DAE / IIA - 2016

MATH - 123 APPLIED MATHEMATICS - I

PAPER 'A' PART - A (OBJECTIVE)

Time : 30 Minutes Marks : 15

Q.1: *Encircle the correct answer.*

1. To make $x^2 - 5x$ a complete square, we should add:

[a] 25 [b] $\frac{25}{4}$

[c] $\frac{25}{9}$ [d] $\frac{25}{16}$

2. If discriminant $b^2 - 4ac$ is a perfect square, its roots will be:

[a] Imaginary [b] Rational

[c] Equal [d] Irrational

3. ~~If ± 3 are the roots of the equation, then equation is:~~

[a] $x^2 - 3 = 0$ [b] $x^2 - 9 = 0$

[c] $x^2 + 3 = 0$ [d] $x^2 + 9 = 0$

4. Third term of $(x+y)^4$ is:

[a] $4x^2y^2$ [b] $4x^3y$

[c] $6x^2y^2$ [d] $6x^3y$

5. $\binom{3}{0}$ will have the value:

[a] 0 [b] 1 [c] 2 [d] 3

6. In the expansion of $(a+b)^n$; the term $\binom{n}{r}a^{n-r}b^r$ will be:

[a] nth term [b] rth term

[c] $(r+1)$th term

[d] $(r-1)$th term

7. ~~One radian is equal to:~~

[a] $90°$ [b] $\left(\frac{90}{\pi}\right)^\circ$

[c] $180°$ [d] $\left(\frac{180}{\pi}\right)^\circ$

8. If $\sin\theta = \frac{3}{5}$ and the terminal side of the angle lies in 2nd quadrant, then tan θ is equal to:

[a] $\frac{4}{5}$ [b] $-\frac{4}{5}$ [c] $\frac{5}{4}$ [c] $-\frac{3}{4}$

9. $\cos\left(\frac{\pi}{2}+\theta\right)$ is equal to:

[a] $\cos\theta$ [b] $-\cos\theta$

[c] $\sin\theta$ [d] $-\sin\theta$

10. sin (A + B) – sin (A – B) is equal to:

[a] 2 sin A cos B [b] 2 cos A cos B

[c] -2 sin A sin B [d] 2 cos A sin B

11. ~~If C = 90º, b = 1, c = $\sqrt{2}$, then Bº is:~~

[a] $90°$ [b] $60°$

[c] $45°$ [d] $30°$

12. If in a triangle ABC, b = 2, c = 2, A = 60º then side a is:

[a] 2 [b] 3

[c] 4 [d] 5

13. Magnitude of the vector $2\mathbf{i} - 2\mathbf{j} - \mathbf{k}$ is:

[a] 4 [b] 3 [c] 2 [d] 1

14. ~~If 2(cos 60º – j sin 60º) then the exponent form is:~~

[a] $2e^{j60º}$ [b] $2e^{-j60º}$

[c] $2e^{60º}$ [d] $2e^{-60º}$

15. ~~If~~ $Z_1 = 8\angle 30°$ ~~and~~ $Z_2 = 2\angle -60°$, ~~then $\frac{Z_1}{Z_2}$ is:~~

[a] $4\angle 30º$ [b] $4\angle 30°$

[c] $4\angle 90º$ [d] $4\angle 90º$

Answer Key

1	b	2	b	3	b	4	c	5	b
6	c	7	d	8	b	9	d	10	a
11	c	12	a	13	b	14	a	15	a

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DAE / IIA - 2016

MATH - 123 APPLIED MATHEMATICS - I

PAPER 'A' PART - B (SUBJECTIVE)

Time: 2:30 Hrs Marks: 60

Section - I

Q.1. Write short answers to any Eighteen (18) questions.

1. Solve the equation by factorization: $3x^2 + 5x = 2$

Sol.

$$3x^2 + 5x = 2$$
$$3x^2 + 5x - 2 = 0$$
$$3x^2 + 6x - x - 2 = 0$$
$$3x(x+2) - 1(x+2) = 0$$
$$(x+2)(3x-1) = 0$$

Either | **OR**

$x + 2 = 0$, $x = -2$ | $3x - 1 = 0$, $3x = 1 \Rightarrow x = \frac{1}{3}$

$$\text{S.S.} = \boxed{\left\{-2, \frac{1}{3}\right\}}$$

2. Solve the equation by completing square: $x^2 + 5x - 6 = 0$

Sol.

$$x^2 + 5x - 6 = 0$$
$$x^2 + 5x = 6$$

Adding the square of one half of the coefficient of x i.e., $\left(\frac{5}{2}\right)^2$ on both sides:

$$x^2 + 5x + \left(\frac{5}{2}\right)^2 = 6 + \left(\frac{5}{2}\right)^2$$
$$\left(x + \frac{5}{2}\right)^2 = 6 + \frac{25}{4}$$
$$\left(x + \frac{5}{2}\right)^2 = \frac{24 + 25}{4} = \frac{49}{4}$$
$$\sqrt{\left(x + \frac{5}{2}\right)^2} = \pm\sqrt{\frac{49}{4}}$$
$$x + \frac{5}{2} = \pm\frac{7}{2} \Rightarrow x = \pm\frac{7}{2} - \frac{5}{2}$$
$$x = \frac{\pm 7 - 5}{2}$$

Either | **OR**

$x = \frac{7-5}{2}$, $x = \frac{2}{2} = 1$ | $x = \frac{-7-5}{2}$, $x = \frac{-12}{2} = -6$

$$\text{S.S.} = \boxed{\{1, -6\}}$$

3. Discuss the nature of the roots of the equation $9x^2 + 6x + 1 = 0$

Sol. Here: $a = 9,\ b = 6,\ c = 1$

$$\text{Disc.} = b^2 - 4ac$$
$$= (6)^2 - 4(9)(1) = 36 - 36 = 0$$

∴ The roots are **Equal and Real.**

4. For what value of 'k' the roots of the given equation $2x^2 + 5x + k = 0$ are equal.

Sol. Here: $a = 2,\ b = 5,\ c = k$

As, Roots are equal

So, $\text{Disc.} = 0$

$\Rightarrow\ b^2 - 4ac = 0$

$\Rightarrow\ (5)^2 - 4(2)(k) = 0$

$\Rightarrow\ 25 - 8k = 0$

$\Rightarrow\ -8k = -25$

$\Rightarrow\ k = \frac{-25}{-8} \Rightarrow \boxed{k = \frac{25}{8}}$

5. ~~If α, β are the roots of the equation~~ $x^2-4x+2=0$ ~~find the equation whose roots are:~~ $-\alpha,\ -\beta$

Sol. As α, β are the roots of the given equation. $x^2-4x+2=0$

Here: $a=1,\ b=-4,\ c=2$

Sum of Roots	Products of Roots
$\alpha+\beta=-\frac{b}{a}$	$\alpha\beta=\frac{c}{a}$
$=-\left(-\frac{4}{1}\right)=4$	$=\frac{2}{1}=2$

As, $-\alpha,-\beta$ are the roots of require equation.

$S=-\alpha+(-\beta)$	$P=(-\alpha)(-\beta)$
$S=-\alpha-\beta$	$P=\alpha\beta$
$S=-(\alpha+\beta)=-4$	$P=2$

$$x^2-Sx+P=0$$

$$x^2-(-4)x+2=0\Rightarrow \boxed{x^2+4x+2=0}$$

6. Expand the expression $\left(\frac{x}{y}+\frac{y}{x}\right)^4$

Sol. $\left(\frac{x}{y}+\frac{y}{x}\right)^4$

$$=\binom{4}{0}\left(\frac{x}{y}\right)^4\left(\frac{y}{x}\right)^0+\binom{4}{1}\left(\frac{x}{y}\right)^3\left(\frac{y}{x}\right)^1+\binom{4}{2}\left(\frac{x}{y}\right)^2\left(\frac{y}{x}\right)^2$$

$$+\binom{4}{3}\left(\frac{x}{y}\right)^1\left(\frac{y}{x}\right)^3+\binom{4}{4}\left(\frac{x}{y}\right)^0\left(\frac{y}{x}\right)^4$$

$$=(1)\left(\frac{x^4}{y^4}\right)(1)+4\left(\frac{x^3}{y^3}\right)\left(\frac{y}{x}\right)+6\left(\frac{x^2}{y^2}\right)\left(\frac{y^2}{x^2}\right)$$

$$+4\left(\frac{x}{y}\right)\left(\frac{y^3}{x^3}\right)+1(1)\left(\frac{y^4}{x^4}\right)$$

$$=\boxed{\frac{x^4}{y^4}+4\frac{x^2}{y^2}+6+4\frac{y^2}{x^2}+\frac{y^4}{x^4}}$$

7. Find the 7th term in the expansion of $\left(x-\frac{1}{x}\right)^9$

Sol. Here: $a=x,\ b=-\frac{1}{x},\ n=9\ \&\ r=6$

Using general term formula:

$$T_{r+1}=\binom{n}{r}a^{n-r}b^r$$

$$T_{6+1}=\binom{9}{6}(x)^{9-6}\left(-\frac{1}{x}\right)^6$$

$$T_7=84x^3\left(\frac{1}{x^6}\right)=\frac{84}{x^3}$$

8. ~~Expand~~ $(1+2x)^{-2}$ ~~to three terms.~~

Sol. $(1+2x)^{-2}$

$$=1+(-2)(2x)+\frac{(-2)(-2-1)}{2!}(2x)^2+\ldots$$

$$=1-4x+\frac{(-2)(-3)}{2}(4x^2)+\ldots$$

$$=\boxed{1-4x+12x^2+\ldots}$$

9. ~~Using Binomial series calculate~~ $\sqrt[3]{65}$ ~~to the nearest hundredth.~~

Sol.

$$\sqrt[3]{65}=(64+1)^{1/3}=\left[64\left(1+\frac{1}{64}\right)\right]^{1/3}$$

$$=4\left(1+\frac{1}{64}\right)^{1/3}=4\left[1+\left(\frac{1}{3}\right)\left(\frac{1}{64}\right)+\ldots..\right]$$

$$=4\left[1+\frac{1}{192}+\ldots\right]=4(1.0052)=\boxed{4.02}$$

10. Find the middle term in the expansion of $(2x+3)^{12}$

Sol. As $n=12$ (Even), so

Middle term $= \left(\frac{n}{2}+1\right)^{th} = \left(\frac{12}{2}+1\right)^{th}$

Middle term $= (6+1)^{th} = 7^{th}$ term

$\boxed{\text{Hence } T_7 \text{ is a middle term}}$

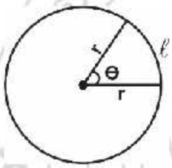
11. ~~**Convert into degree measure: 0.726 radian.**~~

Sol. 0.726 rad

$= 0.726 \times \frac{180}{\pi} = \boxed{41°35'48''}$

12. ~~**Prove that: $\ell = r\theta$.**~~

Sol. The ratio of 'ℓ' to the circumference $2\pi r$ of the circle is same as ratio of the angle θ to 2π.



$\ell : 2\pi r = \theta : 2\pi$

$\frac{\ell}{2\pi r} = \frac{\theta}{2\pi}$

$\ell = \frac{\theta(2\pi r)}{2\pi}$

$\ell = r\theta$ **Proved.**

13. **Prove that:**

$$\tan^2 30° + \tan^2 45° + \tan^2 60° = \frac{13}{3}$$

Sol. L.H.S. $= \tan^2 30° + \tan^2 45° + \tan^2 60°$

$= \left(\frac{1}{\sqrt{3}}\right)^2 + (1)^2 + \left(\sqrt{3}\right)^2 = \frac{1}{3} + 1 + 3$

$= \frac{1+3+9}{3} = \frac{13}{3} = \text{R.H.S.}$ **Proved.**

14. **Prove that:**

$$\frac{1}{1+\sin\theta} + \frac{1}{1-\sin\theta} = 2\sec^2\theta$$

Sol. L.H.S. $= \frac{1}{1+\sin\theta} + \frac{1}{1-\sin\theta}$

$= \frac{1(1-\sin\theta)+1(1+\sin\theta)}{(1+\sin\theta)(1-\sin\theta)}$

$= \frac{1-\sin\theta+1+\sin\theta}{(1)^2-(\sin\theta)^2}$

$= \frac{2}{1-\sin^2\theta} = \frac{2}{\cos^2\theta}$

$= 2\sec^2\theta = \text{R.H.S.}$ **Proved.**

15. **Prove that:** $\cos\left(\frac{\pi}{2}-\theta\right) = \sin\theta$

Sol. L.H.S. $= \cos\left(\frac{\pi}{2}-\theta\right)$

$= \cos(90° - \theta) \because \left\{\frac{\pi}{2} \times \frac{180°}{\pi} = 90°\right\}$

$= \cos 90° \cos\theta + \sin 90° \sin\theta$

$= (0)\cos\theta + (1)\sin\theta \because \left\{\begin{matrix}\text{Using calculator} \\ \cos 90° = 0 \text{ \& } \sin 90° = 1\end{matrix}\right\}$

$= 0 + \sin\theta = \sin\theta = \text{R.H.S.}$ **Proved.**

16. **Express $\sin x \cos 2x - \sin 2x \cos x$ as single term.**

Sol. $\sin x \cos 2x - \sin 2x \cos x$

$= \sin x \cos 2x - \cos x \sin 2x$

$= \sin(x - 2x) \quad \because \left\{\begin{matrix}\sin(\alpha-\beta) \\ = \sin\alpha\cos\beta - \cos\alpha\sin\beta\end{matrix}\right\}$

$= \sin(-x) = \boxed{-\sin x}$

17. **Prove that:** $\tan 2x = \frac{2\tan x}{1-\tan^2 x}$

Sol. L.H.S. $= \tan 2x$

$= \tan(x + x)$

$= \frac{\tan x + \tan x}{1 - \tan x \tan x}$

$= \frac{2\tan x}{1-\tan^2 x} = \text{R.H.S.}$ **Proved.**

18. ~~**Express $\cos 3\theta \cos\theta$ as sum or difference.**~~

Sol. $\cos 3\theta \cos\theta$

$$= \frac{1}{2}\left[2\cos 3\theta \cos\theta\right]$$

$$= \frac{1}{2}\left[\cos(3\theta+\theta)+\cos(3\theta-\theta)\right]$$

$$= \boxed{\frac{1}{2}\left[\cos 4\theta + \cos 2\theta\right]}$$

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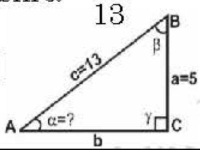
19. ~~**In right triangle ABC, γ = 90º, a = 5, c = 13 then find value of angle ∝.**~~

Sol. We know that, from figure:

$$\sin\alpha = \frac{a}{c} \Rightarrow \sin\alpha = \frac{5}{13}$$

$$\alpha = \sin^{-1}\left(5/13\right)$$

$$\boxed{\alpha = 22^\circ 37'}$$



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20. **In any triangle ABC, if a = 20, c = 32 and γ = 70º find angle ∝.**

Sol. By using law of sines:

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$$

$$\text{we take: } \frac{a}{\sin\alpha} = \frac{c}{\sin\gamma}$$

$$\sin\alpha = \frac{a\sin\gamma}{c} = \frac{20\sin 70^\circ}{32} = 0.5873$$

$$\alpha = \sin^{-1}(0.5873)$$

$$\boxed{\alpha = 35^\circ 57' 58''}$$

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21. **Define angle of elevation and depression.**

Sol. **Angle of Elevation:**

If the line of sight is upward from the horizontal, the angle is called angle of Elevation.

Angle of Depression:

If the line of sight is downward from the horizontal, the angle is called angle of Depression.

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22. **In any triangle ABC in which b = 45, c = 34, ∝ = 52º, find a.**

Sol. By using Law of cosines:

$$a^2 = b^2 + c^2 - 2bc\cos 52^\circ$$

$$a^2 = (45)^2 + (34)^2 - 2(45)(34)\cos 52^\circ$$

$$a^2 = 2025 + 1156 - 1883.92$$

$$a^2 = 1297.07 \Rightarrow \boxed{a = 36.01}$$

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23. **What are parallel vectors?**

Sol. Two vectors $\vec{a}$ and $\vec{b}$ are parallel if there exist a non-zero $k \varepsilon R$, such that $\vec{a} = k\vec{b}$.

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24. **Find unit vector along the vector $\mathbf{4i - 3j - 5k}$**

Sol. Let $\vec{a} = 4i - 3j - 5k$

$$|\vec{a}| = \sqrt{(4)^2 + (-3)^2 + (-5)^2}$$

$$|\vec{a}| = \sqrt{16 + 9 + 25} = \sqrt{50}$$

$$|\vec{a}| = \sqrt{25\times 2} = 5\sqrt{2}$$

$$\text{Unit Vector} = \hat{a} = \frac{\vec{a}}{|\vec{a}|} = \boxed{\frac{4i - 3j - 5k}{5\sqrt{2}}}$$

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25. **For what value of λ, the vectors $\mathbf{2i - j + 2k}$ & $\mathbf{3i + 2\lambda j}$ are perpendicular.**

Sol. Let, $\vec{a} = 2i - j + 2k$ & $\vec{b} = 3i + 2\lambda j$

As given vectors are perpendicular.

So, $\vec{a}.\vec{b} = 0$

$$\Rightarrow (2i - j + 2k).(3i + 2\lambda j) = 0$$

$$\Rightarrow (2)(3) + (-1)(2\lambda) + (2)(0) = 0$$

$$\Rightarrow 6 - 2\lambda + 0 = 0$$

$$\Rightarrow -2\lambda = -6$$

$$\Rightarrow \lambda = \frac{-6}{-2} \Rightarrow \boxed{\lambda = 3}$$

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26. ~~**Find the conjugate and modulus of**~~ $-\frac{2}{3} - j\frac{4}{9}$

Sol. Conjugate $= -\dfrac{2}{3} - j\dfrac{4}{9} = \boxed{-\dfrac{2}{3} + j\dfrac{4}{9}}$

Modulus $= \sqrt{\left(\dfrac{-2}{3}\right)^2 + \left(-\dfrac{4}{9}\right)^2} = \sqrt{\dfrac{4}{9} + \dfrac{16}{81}} = \sqrt{\dfrac{36+16}{81}} = \boxed{\dfrac{\sqrt{52}}{9}}$

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27. ~~Simplify the phasor~~ $\dfrac{1}{4-j5} - \dfrac{1}{5-j4}$ ~~and write the result in Rectangular form.~~

Sol. $\dfrac{1}{4-j5} - \dfrac{1}{5-j4}$

$$= \frac{1(5-j4) - 1(4-j5)}{(4-j5)(5-j4)}$$

$$= \frac{5-j4-4+j5}{20-j16-j25+j^2 20} = \frac{1+j}{20-j41-20}$$

$$= \frac{1+j}{-j41} = \frac{1+j}{-j41} \times \frac{j41}{j41} = \frac{j41+j^2 41}{-j^2 41^2}$$

$$= \frac{j41-41}{1681} = \frac{-41+j41}{1681}$$

$$= \frac{41(-1+j)}{1681} = \frac{-1+j}{41} = \boxed{-\frac{1}{41} + j\frac{1}{41}}$$

Section - II

Note : Attemp any three (3) questions $3 \times 8 = 24$

Q.2.(a) Solve the equation

$$\frac{1}{a+b+x} = \frac{1}{a} + \frac{1}{b} + \frac{1}{x}$$

Sol. See Q.1(vi) of Ex # 1.1 (Page # 08)

(b) ~~If the difference of roots of equation $x^2 - 7x + k - 4 = 0$ is 5, find the value of K and the roots.~~

Sol. See Q.3(i) of Ex # 1.3 (Page # 42)

Q.3. Find the constant term in the expression of $\left(\sqrt{x} + \dfrac{1}{3x^2}\right)$

Sol. See Q.9(ii) of Ex # 2.1 (Page # 95)

Q.4.(a) Prove that:

$$\sin^2\frac{\pi}{6} : \sin^2\frac{\pi}{4} : \sin^2\frac{\pi}{3} : \sin^2\frac{\pi}{2} = 1:2:3:4$$

Sol. See Q.12 of Ex # 3.2 (Page # 129)

(b) Prove that:

$$\frac{\sec\theta - \tan\theta}{\sec\theta + \tan\theta} = 1 - 2\sec\theta\tan\theta + 2\tan^2\theta$$

Sol. See Q.13 of Ex # 3.3 (Page # 136)

Q.5.(a) If $\tan\beta = \dfrac{n\sin\alpha\cos\alpha}{1-n\sin^2\alpha}$, prove that $\tan(\alpha-\beta) = (1-n)\tan\alpha$.

Sol. See Q.11 of Ex # 4.1 (Page # 152)

(b) ~~Prove that:~~

$$\sin 5\theta + 2\sin 3\theta + \sin\theta = 4\sin 3\theta\cos^2\theta$$

Sol. See Q.12 of Ex # 4.3 (Page # 190)

Q.6.(a) Given the vector $\vec{a} = 2i - 2j + 4k$ and $\vec{b} = 2i + j + 3k$. Find the magnitude and direction cosines of $3\vec{a} - 2\vec{b}$

Sol. See Q.9(ii) of Ex # 6.1 (Page # 250)

(b) If $\vec{a} = 2i - 3j + 4k$ & $\bar{b} = 2j + 4k$. Find the component (or projection) of $\vec{a}$ along $\vec{b}$.

Sol. See Q.10 of Ex # 6.2 (Page # 260)

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