$= \begin{vmatrix} \alpha+\beta+\gamma & \beta+\gamma & 1 \\ \alpha+\beta+\gamma & \gamma+\alpha & 1 \\ \alpha+\beta+\gamma & \alpha+\beta & 1 \end{vmatrix} \quad \text{By } C_1 + C_2$$

$$= (\alpha+\beta+\gamma)\begin{vmatrix} 1 & \beta+\gamma & 1 \\ 1 & \gamma+\alpha & 1 \\ 1 & \alpha+\beta & 1 \end{vmatrix} \quad \text{By taking } (\alpha+\beta+\gamma) \text{ common from } C_1$$

$$= (\alpha+\beta+\gamma)(0) \quad \because C_1 \equiv C_3$$

$$= 0 = \text{R.H.S.}$$ **Proved.**

26. If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$ **then find AB.**

Sol. $$AB = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 2+8 & 3+10 \\ 6+16 & 9+20 \end{bmatrix}$$

$$= \boxed{\begin{bmatrix} 10 & 13 \\ 22 & 29 \end{bmatrix}}$$

27. If $\begin{bmatrix} 2 & 3 \\ 4 & k \end{bmatrix}$ is singular, then find k.

Sol. As given matrix is singular, so

$$\begin{vmatrix} 2 & 3 \\ 4 & k \end{vmatrix} = 0$$

$\Rightarrow \quad 2k - 12 = 0$

$\Rightarrow \quad 2k = 12$

$\Rightarrow \quad k = \frac{12}{2}$

$\Rightarrow \quad \boxed{k = 6}$

Section - II

Note : Attemp any three (3) questions $3 \times 8 = 24$

Q.2.(a) Given the vectors $\vec{a} = 3i - 2j + 4k$ and $\vec{b} = 3i - 2j + 4k$ find the magnitude and direction cosines of $3\vec{a} - 2\vec{b}$

Sol. See Q.9(ii) of Ex # 8.1 (Page # 374)

(b) Find the sine and the unit vector perpendicular to each : $\vec{a} = i + j + k$ and $\vec{b} = 2i + 3j - 4k$

Sol. See Q.19(i) of Ex # 8.2 (Page # 388)

Q.3. Solve by Cramer's Rule

$$x - 2y + z = -1$$
$$3x + y - 2z = 4$$
$$y - z = 1$$

Sol. See Q.8(iii) of Ex # 9.2 (Page # 429)

Q.4.(a) From the point within an Equilateral triangle perpendicular are drawn to the three sides are 6, 7 and 8cm respectively. Find the area of triangle.

Sol. See Q.2 of Ex # 10 (Page # 462)

(b) Find area of an irregular figure by Simpson's Rule if the ordinates are 9, 11, 13, 12, 10, 13, 15, 17, 14, 12, 7 meters and base is 73 meters.

Sol. See Q.6 of Ex # 14 (Page # 509)

Q.5.(a) A regular decagon is inscribed in a circle the radius of which is 10cm. Find the area of the decagon.

Sol. See Q.2 of Ex # 12 (Page # 486)

(b) The radius of a right circular cylinder is 25cm and its height is 15cm. Find its volume, lateral surface and the whole surface area.

Sol. See example # 02 of Ch # 16

Q.6.(a) Find the volume and the total surface area of a cone of radius 6.6cm and height of 12.5cm.

Sol. See example # 01 of Ch # 18

(b) Two spheres each a 10m diameter are melted down and recast into a cone with a height equal to the radius of its base. Find the height of the cone.

Sol. See example # 02 of Ch # 19

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