DAE / IIA - 2017

MATH-123 APPLIED MATHEMATICS - I

PAPER 'B' PART - A (OBJECTIVE)

Time: 30 Minutes Marks: 15

Q.1: *Encircle the correct answer.*

1. The Fractions $\frac{2x+5}{x^2+5x+6}$ is known as:

[a] Proper [b] Improper
[c] Neither proper nor improper
[d] None of these

2. If the degree of numerator is less than the degree of denominator, then fraction is:

[a] Proper [b] Improper
[c] Neither a and b
[d] Both a & b

3. Modulus of $3+4i$ is:

[a] 47 [b] 16
[c] 5 [d] 3

4. If $z = a + bi$ then $z + \bar{z}$ is equal to:

[a] 2a [b] 2b
[c] 0 [d] $2a + 2bi$

5. The additive inverse of $a + ib$ is:

[a] $-a + ib$ [b] $a - ib$
[c] $-a - ib$ [d] $a + ib$

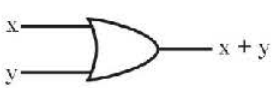
6. In Boolean Algebra $X + \bar{X}Y$ is equal to:

[a] X [b] $\bar{X}$
[c] X+ Y [d] $\bar{X} + Y$

7. $X(\bar{X} + Y)$ equal to:

[a] X [b] $X.\bar{X}$
[c] $X.\bar{X} + XY$ [d] X+ Y

8.



is the symbol for the logic:

[a] OR gate [b] NOR gate
[c] NAND gate
[d] AND gate

9. Slope of the line $\frac{x}{a} + \frac{y}{b} = 1$ is:

[a] $\frac{a}{b}$ [b] $\frac{b}{a}$
[c] $-\frac{b}{a}$ [d] $\frac{-a}{b}$

10. $y = 2$ is a line parallel to:

[a] x − axis [b] y − axis
[c] y = x [d] x = 3

11. ~~Slope of line through (x_1, y_1) and (x_2, y_2) is:~~

[a] $\frac{x_1 + x_2}{y_1 + y_2}$ [b] $\frac{y_2 + y_1}{x_2 + y_1}$
[c] $\frac{y_2 - y_1}{x_2 - x_1}$ [d] None of these

12. Given three points are collinear if their slopes are:

[a] equal [b] unequal
[c] $m_1 m_2 = -1$ [d] None of these

13. When two lines are perpendicular then:

[a] $m_1 = m_2$ [b] $m_1 m_2 = -1$
[c] $m_1 = -m_2$ [d] None of these

14. Straight line from center to the circumference of a circle is:

[a] Circle [b] Radius
[c] Diameters [d] None of these

15. Center of the circle $x^2 + y^2 - 2x - 4y = 8$ is:

[a] (1, 2) [b] (2, 4)
[c] (1, 3) [d] None of these

Answer Key

1	a	2	a	3	c	4	a	5	c
6	a	7	c	8	a	9	c	10	a
11	c	12	a	13	b	14	b	15	a

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DAE / IIA - 2017

MATH-123 APPLIED MATHEMATICS-I

PAPER 'B' PART-B (SUBJECTIVE)

Time: 2:30 Hrs Marks: 60

Section - I

Q.1. Write short answers to any Eighteen (18) questions.

1. Resolve into partial fractions $\frac{1}{x^2-1}$

Sol. $\frac{1}{x^2-1}=\frac{1}{(x)^2-(1)^2}=\frac{1}{(x-1)(x+1)}$

$\frac{1}{(x-1)(x+1)}=\frac{A}{x-1}+\frac{B}{x+1}\rightarrow(i)$

$1=A(x+1)+B(x-1)\rightarrow(ii)$

Put $x=1$ in eq. (ii)

$1=A(1+1)+B(1-1)$

$1=A(2)+B(0)$

$1=2A+0 \Rightarrow \boxed{A=\frac{1}{2}}$

Put $x=-1$ in eq. (ii)

$1=A(-1+1)+B(-1-1)$

$1=A(0)+B(-2)$

$1=0-2B \Rightarrow \boxed{B=-\frac{1}{2}}$

Put values of A, & B in eq. (i),

we get: $\boxed{\frac{1}{2(x-1)}-\frac{1}{2(x+1)}}$

2. ~~Form of partial fractions of~~ ~~$\frac{2x^4-3x^2-4x}{(x+1)(x^2+2)^2}$~~

Sol.

$\frac{2x^4-3x^2-4x}{(x+1)(x^2+2)^2}=\boxed{\frac{A}{(x+1)}+\frac{Bx+C}{(x^2+2)}+\frac{Dx+E}{(x^2+2)^2}}.$

3. Write an identity equation of $\frac{2x}{(x-2)(x+5)}$

Sol. $\frac{2x}{(x-2)(x+5)}=\frac{A}{x-2}+\frac{B}{x+5}\rightarrow(i)$

$2x=A(x+5)+B(x-2)\rightarrow(ii)$

Put $x=2$ in eq. (ii)

$2(2)=A(2+5)+B(2-2)$

$4=A(7)+B(0)$

$4=7A+0 \Rightarrow \boxed{A=\frac{4}{7}}$

Put $x=-5$ in eq. (ii)

$2(-5)=A(-5+5)+B(-5-2)$

$-10=A(0)+B(-7)$

$-10=0-7B \Rightarrow \boxed{B=\frac{10}{7}}$

Put values of A, & B in eq. (i),

we get: $\boxed{\frac{4}{7(x-2)}+\frac{10}{7(x+5)}}$

4. **What is partial fractions.**

Sol. The process, which convert a single rational fraction, into the sum of two or more single rational fractions is called partial fractions.

5. Factorize $9a^2+64b^2$

Sol. $9a^2+64b^2=9a^2-64b^2i^2$

$=(3a)^2-(8bi)^2$

$=\boxed{(3a-8bi)(3a+8bi)}$

6. **Simplify the complex numbers:** $(2+5i)+(-3+i)$

Sol. $(2+5i)+(-3+i)$

$=2+5i-3+i=\boxed{-1+6i}$

7. **Prove that if $Z=\overline{Z}$ then $\overline{Z}$ is real.**

Sol. Let $Z=a+bi \rightarrow (i)$

then $\overline{Z}=a-bi \rightarrow (ii)$

Given that : $Z=\overline{Z}$

$\Rightarrow a+bi=a-bi$ {By using eq.(i) & eq.(ii)}

$\Rightarrow a+bi-a+bi=0$

$\Rightarrow 2bi=0$

$\Rightarrow b=0 \quad \because 2i\neq 0$

Put $b=0$ in eq.(ii)

$\overline{Z}=a-0i=a\in\mathbb{R}$

Hence $\overline{Z}$ is real. **Proved.**

8. **Find the multiplicative of $(-3\ ,\ 4)$.**

Sol. Let $z=(-3,4)=-3+4i$

Multiplicative Inverse of $Z=\frac{1}{Z}$

$$=\frac{1}{-3+4i}=\frac{1}{-3+4i}\times\frac{-3-4i}{-3-4i}$$

$$=\frac{-3-4i}{(-3)^2-(4i)^2}=\frac{-3-4i}{9+16}$$

$$=\frac{-3-4i}{25}=\boxed{-\frac{3}{25}-\frac{4}{25}i}$$

9. **Show that $\left|\frac{1+2i}{2-i}\right|=1$**

Sol. $\text{L.H.S.}=\frac{1+2i}{2-i}$

$$=\frac{\sqrt{(1)^2+(2)^2}}{\sqrt{(2)^2+(-1)^2}}$$

$$=\frac{\sqrt{1+4}}{\sqrt{4+1}}=\frac{\sqrt{5}}{\sqrt{5}}=1=\text{R.H.S.}$$ **Proved.**

10. **Multiply the binary numbers**

$111_2\times 101_2$

Sol.

$$\begin{array}{r} 111 \\ 101 \\ \hline 111 \\ 000\times \\ 111\times\times \\ \hline 100011 \end{array}$$

$111_2\times 101_2=\boxed{(100011)_2}$

11. **Convert octal number $(107)_8$ to binary number.**

Sol.

$$\begin{array}{c|l} 2 & 1 \\ \hline & 0\cdot 1 \end{array} \qquad \begin{array}{c|l} 2 & 0 \\ \hline & 0\cdot 0 \end{array} \qquad \begin{array}{c|l} 2 & 7 \\ \hline 2 & 3\cdot 1 \\ \hline & 1\cdot 1 \end{array}$$

001 000 111

$(107)_8=\boxed{(001000111)_2}$

12. **Convert the decimal number $(932)_{10}$ to octal number.**

Sol.

$$\begin{array}{c|r} 8 & 932 \\ \hline 8 & 116-4 \\ \hline 8 & 14-4 \\ \hline & 1-6 \end{array}$$

$\boxed{(1644)_8}$

13. **Define logic gate.**

Sol. A logic gate is defined as an electronics circuit with two or more input signals and one output signal.

14. **Prove by Boolean Algebra rules:**

$\mathbf{X+XZ=X}$

Sol. $\text{L.H.S.}=X+XZ$

$=X(1+Z)$

$=X(1) \quad \because 1+Z=1$

$=X=\text{R.H.S.}$ **Proved.**

15. Write distance formula between two points and give one example.

Sol. Let $A(x_1, y_1)$ & $B(x_2, y_2)$ be two different points, then,

$$\text{Distance} = |AB| = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

Example: Let A(0, 0), B(1, 1) be two points, then

$$|\overline{AB}| = \sqrt{(0-1)^2 + (0-1)^2}$$

$$|\overline{AB}| = \sqrt{1+1} = \sqrt{2}$$

16. Show that the points $(1, 0)$, $(4, -12)$ and $(2, -4)$ are collinear.

Sol.
$$\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 1 \\ 4 & -12 & 1 \\ 2 & -4 & 1 \end{vmatrix}$$

$$= 1\begin{vmatrix} -12 & 1 \\ -4 & 1 \end{vmatrix} - 0\begin{vmatrix} 4 & 1 \\ 2 & 1 \end{vmatrix} + 1\begin{vmatrix} 4 & -12 \\ 2 & -4 \end{vmatrix}$$

$$= 1(-12 - (-4)) - 0(4-2) + 1(-16 - (-24))$$

$$= 1(-12+4) - 0(2) + 1(-16+24)$$

$$= 1(-8) - 0 + 1(8) = -8 + 8 = 0$$

Hence given points are collinear. **Proved.**

17. ~~Find the equation of a line through the points $(-1, 2)$ and $(3, 4)$.~~

Sol. $\text{Slope} = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{4-2}{3-(-1)} = \dfrac{2}{4} = \dfrac{1}{2}$

Equation of line in point-slope form:

$$y - y_1 = m(x - x_1)$$

$$y - 2 = \frac{1}{2}(x - (-1))$$

$$2(y-2) = 1(x+1)$$

$$2y - 4 = x + 1$$

$$2y - 4 - x - 1 = 0$$

$$-x + 2y - 5 = 0 \Rightarrow \boxed{x - 2y + 5 = 0}$$

18. ~~Find the distance from the point $(-2, 1)$ to the line $3x + 4y - 12 = 0$~~

Sol.

Distance between point & line

$$D = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

$$D = \frac{|3(-2) + 4(1) - 12|}{\sqrt{(3)^2 + (4)^2}}$$

$$D = \frac{|-6+4-12|}{\sqrt{9+16}} = \frac{|-14|}{\sqrt{25}} = \boxed{\frac{14}{5}}$$

19. Find the coordinates of the mid-point of the segment $P_1(3, 7)$ and $P_2(-2, 3)$.

Sol. $\text{Mid-point} = \left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$

$$= \left(\frac{3 + (-2)}{2}, \frac{7+3}{2}\right)$$

$$= \left(\frac{3-2}{2}, \frac{10}{2}\right) = \boxed{\left(\frac{1}{2}, 5\right)}$$

20. Reduce the equation $3x + 4y - 2 = 0$ into intercept form.

Sol. $3x + 4y - 2 = 0$

$$3x + 4y = 2$$

Dividing both sides by 2, we have:

$$\frac{3x}{2} + \frac{4y}{2} = \frac{2}{2}$$

$$\frac{x}{2/3} + \frac{y}{2/4} = 1 \Rightarrow \boxed{\frac{x}{2/3} + \frac{y}{1/2} = 1}$$

21. ~~Find equation of line when $\theta = 45°$ and $P = \frac{1}{\sqrt{2}}$.~~

Sol. Since the equation of straight line in the Normal form is

$$x\cos\theta + y\sin\theta = p$$

Put $\theta = 45°$ & $p = \frac{1}{\sqrt{2}}$

$$x\cos 45° + y\sin 45° = \frac{1}{\sqrt{2}}$$

$$x\left(\frac{1}{\sqrt{2}}\right) + y\left(\frac{1}{\sqrt{2}}\right) = \frac{1}{\sqrt{2}}$$

Multipling both by $\sqrt{2}$, we get:

$$x + y = 1 \Rightarrow \boxed{x + y - 1 = 0}$$

22. **Find 'k' so that the lines $\mathbf{x - 2y + 1 = 0}$, $\mathbf{2x - 5y + 3 = 0}$, $\mathbf{5x + 9y + k = 0}$ are concurrent.**

Sol.

$$\begin{vmatrix} 1 & -2 & 1 \\ 2 & -5 & 3 \\ 5 & 9 & k \end{vmatrix} = 0$$

$$1\begin{vmatrix} -5 & 3 \\ 9 & k \end{vmatrix} - (-2)\begin{vmatrix} 2 & 3 \\ 5 & k \end{vmatrix} + 1\begin{vmatrix} 2 & -5 \\ 5 & 9 \end{vmatrix} = 0$$

$$1(-5k - 27) + 2(2k - 15) + 1(18 + 25) = 0$$

$$-5k - 27 + 4k - 30 + 43 = 0$$

$$-k - 14 = 0$$

$$-k = 14 \Rightarrow \boxed{k = -14}$$

23. ~~Find the equation of line having x-intercept -2 and y-intercept 3.~~

Sol. Let, $x - \text{intercept} = a = -2$

& $y - \text{intercept} = b = 3$

Equation of line in intercept form: $\frac{x}{a} + \frac{y}{b} = 1$

$$\Rightarrow \frac{x}{-2} + \frac{y}{3} = 1$$

$$\Rightarrow \frac{-3x + 2y}{6} = 1$$

$$\Rightarrow -3x + 2y = 6$$

$$\Rightarrow -3x + 2y - 6 = 0$$

$$\Rightarrow \boxed{3x - 2y + 6 = 0}$$

24. **Find the equation of circle with center on origin and radius is ½.**

Sol. Standard form of eq. of circle:

$$(x - h)^2 + (y - k)^2 = r^2$$

Put $h = 0$, $k = 0$ & $r = \frac{1}{2}$

$$(x - 0)^2 + (y - 0)^2 = \left(\frac{1}{2}\right)^2$$

$$\boxed{x^2 + y^2 - \frac{1}{4} = 0}$$

25. **Reduce the equation of the circle into standard form.**

$$\mathbf{x^2 + y^2 - 4x + 6y - 12 = 0}$$

Sol. As given equation:

$$x^2 + y^2 - 4x + 6y - 12 = 0$$

$$x^2 - 4x + y^2 + 6y = 12$$

Adding the square of one half of the coefficient of x & y on both sides:

$$x^2 - 4x + (2)^2 + y^2 + 6y + (3)^2 = 12 + (2)^2 + (3)^2$$

$$(x-2)^2+(y+3)^2=12+4+9$$
$$(x-2)^2+(y+3)^2=25$$
$$\boxed{(x-2)^2+(y+3)^2=(5)^2}$$

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26. **What type of circle is represented by $x^2+y^2+2x-4y+8=0$**

Sol. $x^2+y^2+2x-4y+8=0$

Comparing this equation with general form of equation of circle:

$$x^2+y^2+2gx+2fy+c=0$$

$2g=2$	$2f=-4$	
$g=\frac{2}{2}$	$f=-\frac{4}{2}$	$c=8$
$g=1$	$f=-2$	

$$\text{Radius}=r=\sqrt{g^2+f^2-c}$$
$$r=\sqrt{(-1)^2+(2)^2-8}$$
$$r=\sqrt{1+4-8}=\sqrt{-3}=\boxed{\sqrt{3}i}$$

So, it is an **Imaginary circle**.

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27. **Define the circle.**

Sol. A circle is the set of all points in a plane that are equally distance from a fixed point.

Section - II

Note : Attemp any three (3) questions $3\times 8=24$

Q.2. **Resolve into partial fraction**

(i) $\dfrac{x+4}{(x-2)^2(x+1)}$

Sol. See Q.1 of Ex # 9.2 (Page # 358)

(ii) $\dfrac{2x+5}{x^2+5x+6}$

Sol. See Q.2 of Ex # 9.1 (Page # 346)

Q.3.(i) ~~Find magnitude (modulus) and argument of~~ $(4+7i)(3-2i)$

Sol. See example 14 of chapter 08

(ii) ~~Express the complex number in the polar form~~ $Z=2+2\sqrt{3}\,i$

Sol. See Q.2(i) of Ex # 8.2 (Page # 318)

Q.4.(i) **Minimize the expression**

$$X=W\overline{Z}(W+Y)+WY(\overline{Z}+\overline{W})$$

Sol. See Q.3[a] of Ex # 11 (Page # 429)

(ii) **Prepare truth table for the Boolean Expression $AB+\overline{A}\,\overline{B}$**

Sol. See Q.1(v) of Ex # 11 (Page # 427)

Q.5.(i) ~~Find the points trisecting the join of~~ $A(-1, 4)$ & $B(6, 2)$.

Sol. See Q.9 of Ex # 12.2 (Page # 461)

(ii) ~~Find the equation of the line which is perpendicular to the line~~ $4x+7y=5$ ~~and pass through~~ $(-1, 2)$.

Sol. See Q.10 of Ex # 12.4 (Page # 480)

Q.6. **Find the equation of circle passing through the points $(1, 2)$, $(0, -1)$ and $(-1, 1)$.**

Sol. See Q.3[a] of Ex # 13 (Page # 524)

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