DAE / IIA - 2016

MATH - 113 APPLIED MATHEMATICS - I

PAPER 'A' PART - A (OBJECTIVE)

Time : 30 Minutes Marks : 15

Q.1: *Encircle the correct answer.*

1. **The quadratic formula is:**

[a] $\frac{b \pm \sqrt{b^2 - 4ac}}{2a}$ [b] $\frac{\pm\sqrt{b^2 - 4ac}}{2a}$

[c] $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

[d] $\frac{-b \pm \sqrt{b^2 + 4ac}}{2a}$

2. **If the discriminant $b^2 - 4ac$ is negative, the roots are:**

[a] Real [b] Rational
[c] Irrational [d] Imaginary

3. **The 7^{th} term of an A.P 1,4,7, …… is:**

[a] 7 [b] 19
[c] 21 [d] 23

4. **The G.M between 'a' and 'b' is:**

[a] $\pm ab$ [b] ab
[c] $\pm\sqrt{ab}$ [d] $\sqrt{ab}$

5. **If an = $n^2 + n + 1$, then its 4^{th} term will be:**

[a] 21 [b] 40 [c] 41 [d] 101

6. **The number of terms in the expansion of $(a+b)^{13}$ are:**

[a] 12 [b] 13 [c] 15 [d] 14

7. **In the expansion of $(a+b)^n$, the general term is:**

[a] $\binom{n}{r}a^r b^r$ [b] $\binom{n}{r}a^{n-r}b^r$

[c] $\binom{n}{r-1}a^{n-r}b^{r+1}$

[d] $\binom{n}{r}a^{n+r-1}b^{r+1}$

8. **The number of partial fractions $\frac{6x+27}{4x^3-9x}$ are:**

[a] 2 [b] 3
[c] 4 [c] None of these

9. **π radian is equal to:**

[a] $360°$ [b] $270°$
[c] $90°$ [d] $180°$

10. **If an arc of a circle has length 1 and subtends are angle of θ, then its radius will be:**

[a] $\frac{\theta}{\ell}$ [b] $\frac{\ell}{\theta}$
[c] $\ell\theta$ [d] $\ell + \theta$

11. **$\cos(\alpha - \beta)$ is equal to:**

[a] $\cos\alpha\cos\beta - \sin\alpha\sin\beta$
[b] $\cos\alpha\cos\beta + \sin\alpha\sin\beta$
[c] $\cos\alpha\sin\beta - \sin\alpha\cos\beta$
[d] $\sin\alpha\cos\beta + \cos\alpha\sin\beta$

12. **$\sin(\pi - x)$ is equal to:**

[a] $-\sin x$ [b] $\sin x$
[c] $\cos x$ [d] $-\cos x$

13. **$\frac{\sin(\alpha+\beta)}{\cos\alpha\cos\beta}$ is equal to:**

[a] $\tan\alpha + \tan\beta$ [b] $\tan\alpha - \tan\beta$
[c] $\sin\alpha + \sin\beta$ [d] $\sin\alpha - \sin\beta$

14. **In a triangle ABC, ∠A = 70º, ∠B = 60º then ∠C is:**

[a] $30°$ [b] $40°$ [c] $50°$ [d] $60°$

15. **Find the distance of a man from the foot of a tower, 100m high if the angle of elevation of its top as observed by the man is 30º.**

[a] 50m [b] 100m
[c] 150m [d] 173m

Answer Key

1	c	2	d	3	c	4	c	5	a
6	d	7	b	8	b	9	d	10	b
11	b	12	b	13	a	14	c	15	d

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DAE / IIA - 2016

MATH - 113 APPLIED MATHEMATICS - I

PAPER 'A' PART - B (SUBJECTIVE)

Time: 2:30 Hrs Marks: 60

Section - I

Q.1. Write short answers to any Eighteen (18) questions.

1. Solve the quadratic equation

$$x^2 - x = 2$$

Sol.
$$x^2 - x = 2$$
$$x^2 - x - 2 = 0$$
$$x^2 - 2x + x - 2 = 0$$
$$x(x-2) + 1(x-2) = 0$$
$$(x-2)(x+1) = 0$$

Either $x - 2 = 0$, $x = 2$ OR $x + 1 = 0$, $x = -1$

$$S.S. = \{-1, 2\}$$

2. ~~Find the sum and the product of roots of~~ $x^2 - 9 = 0$.

Sol. Here: $a = 1,\ b = 0,\ c = -9$

$$\text{Sum of the Roots} = S = -\frac{b}{a} = -\frac{0}{1} = \boxed{0}$$

$$\text{Product of the Roots} = P = \frac{c}{a} = \frac{-9}{1} = \boxed{-9}$$

3. ~~From the quadratic equation whose roots are~~ -2 and -3.

Sol.

$S = -2 + (-3)$, $S = -2 - 3$, $S = -5$ | $P = (-2)(-3)$, $P = 6$

$$x^2 - Sx + P = 0$$

$$x^2 - (-5)x + (6) = 0 \Rightarrow \boxed{x^2 + 5x + 6 = 0}$$

4. Find the 7^{th} term of an A.P. $1, 4, 7, \ldots$

Sol. Here: $a_1 = 1$ & $d = 4 - 1 = 3$

$$a_7 = a_1 + 6d$$
$$a_7 = 1 + 6(3)$$
$$a_7 = 1 + 18 \Rightarrow \boxed{a_7 = 19}$$

5. ~~Define a series.~~

Sol. The sum of the terms of a sequence is called a series.

6. Write the nth term of a Geometric progression.

Sol. $\boxed{a_n = ar^{n-1}}$ Where $a = 1^{st}$ term, r = common Ratio and n = No. of terms.

7. ~~Find the sum of the series 3 + 11 + 19 + ... to 16 terms.~~

Sol. Here: $a_1 = 3,\ d = 11 - 3 = 8$ & $n = 16$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{16} = \frac{16}{2}[2(3) + (16-1)(8)]$$

$$S_{16} = 8[6 + 120] = 8(126) = \boxed{1008}$$

8. Find the geometric mean between $\frac{4}{3}$ and 243.

Sol. Here: $a = \frac{4}{3}$, $b = 243$

$$G = \pm\sqrt{ab} = \pm\sqrt{\frac{4}{3} \times 243}$$

$$G = \pm\sqrt{324} \Rightarrow \boxed{G = \pm 18}$$

9. ~~Find the sum of the series~~ $1 + \frac{1}{3} + \frac{1}{9} + \ldots$ ~~to 6 terms.~~

Sol. Here: $a_1 = 1$, $r = \frac{1}{3} \div 1 = \frac{1}{3}$,

$n = 6$ & $S_6 = ?$

$$S_n = \frac{a(1 - r^n)}{1 - r}$$

$$S_6=\frac{1\left(1-\left(\frac{1}{3}\right)^6\right)}{1-\frac{1}{3}}=\frac{1-\frac{1}{729}}{\frac{3-1}{3}}$$

$$S_6=\frac{\frac{729-1}{729}}{\frac{2}{3}}=\frac{728}{729}\times\frac{3}{2}=\boxed{\frac{364}{243}}$$

10. **Expand $(2x-3y)^4$.**

Sol. $(2x-3y)^4$

$$=\binom{4}{0}(2x)^4(3y)^0-\binom{4}{1}(2x)^3(3y)^1+\binom{4}{2}(2x)^2(3y)^2-\binom{4}{3}(2x)^1(3y)^3+\binom{4}{4}(2x)^0(3y)^4$$

$$=(1)(16x^4)(1)-4(8x^3)(3y)+6(4x^2)(9y^2)-4(2x)(27y^3)+(1)(1)(81y^4)$$

$$=\boxed{16x^4-96x^3y+216x^2y^2-216xy^3+81y^4}$$

11. **Find the 6th term in the expansion of $(x+3y)^{10}$.**

Sol. Here: $a=x$, $b=3y$, $n=10$ & $r=5$

Using general term formula,

$$T_{r+1}=\binom{n}{r}a^{n-r}b^r$$

$$T_{5+1}=\binom{10}{5}(x)^{10-5}(3y)^5$$

$$T_6=(252)(x^5)(243y^5)$$

$$\boxed{T_6=61236x^5y^5}$$

12. ~~**Expand to three terms**~~ $\frac{1}{(1+x)^2}$

Sol. $\frac{1}{(1+x)^2}=(1+x)^{-2}$

Put $b=x$ & $n=-2$ in Binomial series Formula, we have:

$$=1+(-2)(x)+\frac{(-2)(-2-1)}{2!}(x)^2+\dots$$

$$=1-2x+\frac{(-2)(-3)}{2}x^2+\dots$$

$$=\boxed{1-2x+3x^2+\dots}$$

13. **Give an example of improper fraction.**

Sol. Example: $\frac{x^2+1}{(x+1)(x-1)}$

14. **Resolve $\frac{x^2+1}{(x+1)(x-1)}$ into partial fractions.**

Sol. $\frac{x^2+1}{(x+1)(x-1)}$ {Improper Fraction}

$$=1+\frac{2}{(x+1)(x-1)}$$

(Division: $x^2-1 \overline{) x^2+1}$, quotient 1; subtract $\pm x^2 \mp 1$; remainder 2)

Take $\frac{2}{(x+1)(x-1)}=\frac{A}{x+1}+\frac{B}{x-1}\rightarrow(i)$

$$2=A(x-1)+B(x+1)\rightarrow(ii)$$

Put $x=-1$ in eq.(ii)

$$2=A(-1-1)+B(-1+1)$$

$$2=A(-2)+B(0)$$

$$2=-2A+0\Rightarrow\boxed{A=-1}$$

Put $x=1$ in eq.(ii)

$$2=A(1-1)+B(1+1)$$

$$2=A(0)+B(2)$$

$$2=0+2B\Rightarrow\boxed{B=1}$$

Put values of A & B in eq.(i),

we get: $\boxed{1-\frac{1}{x+1}+\frac{1}{x-1}}$

15. Write an identity equation of $\dfrac{2x+5}{x^2+5x+6}$

Sol. $\dfrac{2x+5}{x^2+5x+6}$

$=\dfrac{2x+5}{(x+2)(x+3)}$

$=\boxed{\dfrac{A}{x+2}+\dfrac{B}{x+3}}$

x^2+5x+6
$=x^2+3x+2x+6$
$=x(x+3)+2(x+3)$
$=(x+3)(x+2)$

16. ~~Convert $\dfrac{2\pi}{3}$ radians into degree measure.~~

Sol. $\dfrac{2\pi}{3}\text{rad}=\dfrac{2\pi}{3}\times\dfrac{180}{\pi}=\boxed{120°}$

17. ~~Find 'r' when ℓ = 33 cm and θ = 6 radian.~~

Sol. By using formula : $\ell = r\theta$

$r=\dfrac{\ell}{\theta}=\dfrac{33}{6}=\boxed{5.5\text{cm}}$

18. Prove that:

$$(1+\sin\theta)(1-\sin\theta)=\frac{1}{\sec^2\theta}$$

Sol. L.H.S. $=(1+\sin\theta)(1-\sin\theta)$

$=(1)^2-(\sin\theta)^2=1-\sin^2\theta$

$=\cos^2\theta=\dfrac{1}{\sec^2\theta}=$ R.H.S. **Proved.**

19. Prove that:

$$\frac{1}{1+\sin\theta}+\frac{1}{1-\sin\theta}=2\sec^2\theta$$

Sol. L.H.S. $=\dfrac{1}{1+\sin\theta}+\dfrac{1}{1-\sin\theta}$

$=\dfrac{1(1-\sin\theta)+1(1+\sin\theta)}{(1+\sin\theta)(1-\sin\theta)}$

$=\dfrac{1-\sin\theta+1+\sin\theta}{(1)^2-(\sin\theta)^2}$

$=\dfrac{2}{1-\sin^2\theta}=\dfrac{2}{\cos^2\theta}$

$=2\sec^2\theta=$ R.H.S. **Proved.**

20. Prove that: $\cos\left(\dfrac{\pi}{2}-\theta\right)=\sin\theta$

Sol. L.H.S. $=\cos\left(\dfrac{\pi}{2}-\theta\right)$

$=\cos(90°-\theta)\ \because\left\{\dfrac{\pi}{2}\times\dfrac{180°}{\pi}=90°\right\}$

$=\cos 90°\cos\theta+\sin 90°\sin\theta$

$=(0)\cos\theta+(1)\sin\theta\ \because\left\{\begin{matrix}\text{Using calculator}\\ \cos 90°=0\ \&\ \sin 90°=1\end{matrix}\right\}$

$=0+\sin\theta=\sin\theta=$ R.H.S. **Proved.**

21. Prove that:

$$\sin(\alpha+\beta)+\sin(\alpha-\beta)=2\sin\alpha\cos\beta$$

Sol. L.H.S. $=\sin(\alpha+\beta)+\sin(\alpha-\beta)$

$=\sin\alpha\cos\beta+\cancel{\cos\alpha\sin\beta}+\sin\alpha\cos\beta-\cancel{\cos\alpha\sin\beta}$

$=2\sin\alpha\cos\beta=$ R.H.S. **Proved.**

22. Prove that:

$$\cos 2\alpha=\cos^2\alpha-\sin^2\alpha$$

Sol. L.H.S. $=\cos 2\alpha=\cos(\alpha+\alpha)$

$=\cos\alpha\cos\alpha-\sin\alpha\sin\alpha$

$=\cos^2\alpha-\sin^2\alpha=$ R.H.S. **Proved.**

23. ~~Express as sum or difference~~ $\cos 3\theta\cos\theta$.

Sol. $\cos 3\theta\cos\theta$

$=\dfrac{1}{2}[2\cos 3\theta\cos\theta]$

$=\dfrac{1}{2}[\cos(3\theta+\theta)+\cos(3\theta-\theta)]$

$=\boxed{\dfrac{1}{2}[\cos 4\theta+\cos 2\theta]}$

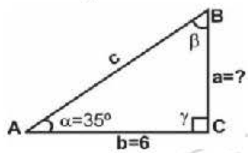
24. **Define law of Sines.**

Sol. In any triangle ABC, with usual notations.

$$\boxed{\frac{a}{\sin\alpha}=\frac{b}{\sin\beta}=\frac{c}{\sin\gamma}}$$

25. ~~**In a right angled triangle ABC,**~~ $\mathbf{b=6,\ a=35^\circ,\ r=90^\circ}$. ~~**Find side 'a'.**~~

Sol. We know that, from figure:



$$\tan\alpha=\frac{a}{b}$$

$$\tan 35^\circ=\frac{a}{6}$$

$$6\tan 35^\circ=a \Rightarrow \boxed{a=4.2}$$

26. **In any triangle ABC in which b = 45, c = 34, ∝ = 52º, find 'a'.**

Sol. By using Law of cosines:

$$a^2=b^2+c^2-2bc\cos 52^\circ$$

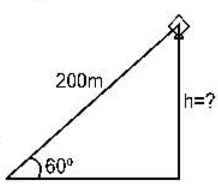
$$a^2=(45)^2+(34)^2-2(45)(34)\cos 52^\circ$$

$$a^2=2025+1156-1883.92$$

$$a^2=1297.07 \Rightarrow \boxed{a=36.01}$$

27. ~~**A string of a flying kite is 200 meters long, and its angle of elevation is 60º. Find the height of the kite above the ground taking the string to be fully stretched.**~~

Sol. In this figure:



$$\sin 60^\circ=\frac{h}{200}$$

$$200\sin 60^\circ=h$$

$$\boxed{h=173.20\,\text{m}}$$

Section - II

Note : Attemp any three (3) questions $3\times 8=24$

Q.2.(a) **Solve the quadratic equation by using quadratic formula** $\mathbf{mx^2+(1+m)x+1=0}$

Sol. See Q.3(vi) of Ex # 1.1 (Page # 22)

(b) **Show that the roots of the equation** $\mathbf{(mx+c)^2=4ax}$ **will be equal; is** $\mathbf{c=\frac{a}{m}}$

Sol. See Q.3(ii) of Ex # 1.2 (Page # 33)

Q.3. **Find five numbers in A.P whose sum is 30 and the sum of whose squares is 190.**

Sol. See Q.12 of Ex # 2.3 (Page # 97)

Q.4. **Find the middle term in the expansion of** $\mathbf{\left(\frac{a}{2}-\frac{b}{3}\right)^{11}}$.

Sol. See Q.6(ii) of Ex # 3.1 (Page # 151)

Q.5.(a) ~~**A horse moves in a circle, at one end of rope 27m long; the other end being fixed. How far does the horse move when the rope traces an angle of 70º at the center.**~~

Sol. See Q.7 of Ex # 5.1 (Page # 240)

(b) **If** $\mathbf{\tan\beta=\frac{n\sin\alpha\cos\alpha}{1-\sin^2\alpha}}$; **prove that tan (α – β) = (1 – n) tan α**

Sol. See Q.11 of Ex # 6.1 (Page # 286)

Q.6.(a) **Prove that:**

$$\mathbf{\sin 3\theta=3\sin\theta-4\sin^3\theta}$$

Sol. See Triple angles proof (Page # 294)

(b) **Find the cosine of the smallest measure of an angle of a triangle with 12, 13 and 14 meters as the measure of its sides.**

Sol. See Q.10 of Ex # 7.4 (Page # 352)

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