DAE / IIA - 2017

MATH - 113 APPLIED MATHEMATICS - I

PAPER 'A' PART - A (OBJECTIVE)

Time: 30 Minutes Marks: 15

Q.1: *Encircle the correct answer.*

1. The roots of the equation $x^2 + 4x - 21 = 0$ are:

[a] $(7, 3)$ [b] $(-7, 3)$

[c] $(-7, -3)$ [d] $(7, -3)$

2. A second degree equation is known as:

[a] Linear [b] Quadratic

[c] Cubic [d] None of these

3. The 10^{th} term in $7, 17, 27, \ldots$ is:

[a] 97 [b] 98 [c] 99 [d] 100

4. Arithmetic mean between -7 and 7 is:

[a] $\frac{7}{2}$ [b] $-\frac{7}{2}$

[c] 0 [d] 14

5. The sum of infinite geometric series $1 + \frac{1}{3} + \frac{1}{9} + \ldots$ is

[a] $\frac{2}{3}$ [b] $-\frac{2}{3}$

[c] $\frac{3}{2}$ [d] $-\frac{3}{2}$

6. The second last term in the expansion $(a + b)^7$ is:

[a] $7a^2b$ [b] $7ab^6$

[c] $7b^7$ [d] 15

7. $\binom{3}{0}$ will have the value:

[a] 0 [b] 1

[c] 2 [d] 3

8. The number of partial fraction is $\frac{x^3 - 3x^2 - 1}{(x-1)(x+1)(x^2-1)}$ are:

[a] 2 [b] 3

[c] 4 [d] 5

9. $\frac{2\pi}{3}$ radians are equal to:

[a] $60°$ [b] $90°$

[c] $120°$ [d] $150°$

10. The terminal side of 'θ' lies in 4^{th} quadrant, the sign of sinθ will be

[a] Positive [b] Negative

[c] Both a and b [d] None of these

11. $\tan(45° - x)$ is equal to:

[a] $\frac{\cos x + \sin x}{\cos x - \sin x}$ [b] $\frac{1 + \tan x}{1 - \tan x}$

[c] $\frac{1 + \cot x}{1 - \cot x}$ [d] $\frac{\cos x - \sin x}{\cos x + \sin x}$

12. $\cos(\pi + \theta)$ is equal to:

[a] $\cos\theta$ [b] $-\sin\theta$

[c] $-\cos\theta$ [d] $\sin\theta$

13. $\sin(A + B) - \sin(A - B)$ is equal to:

[a] $2\sin A\cos B$

[b] $2\cos A\sin B$

[c] $-2\sin A\sin B$

[d] $2\cos A\cos B$

14. If a = 2, b = 2, ∠A = 30º then ∠B is equal to:

[a] $30°$ [b] $45°$

[c] $60°$ [d] $75°$

15. In a right angled triangle, if one angle is 30º the other will be:

[a] $45°$ [b] $50°$

[c] $60°$ [d] $75°$

Answer Key

1	b	2	b	3	a	4	c	5	c
6	b	7	b	8	c	9	c	10	b
11	d	12	c	13	b	14	a	15	c

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DAE / IIA - 2017

MATH - 113 APPLIED MATHEMATICS - I

PAPER 'A' PART - B (SUBJECTIVE)

Time: 2:30 Hrs Marks: 60

Section - I

Q.1. Write short answers to any Eighteen (18) questions.

1. Solve the quadratic equation:

$$x^2+7x+12=0$$

Sol. $x^2+7x+12=0$

$x^2+4x+3x+12=0$

$x(x+4)+3(x+4)=0$

$(x+4)(x+3)=0$

Either $x+4=0$, $x=-4$ OR $x+3=0$, $x=-3$

$S.S.=\{-3,-4\}$

2. ~~Find the sum and product of the roots of the equation~~

$$9x^2+6x+1=0$$

Sol. Here: $a=9,\ b=6,\ c=1$

Sum of Roots: $S=-\frac{b}{a}=-\frac{6}{9}=\boxed{-\frac{2}{3}}$

Product of Roots: $P=\frac{c}{a}=\boxed{\frac{1}{9}}$

3. ~~Form the quadratic equation whose roots are~~ $-2+\sqrt{3},\ -2-\sqrt{3}$

Sol. $S=-2+\sqrt{3}+\left(-2-\sqrt{3}\right)$

$S=-2+\sqrt{3}-2-\sqrt{3}=-4$

$P=\left(-2+\sqrt{3}\right)\left(-2-\sqrt{3}\right)$

$P=(-2)^2-\left(\sqrt{3}\right)^2$

$P=4-(3)=1$

$x^2-Sx+P=0$

$x^2-(-4)x+1=0\Rightarrow\boxed{x^2+4x+1=0}$

4. Define a sequence.

Sol. A set of numbers arranged in order by some fixed rule is called a sequence. For examples:
(i) $2, 4, 6,\ldots$ **(ii)** $3, 9, 27,\ldots$

5. Find the 7th term of A.P., in which the first term is 7 and the common difference is -3.

Sol. Here: $a_7=?$, $a_1=7$ & $d=-3$

$a_7=a+6d$

$a_7=7+6(-3)$

$a_7=7-18\quad\Rightarrow\quad\boxed{a_7=-11}$

6. ~~Write the formula to find the sum of 'n' terms of an arithmetic progression.~~

Sol. $\boxed{S_n=\frac{n}{2}\left[2a_1+(n-1)d\right]}$

7. Find the A.M between $\sqrt{5}-4$ and $\sqrt{5}+4$

Sol. Let $a=\sqrt{5}-4$ and $b=\sqrt{5}+4$

$A.M.=A=\frac{a+b}{2}$

$A=\frac{\sqrt{5}-4+\sqrt{5}+4}{2}=\frac{2\sqrt{5}}{2}=\boxed{\sqrt{5}}$

8. Write down the geometric sequence in which the 1st term is 2 and second term is -6 and n = 5.

Sol. Here: $a_1=2,\ a_2=-6$ & $n=5.$

$r=\frac{a_2}{a_1}=\frac{-6}{2}=-3$

$a_3=ar^2=2(-3)^2=2(9)=\boxed{18}$

$a_4=ar^3=2(-3)^3=2(-27)=\boxed{-54}$

$a_5=ar^4=2(-3)^4=2(81)=\boxed{162}$

Hence $\boxed{2,-6,18,-54,162,\ldots}$ is required G.P.

9. Find the Geometric mean between 8 and 72.

Sol. Let : $a = 8$ & $b = 72$

As, $G = \pm\sqrt{ab}$

$G = \pm\sqrt{8 \times 72} = \pm\sqrt{576} = \boxed{\pm 24}$

10. **Expand** $\left(\mathbf{x} + \frac{\mathbf{1}}{\mathbf{x}}\right)^4$.

Sol. By using binomial theorem.

$$= \binom{4}{0}(x)^4\left(\frac{1}{x}\right)^0 + \binom{4}{1}(x)^3\left(\frac{1}{x}\right)^1 + \binom{4}{2}(x)^2\left(\frac{1}{x}\right)^2 + \binom{4}{3}(x)^1\left(\frac{1}{x}\right)^3 + \binom{4}{4}(x)^0\left(\frac{1}{x}\right)^4$$

$$= (1)(x^4)(1) + (4)(x^3)\left(\frac{1}{x}\right) + (6)(x^2)\left(\frac{1}{x^2}\right) + (4)(x)\left(\frac{1}{x^3}\right) + (1)(1)\left(\frac{1}{x^4}\right)$$

$$= \boxed{x^4 + 4x^2 + 6 + \frac{4}{x^2} + \frac{1}{x^4}}$$

11. **Find the 7th term in the expansion of** $\left(\mathbf{x} - \frac{\mathbf{1}}{\mathbf{x}}\right)^9$

Sol. Here : $a = x$, $b = -\frac{1}{x}$, $n = 9$ & $r = 6$

Using general term formula:

$$T_{r+1} = \binom{n}{r} a^{n-r} b^r$$

$$T_{6+1} = \binom{9}{6}(x)^{9-6}\left(-\frac{1}{x}\right)^6$$

$$T_7 = 84x^3\left(\frac{1}{x^6}\right) = \frac{84}{x^3}$$

12. ~~**Expand to three terms,**~~ $(\mathbf{1} + \mathbf{2x})^{-2}$

Sol. $(1 + 2x)^{-2}$

$$= 1 + (-2)(2x) + \frac{(-2)(-2-1)}{2!}(2x)^2 + \dots$$

$$= 1 - 4x + \frac{(-2)(-3)}{2}(4x^2) + \dots$$

$$= \boxed{1 - 4x + 12x^2 + \dots}$$

13. **Define an example of proper fraction.**

Sol. **Example:** $\frac{2x}{(x-2)(x+5)}$

14. **Resolve into partial fractions** $\frac{\mathbf{2x}}{(\mathbf{x}-\mathbf{2})(\mathbf{x}+\mathbf{5})}$

Sol. $\frac{2x}{(x-2)(x+5)} = \frac{A}{x-2} + \frac{B}{x+5} \rightarrow (i)$

$2x = A(x+5) + B(x-2) \rightarrow (ii)$

Put $x = 2$ in eq. (ii)

$2(2) = A(2+5) + B(2-2)$

$4 = A(7) + B(0)$

$4 = 7A + 0 \Rightarrow \boxed{A = 4/7}$

Put $x = -5$ in eq. (ii)

$2(-5) = A(-5+5) + B(-5-2)$

$-10 = A(0) + B(-7)$

$-10 = 0 - 7B \Rightarrow \boxed{B = 10/7}$

Put values of A, & B in eq. (i),

we get: $\boxed{\frac{4}{7(x-2)} + \frac{10}{7(x+5)}}$

15. **Write identity equation of** $\frac{\mathbf{x}-\mathbf{5}}{(\mathbf{x}+\mathbf{1})(\mathbf{x}^2+\mathbf{3})}$

Sol. $\frac{x-5}{(x+1)(x^2+3)} = \frac{A}{(x+1)} + \frac{Bx+C}{(x^2+3)}$

16. ~~Convert 120° into radians measure.~~

Sol. $120° = 120 \times \frac{\pi}{180} = \frac{2\pi}{3} = \boxed{2.09\,\text{rad}}$

17. ~~What is the length of an arc of a circle of radius 5 cm whose central angle is 140°.~~

Sol. Here: $\ell = ?$, $r = 5\text{cm}$, $\theta = 140°$

$\theta = 140° = 140 \times \frac{\pi}{180} = 2.44\text{rad}.$

By using formula: $\ell = r\theta$

$\ell = r\theta = (5)(2.44) = \boxed{12.22\text{cm}}$

18. **Prove that:**

$$\tan^2 30° + \tan^2 45° + \tan^2 60° = \frac{13}{3}$$

Sol. $\text{L.H.S.} = \tan^2 30° + \tan^2 45° + \tan^2 60°$

$= \left(\frac{1}{\sqrt{3}}\right)^2 + (1)^2 + (\sqrt{3})^2 = \frac{1}{3} + 1 + 3$

$= \frac{1+3+9}{3} = \frac{13}{3} = \text{R.H.S.}$ **Proved.**

19. **Prove that:**

$$\cos^4\theta - \sin^4\theta = 1 - 2\sin^2\theta$$

Sol. $\text{L.H.S.} = \cos^4\theta - \sin^4\theta$

$= (\cos^2\theta)^2 - (\sin^2\theta)^2$

$= (\cos^2\theta - \sin^2\theta)(\cos^2\theta + \sin^2\theta)$

$= (1 - \sin^2\theta - \sin^2\theta)(1)$

$= 1 - 2\sin^2\theta = \text{R.H.S.}$ **Proved.**

20. **Prove that:** $\sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta$

Sol. $\text{L.H.S.} = \sin\left(\frac{\pi}{2} - \theta\right)$

$= \sin(90° - \theta) \because \left\{\frac{\pi}{2} \times \frac{180°}{\pi} = 90°\right\}$

$= \sin 90° \cos\theta + \cos 90° \sin\theta$

$= (1)\cos\theta + (0)\sin\theta \because \left\{\begin{array}{c}\text{Using calculator}\\ \cos 90°=0 \;\&\; \sin 90°=1\end{array}\right\}$

$= \cos\theta + 0 = \cos\theta = \text{R.H.S.}$ **Proved.**

21. **Show that:**

$$\cos(\alpha+\beta) - \cos(\alpha-\beta) = -2\sin\alpha\sin\beta$$

Sol. $\text{L.H.S.} = \cos(\alpha+\beta) - \cos(\alpha-\beta)$

$= [\cos\alpha\cos\beta - \sin\alpha\sin\beta] - [\cos\alpha\cos\beta + \sin\alpha\sin\beta]$

$= \cancel{\cos\alpha\cos\beta} - \sin\alpha\sin\beta - \cancel{\cos\alpha\cos\beta} - \sin\alpha\sin\beta$

$= -2\sin\alpha\sin\beta = \text{R.H.S.}$ **Proved.**

22. **Prove that:**

$$\tan(45° + \theta)\tan(45° - \theta) = 1$$

Sol. $\text{L.H.S.} = \tan(45° + \theta)\tan(45° - \theta)$

$= \frac{\tan 45° + \tan\theta}{1 - \tan 45° \tan\theta} \times \frac{\tan 45° - \tan\theta}{1 + \tan 45° \tan\theta}$

$= \frac{1 + \tan\theta}{1 - (1)\tan\theta} \times \frac{1 - \tan\theta}{1 + (1)\tan\theta} \because \left\{\begin{array}{c}\text{Using calculator}\\ \tan 45°=1\end{array}\right\}$

$= \frac{1 + \tan\theta}{1 - \tan\theta} \times \frac{1 - \tan\theta}{1 + \tan\theta} = 1 = \text{R.H.S.}$ **Proved.**

23. **Find cosθ, if $\sin\theta = \frac{7}{25}$ and angle θ is an acute angle.**

Sol. As, we know that: $\cos^2\theta = 1 - \sin^2\theta$

$\cos^2\theta = 1 - \left(\frac{7}{25}\right)^2 = 1 - \frac{49}{625} = \frac{625 - 49}{625}$

$\cos^2\theta = \frac{576}{625} \Rightarrow \sqrt{\cos^2\theta} = \pm\sqrt{\frac{576}{625}}$

As, θ is an acute angle so, $\boxed{\cos\theta = \frac{24}{25}}$

24. **Define the laws of cosines.**

Sol.
i. $a^2 = b^2 + c^2 - 2bc\cos\alpha$
ii. $b^2 = c^2 + a^2 - 2ca\cos\beta$
iii. $c^2 = a^2 + b^2 - 2ab\cos\gamma$

25. **The sides of a triangle are 16, 20 and 33 meters respectively. Find its greatest angle.**

Sol. Let $a = 16$, $b = 20$, $c = 33$

As side 'c' is greatest so, we will find angle 'γ'.

Using law of cosines:

$$\cos\gamma = \frac{a^2+b^2-c^2}{2ab}$$

$$\cos\gamma = \frac{(16)^2+(20)^2-(33)^2}{2(16)(20)}$$

$$\cos\gamma = -0.6765$$

$$\gamma = \cos^{-1}(-0.6765) \Rightarrow \boxed{\gamma = 132°34'}$$

26. **In any triangle ABC in which a = 16, b = 17, γ = 25º, find 'c'.**

Sol. By using law of cosines:

$$c^2 = a^2+b^2-2ab\cos\gamma$$

$$c^2 = (16)^2+(17)^2-2(16)(17)\cos 25°$$

$$c^2 = 256+289-493.03$$

$$c^2 = 51.97 \Rightarrow \sqrt{c^2} = \sqrt{51.97} \Rightarrow \boxed{c = 7.2}$$

27. **In any triangle ABC if a = 3, b = 7, β = 85º find α.**

Sol. By using law of sines:

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$$

$$\text{We take: } \frac{a}{\sin\alpha} = \frac{b}{\sin\beta}$$

$$\sin\alpha = \frac{a\sin\beta}{b}$$

$$\sin\alpha = \frac{3\sin 85°}{7}$$

$$\sin\alpha = 0.4269$$

$$\alpha = \sin^{-1}(0.4269) \Rightarrow \boxed{\alpha = 25°16'}$$

Section - II

Note: Attemp any three (3) questions $3\times 8 = 24$

Q.2.(a) Solve the equation $\frac{1}{x+1}+\frac{1}{x+2}+\frac{1}{x+3}=\frac{3}{x}$ by using quadratic formula.

Sol. See Q.3(iii) of Ex # 1.1 (Page # 20)

(b) **For what value of k the roots of the equation $x^2 + 2(k-2)x - 8k = 0$ are equal.**

Sol. See Q.2(ii) of Ex # 1.2 (Page # 31)

Q.3.(a) The 9th term of an A.P is 30 and the 17th term is 50. Find the first three terms.

Sol. See Q.8 of Ex # 2.1 (Page # 80)

(b) ~~**Sum the series: 5 + 3 + 1 – 1 – ...... to 10 terms.**~~

Sol. See Q.1(i) of Ex # 2.3 (Page # 89)

Q.4. **Find the term independent of x in the expansion of $\left(2x^2+\frac{1}{x}\right)^9$.**

Sol. See Q.10(ii) of Ex # 3.1 (Page # 162)

Q.5.(a) Prove that:

$$\sqrt{\frac{1-\sin\theta}{1+\sin\theta}} = \sec\theta - \tan\theta$$

Sol. See Q.9 of Ex # 5.3 (Page # 256)

(b) **Show that:**

$$\sqrt{3}\cos\theta - \sin\theta = 2\cos(\theta+30°)$$

Sol. See Q.5(ii) of Ex # 6.1 (Page # 279)

Q.6.(a) Prove that:

$$\cos 3\theta = 4\cos^3\theta - 3\cos\theta$$

Sol. See Triple angles proof (Page # 294)

(b) **Solve the triangle ABC with given data: $c = 4,\ \alpha = 70°,\ \gamma = 42°$.**

Sol. See Q.1 of Ex # 7.5 (Page # 354)

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