DAE / IIA - 2016

MATH-113 APPLIED MATHEMATICS-I

PAPER 'B' PART - A (OBJECTIVE)

Time: 30 Minutes Marks: 15

Q.1: *Encircle the correct answer.*

1. Area of right triangle if base = 4cm and height = 6 is:
[a] 24 sq.cm [b] 12 sq.cm
[c] 6 sq.cm [d] 10 sq.cm

2. Perimeter of square having a side 5cm is:
[a] 20cm [b] 10cm
[c] 15cm [d] 24cm

3. A decagon has number of sides;
[a] 8 [b] 9 [c] 10 [d] 12

4. The space enclosed between two concentric circles is called:
[a] Cone [b] Ellipse
[c] Annulus [d] None of these

5. In Simpson's rule, number of strips should be:
[a] Even [b] Odd
[c] Double [d] Half

6. If volume of a cube is 27, then side of cube is:
[a] 3 **[b]** 9 **[c]** $\sqrt{3}$ **[d]** $\sqrt{27}$

7. Volume of circular cylinder of height 'h' and radius 'r' is:
[a] $2\pi r^2 h$ [b] $\pi r^2 h$
[c] $2\pi rh$ [d] $2\pi r^2 h$

8. If area of base of pyramid is 'A' and height 'h', then volume of pyramid.
[a] $\frac{1}{3}Ah$ **[b]** $\frac{1}{2}Ah$
[c] $\frac{1}{6}Ah$ **[c]** Ah

9. Volume of a cone of height 'h' and base radius 'r' is:
[a] $\frac{1}{3}\pi r^2 h$ **[b]** $\frac{1}{3}\pi rh$
[c] $\pi r^2 h$ **[d]** $\frac{1}{2}\pi r^2$

10. Magnitude of the vector $2\underline{i} - 2\underline{j} - \underline{k}$ is:
[a] 4 **[b]** 3 **[c]** 2 **[d]** 1

11. If $\hat{i}$, $\hat{j}$ and $\hat{k}$ are orthogonal unit vector, then $\hat{j} \times \hat{i} = ?$
[a] $\hat{k}$ **[b]** $-\hat{k}$
[c] 1 **[d]** -1

12. $|\vec{a} \times \vec{b}|$ is a
[a] Vector quantity
[b] Unity
[c] Scalar quantity
[d] None of these

13. The value of m for which matrix is $\begin{bmatrix} 2 & 3 \\ 6 & m \end{bmatrix}$ singular is:
[a] 6 [b] 3 [c] 8 [d] 9

14. ~~If two rows of a determinant are identical, then its value is:~~
[a] 0 **[b]** 1 **[c]** -1 **[d]** 2

15. The matrix $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ is called:
[a] Identity [b] Null
[c] Scalar [d] Diagonal

Answer Key

1	b	2	a	3	c	4	c	5	c
6	a	7	b	8	a	9	a	10	b
11	b	12	c	13	d	14	a	15	b

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DAE / IIA - 2016

MATH - 113 APPLIED MATHEMATICS - I

PAPER 'B' PART - B (SUBJECTIVE)

Time : 2 : 30 Hrs Marks : 60

Section - I

Q.1. Write short answers to any Eighteen (18) questions.

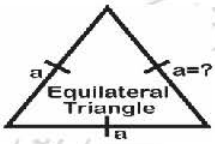
1. **What is the side of the equilateral triangle whose area is $9\sqrt{3}$ sq.cm.**

Sol. Let 'a' be length of each side of an equilateral triangle.

As, Area of equilateral triangle $= 9\sqrt{3}$ sq.cm

$$\Rightarrow \quad \frac{\sqrt{3}}{4}a^2 = 9\sqrt{3}$$

$$\Rightarrow \quad a^2 = 9\sqrt{3}\left(\frac{4}{\sqrt{3}}\right)$$



$$\Rightarrow \quad a^2 = 36 \Rightarrow \sqrt{a^2} = \sqrt{36} \Rightarrow \boxed{a = 6\,cm}$$

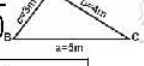
2. **Find the area of triangle with sides 5, 4 and 3 meters respectively.**

Sol. Let, a = 5m, b = 4m, c = 3m

$$s = \frac{a+b+c}{2} = \frac{5+4+3}{2} = \frac{12}{2} = 6\,m$$

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$A = \sqrt{6(6-5)(6-4)(6-3)}$$



$$A = \sqrt{6(1)(2)(3)} = \sqrt{36} = \boxed{6\ sq.m}$$

3. **The sides of a cyclic quadrilateral are 75, 55, 140 and 40m, find its area.**

Sol. Let, $a = 75m, b = 55m, c = 140m, d = 40m$

$$s = \frac{a+b+c+d}{2} = \frac{75+55+140+40}{2} = 155m$$

$$\text{Area} = \sqrt{(s-a)(s-b)(s-c)(s-d)}$$

$$= \sqrt{(155-75)(155-55)(155-140)(155-40)}$$

$$= \sqrt{80(100)(15)(115)} = \sqrt{13800000} = \boxed{3714.8\,sq.m}$$

4. **The area of a rectangle is 20 sq.cm and one of its side is 4cm long. Find its breadth.**

Sol. As, Area of rectangle = $20cm^2$

$$\text{Length} \times \text{Breadth} = 20$$

$$4 \times \text{Breadth} = 20$$

$$\text{Breadth} = 20/4$$

$$\boxed{\text{Breadth} = 5\,cm}$$

5. **Write the formula to find the angle of a regular polygon of n sides.**

Sol. Interior angle $= \boxed{\frac{2n-4}{n} \times 90^\circ}$

6. **Define inscribed polygon.**

Sol. If a circle passes through the corners of a polygon, then this polygon is called inscribed polygon.

7. **A path 14cm wide, surrounds a circular lawn whose diameter is 360cm. Find the area of the path.**

Sol. $d = 360cm \Rightarrow r = 360/2 = 180cm$

$$R = 180 + 14 = 194cm$$

$$\text{Area of path} = \pi\left[R^2 - r^2\right]$$

$$A = \pi\left[(194)^2 - (180)^2\right]$$

$$A = \boxed{16449\,cm^2}$$

8. **Find the area of a segment, the chord of which is 8cm with a height of 2cm.**

Sol. Given that: $c = 8cm,\ h = 2cm$

$$\text{Area of Segment} = \frac{h}{6c}\left[3h^2 + 4c^2\right]$$

$$A = \frac{2}{6(8)}\left(3(2)^2 + 4(8)^2\right)$$

$$A = \boxed{11.16\ sq.cm}$$

9. **The dimension of a marriage hall are 100m, 50m and 18m**

respectively, find volume of the hall.

Sol. Here: ℓ = 100m, b = 50m & h = 18m

$$\text{Volume} = \ell bh$$

$$V = 100 \times 50 \times 18 = \boxed{90000\,m^3}$$

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10. **Find surface area of cube of volume 64cm³.**

Sol. Let 'a' be length of one side of cube

As, Volume of cube $= 64\,cm^3$

$$a^3 = 64$$

$$(a^3)^{1/3} = (64)^{1/3}$$

$$\boxed{a = 4cm}$$

Surface area of cube $= 6a^2$

$$S.A. = 6(4)^2 = \boxed{96\,cm^2}$$

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11. **Find the diameter of the cylinder if its volume is 704cm³ and height is 14cm.**

Sol. Here : $d = ?, V = 704cm^3$ & $h = 14cm$

As, Volume of Cylinder $= 704cm^3$

$$\Rightarrow \quad \pi r^2 h = 704$$

$$\Rightarrow \quad r^2 = \frac{704}{\pi h}$$

$$\Rightarrow \quad r^2 = \frac{704}{\pi(14)}$$

$$\Rightarrow \quad r^2 = 16$$

$$\Rightarrow \quad r = 4\,cm$$

Diameter $= d = 2r = 2(4) = \boxed{8\,cm}$

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12. **A square pyramid has a volume of 60cu.cm and the side of the base is 6cm. Find the height of the pyramid.**

Sol. Here : $V = 60cm^3$, $a = 6cm$ & $h = ?$

Area of base (square) $= a^2 = (6)^2 = 36cm^2$

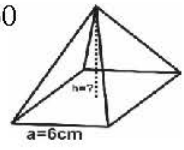
As, Volume of Pyramid $= 60cm^3$

$$\frac{1}{3} \times \text{Area of base} \times \text{Height} = 60$$

$$\frac{1}{3} \times 36 \times h = 60$$

$$12h = 60$$

$$h = \frac{60}{12} \Rightarrow \boxed{h = 5cm}$$



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13. **Find the volume of the largest cone that can be cut out of a cube whose edge is 3cm.**

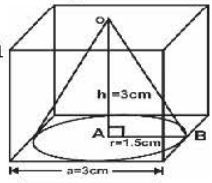
Sol. Let 'a' = edge of the cube = 3cm

Then h = 3cm &

$$r = \frac{a}{2} = \frac{3}{2} = 1.5cm$$

$$\text{Volume} = \frac{1}{3}\pi r^2 h$$

$$V = \frac{1}{3}\pi(1.5)^2(3) = \boxed{7.069\,cm^3}$$



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14. **Write formula of curved surface area of cone and slant height of cone.**

Sol. Curved surface area $= \boxed{\pi r \ell \text{ sq.unit}}$

Slant height $= \ell = \boxed{\sqrt{r^2 + h^2}\text{ unit}}$

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15. **How many square meter of copper will be required to cover a hemi-spherical dome of 30m diameter.**

Sol. Let d = 30m

Surface area of hemi-sphere dome $= \frac{1}{2}\pi d^2$

$$= \frac{1}{2}\pi(30)^2 = \boxed{1413.72\,m^2}$$

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16. **Write the formula of volume of sphere and hemi-sphere.**

Sol. Volume of Shpere $= \boxed{\frac{4}{3}\pi r^3 \text{ cu.unit}}$

Volume of Hemi-Shpere $= \boxed{\frac{2}{3}\pi r^3 \text{ cu.unit}}$

17. **A brick measures 18cm by 9cm by 6cm, find the number of bricks that will be needed to build a wall 4.5cm wide, 18cm thick and 3.6cm high.**

Sol. Let, ℓ = Length of brick = 18cm

b = breath of brick = 9cm

h = Height of brick = 6cm

L = Length of wall = 450cm

B = breath of wall = 18cm

H = Height of wall = 360cm

Volume of brick = $V_1 = \ell bh$

$V_1 = 18 \times 9 \times 6 = 972 cm^3$

Volume of wall = $V_2 = LBH$

$V_2 = 450 \times 18 \times 360 = 2916000 cm^3$

$$\text{No. of Bricks} = \frac{V_2}{V_1}$$

$$= \frac{2916000}{972} = \boxed{3000 \text{Bricks}}$$

18. **Find the unit vector parallel to the sum of the vector.**

$\vec{a} = [2, 4, -5]$ and $\vec{b} = [1, 2, 3]$

Sol. Let $\vec{v}$ = sum of the vectors

$\vec{a}$ & $\vec{b} = \vec{a} + \vec{b} = [2, 4, -5] + [1, 2, 3]$

$\vec{v} = [3, 6, -2] = 3i + 6j - 2k$

$|\vec{v}| = \sqrt{(3)^2 + (6)^2 + (-2)^2}$

$|\vec{v}| = \sqrt{9 + 36 + 4}$

$|\vec{v}| = \sqrt{49} = 7$

Unit vector $= |\hat{v}| = \frac{\vec{v}}{|\vec{v}|} = \boxed{\frac{3i + 6j - 2k}{7}}$

19. **Find $\vec{a}.\vec{b}$ if $\vec{a} = i + 2j + 2k$ & $\vec{b} = 3i - 2j - 2k$**

Sol. $\vec{a}.\vec{b} = (i + 2j + 2k) \bullet (3i - 2j - 4k)$

$= (1)(3) + (2)(-2) + (2)(-4)$

$= 3 - 4 - 8 = \boxed{-9}$

20. **Find the area of parallelogram with adjacent sides,**

$\vec{a} = 7i - j + k$ & $\vec{b} = 2j - 3k$

Sol. $$\vec{a} \times \vec{b} = \begin{vmatrix} i & j & k \\ 7 & -1 & 1 \\ 0 & 2 & -3 \end{vmatrix}$$

$$= i\begin{vmatrix} -1 & 1 \\ 2 & -3 \end{vmatrix} - j\begin{vmatrix} 7 & 1 \\ 0 & -3 \end{vmatrix} + k\begin{vmatrix} 7 & -1 \\ 0 & 2 \end{vmatrix}$$

$= i(3-2) - j(-21-0) + k(14+0)$

$= i + 21j + 14k$

$|\vec{a} \times \vec{b}| = \sqrt{(1)^2 + (21)^2 + (14)^2}$

$|\vec{a} \times \vec{b}| = \sqrt{1 + 441 + 196} = \sqrt{638}$

Area of parallelogram

$= |\vec{a} \times \vec{b}| = \boxed{\sqrt{638} \text{ sq. unit}}$

21. **For what value of λ, the vectors $2i - j + 2k$ & $3i + 2\lambda j$ are perpendicular.**

Sol. Let, $\vec{a} = 2i - j + 2k$ & $\vec{b} = 3i + 2\lambda j$

As given vectors are perpendicular.

So, $\vec{a}.\vec{b} = 0$

$\Rightarrow (2i - j + 2k).(3i + 2\lambda j) = 0$

$\Rightarrow (2)(3) + (-1)(2\lambda) + (2)(0) = 0$

$\Rightarrow 6 - 2\lambda + 0 = 0$

$\Rightarrow -2\lambda = -6$

$\Rightarrow \lambda = \frac{-6}{-2}$

$\Rightarrow \boxed{\lambda = 3}$

22. Given the vectors $\vec{a} = 3i + j - k$ and $\vec{b} = 2i + j - k = 2i + j - k$, find magnitude of $3\vec{a} - \vec{b}$.

Sol. $3\vec{a} - \vec{b}$

$= 3(3i + j - k) - (2i + j - k)$

$= 9i + 3j - 3k - 2i - j + k$

$= 7i + 2j - 2k$

$|3\vec{a} - \vec{b}|$

$= \sqrt{(7)^2 + (2)^2 + (-2)^2}$

$= \sqrt{49 + 4 + 4}$

$= \boxed{\sqrt{57}}$

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23. Define Diagonal matrix.

Sol. A square matrix in which all elements except diagonal elements are zero is called diagonal matrix.

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24. Find x and y if

$$\begin{bmatrix} x+3 & 1 \\ -3 & 3y-4 \end{bmatrix} = \begin{bmatrix} y & 1 \\ -3 & 2x \end{bmatrix}$$

Sol. $\begin{bmatrix} x+3 & 1 \\ -3 & 3y-4 \end{bmatrix} = \begin{bmatrix} y & 1 \\ -3 & 2x \end{bmatrix}$

Comparing corresponding elements of both matrices :

$x + 3 = y$ and $3y - 4 = 2x$

$x - y = -3 \rightarrow (i) \mid 2x - 3y = -4 \rightarrow (ii)$

CMultipling eq.(i) by 2 and subtracting eq.(ii)

$2x - 2y = -6$

$-2x \mp 3y = \mp 4$

$y = -2$

$\boxed{y = -2}$

Put $y = -2$ in eq.(i)

$x - (-2) = -3$

$x = -3 - 2$

$\boxed{x = -5}$

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25. Find the inverse of $\begin{bmatrix} 2 & 1 \\ 6 & 3 \end{bmatrix}$

Sol. Let $A = \begin{bmatrix} 2 & 1 \\ 6 & 3 \end{bmatrix}$

$|A| = \begin{vmatrix} 2 & 1 \\ 6 & 3 \end{vmatrix}$

$|A| = (2)(3) - (6)(1)$

$|A| = 6 - 6$

$|A| = 0$

As, $|A| = 0$ so inverse of A does not exits.

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26. If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$ then find AB.

Sol. $AB = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$

$= \begin{bmatrix} 2+8 & 3+10 \\ 6+16 & 9+20 \end{bmatrix}$

$= \boxed{\begin{bmatrix} 10 & 13 \\ 22 & 29 \end{bmatrix}}$

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27. What is the cofactor of 4 in matrix $\begin{bmatrix} 3 & 1 & -4 \\ 2 & 5 & 4 \\ 1 & 4 & 8 \end{bmatrix}$

Sol. $\begin{bmatrix} 3 & 1 & -4 \\ 2 & 5 & 4 \\ 1 & 4 & 8 \end{bmatrix}$

Here : $4 = a_{32}$, so

Minor of $4 = M_{32}$

$= \begin{vmatrix} 3 & -4 \\ 2 & 6 \end{vmatrix} = 18 + 8 = \boxed{26}$

Cofactor of $4 = C_{32}$

$= (-1)^{3+2} M_{32} = (-1)^5 (26) = \boxed{-26}$

Section - II

Note : Attemp any three (3) questions $3 \times 8 = 24$

Q.2.(a) Find the area of the triangle whose sides are in the ratio

9:40:41 and whose perimeter is 180 meters.

Sol. See Q.7 of Ex # 10 (Page # 466)

(b) A rectangular field is 13m long and 10m wide. It has a cement path 3.5m wide around it. What is the area of the cement path.

Sol. See example # 04 of Ch # 11

Q.3.(a) A regular Octagon circumscribes a circle of 2cm radius. Find the area of the octagon.

Sol. See Q.5 of Ex # 12 (Page # 487)

(b) The area of two concentric circles are 1386sq.cm and 1886.5 sq.cm respectively. Find the width of the ring.

Sol. See Q.3 of Ex # 13 (Page # 496)

Q.4. Find area of an irregular figure by Simpson's rule if the ordinates are 9, 11, 13, 12, 10, 13, 15, 17, 14, 12 and 7 meters and base 73 meters.

Sol. See Q.6 of Ex # 14 (Page # 509)

(b) The curved surface of a cylinder is 1000sq.m and the diameter of the base is 20m. Find the volume and height of the cylinder.

Sol. See example # 01 of Ch # 16

Q.5.(a) Given the vectors

$\vec{a} = 3\underline{i} - 2\underline{j} + 4\underline{k}$ and

$\vec{b} = 2\underline{i} + \underline{j} + 3\underline{k}$ find the magnitude and direction cosines of $3\vec{a} - 2\vec{b}$

Sol. See Q.9(ii) of Ex # 8.1 (Page # 374)

(b) Find the cosine of the angle between the vectors $\vec{a} = 2\underline{i} - 8\underline{j} + 3\underline{k}$ and $\vec{b} = 4\underline{j} + 3\underline{k}$

Sol. See Q.3(i) of Ex # 8.2 (Page # 380)

Q.6.(a) ~~Prove that :~~

$$\begin{bmatrix} a+\lambda & b & c \\ a & b+\lambda & c \\ a & b & c+\lambda \end{bmatrix} = \lambda^2(a+b+c+\lambda)$$

Sol. See Q.6(ii) of Ex # 9.2 (Page # 426)

(b) Find the inverse of

$$\begin{bmatrix} 0 & -2 & -3 \\ 1 & 3 & 3 \\ -1 & -2 & -2 \end{bmatrix}$$

Sol. See Q.4(ii) of Ex # 9.3 (Page # 439)

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