DAE / IIA - 2016

MATH - 123 APPLIED MATHEMATICS - I

PAPER 'B' PART - A (OBJECTIVE)

Time : 30 Minutes Marks : 15

Q.1: *Encircle the correct answer.*

1. Conjugate of $7-2i$ is:
[a] $-7+2i$ [b] $7-2i$
[c] $-7-2i$ [d] $7+2i$

2. The value of $(-i)^5$ is:
[a] i [b] -1
[c] $-i$ [d] 1

3. Which complex number shall be the additive identity:
[a] (1, 1) [b] (1, 0)
[c] (0, 1) [d] (0, 0)

4. In the proper fraction the degree of numerator than the degree of denomination will be:
[a] greater [b] equal
[c] less [d] similar

5. Partial fractions of $\frac{7x-25}{(x-3)(x-4)}$ are:
[a] $\frac{Ax+B}{x^2-7x+12}$ [b] $\frac{A}{x^2-7x+12}$
[c] $\frac{A}{(x-3)}+\frac{B}{(x-4)}$
[d] $\frac{A+Bx}{x^2-7x+12}$

6. The bits 0 and 1 used in the number system:
[a] Octal [b] Binary
[c] Hexadecimal
[d] Decimal

7. If $X=1$, then $\overline{X}$ equal to:
[a] 0 [b] -1
[c] X [d] X

8. In Boolean algebra $X+\overline{X}Y$ is equal to:
[a] $X+\overline{Y}$ [b] $\overline{X}+Y$
[c] $\overline{X}+\overline{Y}$ [c] $X+Y$

9. $X.(\overline{X}+Y)$ equal to:
[a] XY [b] $X.\overline{X}$
[c] $X.\overline{X}+X.Y$ [d] $X+Y$

10. If two lines are parallel, then their slope m_1 and m_2 will be:
[a] $m_1m_2=-1$ [b] $m_1\neq m_2$
[c] $m_1=m_2$ [d] $m_1m_2=1$

11. If the inclination of a line is 45º then its slope is:
[a] 1 [b] -1
[c] 0 [d] $\frac{\pi}{4}$

12. In the equation $y=mx+c$, the slope of the line is:
[a] m [b] c
[c] x [d] y

13. $\frac{x}{a}+\frac{y}{b}=1$ is the equation of straight line is:
[a] Slope intercept
[b] Two points form
[c] Point slope form
[d] Intercept form

14. ~~In the general form of the equation of circle, the coefficient of x^2 and y^2 are:~~
[a] Proportional [b] Equal
[c] Unequal [d] Opposite to

15. The center of $(x-3)^2+(y-4)^2=5$
[a] (–3, –4) [b] (3, 5)
[c] (3, 4) [d] (4, 5)

Answer Key

1	d	2	c	3	d	4	c	5	c
6	b	7	a	8	d	9	a	10	c
11	a	12	a	13	d	14	b	15	c

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DAE / IIA - 2016

MATH-123 APPLIED MATHEMATICS-I

PAPER 'B' PART-B(SUBJECTIVE)

Time: 2:30 Hrs Marks: 60

Section - I

Q.1. Write short answers to any Eighteen (18) questions.

1. Find the value of x and y from the equation $(x, y)(1, 2) = (-1, 8)$

Sol. $(x, y)(1, 2) = (-1, 8)$

$(x + yi)(1 + 2i) = (-1 + 8i)$

$x + 2xi + yi + 2yi^2 = -1 + 8i$

$(x - 2y) + i(2x + y) = -1 + 8i$

Comparing real & imaginary parts:

$x - 2y = -1 \rightarrow (i),\ 2x + y = 8 \rightarrow (ii)$

Multiply eq. (ii) by 2 & adding in eq. (i)

$4x + 2y = 16$

$\begin{array}{l} x - 2y = -1 \\ \hline 5x \quad = 15 \end{array} \Rightarrow x = \frac{15}{5} \Rightarrow \boxed{x = 3}$

Put $x = 3$ in eq.(i), we have:

$3 - 2y = -1 \Rightarrow -2y = -1 - 3$

$-2y = -4 \Rightarrow y = \frac{-4}{-2} \Rightarrow \boxed{y = 2}$

2. Reduce $\frac{\sqrt{-3}}{1-\sqrt{-7}}$ in the form $a + bi$.

Sol. $\frac{\sqrt{-3}}{1-\sqrt{-7}} = \frac{\sqrt{3}i}{1-\sqrt{7}i}$

$= \frac{\sqrt{3}i}{1-\sqrt{7}i} \times \frac{1+\sqrt{7}i}{1+\sqrt{7}i}$

$= \frac{\sqrt{3}i + \sqrt{21}i^2}{(1)^2 - (7i)^2} = \frac{\sqrt{3}i - \sqrt{21}}{1+7}$

$= \frac{-\sqrt{21} + \sqrt{3}i}{8} = \boxed{-\frac{\sqrt{21}}{8} + \frac{\sqrt{3}}{8}i}$

3. Factorize $2x^2 + 5y^2$

Sol. $2x^2 + 5y^2$

$= 2x^2 - 5y^2 i^2$

$= \left(\sqrt{2}x\right)^2 - \left(\sqrt{5}yi\right)^2$

$= \boxed{\left(\sqrt{2}x - \sqrt{5}yi\right)\left(\sqrt{2}x + \sqrt{5}yi\right)}$

4. Find the multiplicative of $(-3, 4)$.

Sol. Let $z = (-3, 4) = -3 + 4i$

Multiplicative Inverse of $Z = \frac{1}{Z}$

$= \frac{1}{-3+4i} = \frac{1}{-3+4i} \times \frac{-3-4i}{-3-4i}$

$= \frac{-3-4i}{(-3)^2 - (4i)^2} = \frac{-3-4i}{9+16}$

$= \frac{-3-4i}{25} = \boxed{-\frac{3}{25} - \frac{4}{25}i}$

5. ~~**Express the complex number $3-\sqrt{3}i$ in polar form.**~~

Sol. Let, $z = 3 - \sqrt{3}i$

Here: $a = 3$ & $b = -\sqrt{3}$

$r = |z| = \sqrt{a^2 + b^2}$

$r = \sqrt{(3)^2 + (-\sqrt{3})^2}$

$r = \sqrt{9+3}$

$r = \sqrt{12}$

$r = \sqrt{4 \times 3}$

$r = 2\sqrt{3}$

$\theta = \tan^{-1}\left(\frac{b}{a}\right)$

$\theta = \tan^{-1}\left(\frac{-\sqrt{3}}{3}\right)$

$\theta = \tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)$

$\theta = -30°$

$z = r\text{cis}\theta = 2\sqrt{3}\text{cis}(-30°)$

$z = \boxed{2\sqrt{3}(\cos 30° - i\sin 30°)}$

6. **Resolve into partial fractions** $\dfrac{2x}{(x-2)(x+5)}$

Sol. $\dfrac{2x}{(x-2)(x+5)} = \dfrac{A}{x-2} + \dfrac{B}{x+5} \rightarrow (i)$

$2x = A(x+5) + B(x-2) \rightarrow (ii)$

Put $x = 2$ in eq. (ii)

$2(2) = A(2+5) + B(2-2)$

$4 = A(7) + B(0)$

$4 = 7A + 0 \Rightarrow \boxed{A = 4/7}$

Put $x = -5$ in eq. (ii)

$2(-5) = A(-5+5) + B(-5-2)$

$-10 = A(0) + B(-7)$

$-10 = 0 - 7B \Rightarrow \boxed{B = 10/7}$

Put values of A, & B in eq. (i),

we get: $\boxed{\dfrac{4}{7(x-2)} + \dfrac{10}{7(x+5)}}$

7. **Resolve into partial fractions.** $\dfrac{1}{x^2 - x}$

Sol. $\dfrac{1}{x^2 - x} = \dfrac{1}{x(x-1)}$

$\dfrac{1}{x(x-1)} = \dfrac{A}{x} + \dfrac{B}{x-1} \rightarrow (i)$

$1 = A(x-1) + Bx \rightarrow (ii)$

Put $x = 0$ in eq. (ii)

$1 = A(0-1) + B(0)$

$1 = -A \Rightarrow \boxed{A = -1}$

Put $x = 1$ in eq. (ii)

$1 = A(1-1) + B(1)$

$1 = A(0) + B \Rightarrow \boxed{B = 1}$

Put values of A, & B

in eq. (i), we get: $\boxed{-\dfrac{1}{x} + \dfrac{1}{x-1}}$

8. **Write an identity equation of** $\dfrac{6x^3 + 5x^2 - 7}{3x^2 - 2x - 1}$

Sol. $\dfrac{6x^3 + 5x^2 - 7}{3x^2 - 2x - 1}$ {Improper Fraction}

$$
\begin{array}{r|l}
 & 2x + 3 \\
3x^2 - 2x - 1 & 6x^3 + 5x^2 - 7 \\
 & \underline{-6x^3 \mp 4x^2 \mp 2x} \\
 & 9x^2 + 2x - 7 \\
 & \underline{-9x^2 \mp 6x \mp 3} \\
 & 8x - 4
\end{array}
$$

$3x^2 - 2x - 1$
$= 3x^2 - 3x + x - 1$
$= 3x(x-1) + 1(x-1)$
$= (x-1)(3x+1)$

$= 2x + 3 + \dfrac{8x - 4}{3x^2 - 2x - 1}$

$= 2x + 3 + \dfrac{8x - 4}{(x-1)(3x+1)}$

$= \boxed{2x + 3 + \dfrac{A}{x-1} + \dfrac{B}{3x+1}}$

9. ~~**Form of partial fractions of**~~ ~~$\dfrac{2x^4 - 3x^2 - 4x}{(x+1)(x^2+2)^2}$~~

Sol. $\dfrac{2x^4 - 3x^2 - 4x}{(x+1)(x^2+2)^2}$

$= \boxed{\dfrac{A}{(x+1)} + \dfrac{Bx + C}{(x^2+2)} + \dfrac{Dx + E}{(x^2+2)^2}}$

10. **Convert binary number** $(11111)_2$ **to decimal number.**

Sol. $(11111)_2$

$= 1 \times 2^4 + 1 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 1 \times 2^0$

$= 16 + 8 + 4 + 2 + 1 = \boxed{31}$

11. **Convert octal numbers to decimal number $(100.24)_8$**

Sol. $(100.24)_8$

$$=1\times 8^2+0\times 8^1+0\times 8^0+2\times 8^{-1}+4\times 8^{-2}$$

$$=64+0+0+\frac{2}{8}+\frac{4}{64}$$

$$=64+0.25+0.0625=\boxed{64.3125}$$

12. **Simplify by use of Boolean rules:**

$$(X+Y)(X+\overline{Y})(\overline{X}+Z)$$

Sol. $(X+Y)(X+\overline{Y})(\overline{X}+Z)$

$$=(XX+X\overline{Y}+XY+Y\overline{Y})(\overline{X}+Z)$$

$$=(X+X\overline{Y}+XY+0)(\overline{X}+Z)\because\left\{\begin{matrix}XX=X\\ Y\overline{Y}=0\end{matrix}\right\}$$

$$=(X(1+\overline{Y})+XY)(\overline{X}+Z)$$

$$=(X(1)+XY)(\overline{X}+Z)\because 1+\overline{Y}=1$$

$$=(X+XY)(\overline{X}+Z)$$

$$=X(1+Y)(\overline{X}+Z)$$

$$=X(1)(\overline{X}+Z)\quad\because 1+Y=1$$

$$=X(\overline{X}+Z)$$

$$=X\overline{X}+XZ=0+XZ=\boxed{XZ}$$

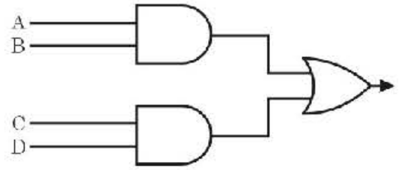
13. **Prove by truth table $\overline{X+Y}=\overline{X}.\overline{Y}$**

Sol. L.H.S. R.H.S.

X	Y	$\overline{X}$	$\overline{Y}$	X+Y	$\overline{X+Y}$	$\overline{X}.\overline{Y}$
0	0	1	1	0	1	1
0	1	1	0	1	0	0
1	0	0	1	1	0	0
1	1	0	0	1	0	0

14. **Construct the logic diagram for expression $AB+CD$**

Sol.



15. **Find the value of 'y' so that the distance between $(1, y)$ and $(-1, 4)$ is 2.**

Sol. Let, $A=(1, y)$ & $B=(-1, 4)$

As, $|\overline{AB}|=2$

$$\Rightarrow\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}=2$$

$$\Rightarrow\sqrt{(1-(-1))^2+(y-4)^2}=2$$

Squaring both sides:

$$\Rightarrow\left(\sqrt{(2)^2+(y-4)^2}\right)^2=(2)^2$$

$$4+(y-4)^2=4$$

$$(y-4)^2=4-4$$

$$(y-4)^2=0$$

Taking square root on both sides:

$$y-4=0\qquad\Rightarrow\qquad\boxed{y=4}$$

16. **Find the point which is two third of the way from the point $(5, 1)$ to the point $(-2, 9)$.**

Sol. Let P(x, y) be require point which is $2/3$ of the way.

Here $r_1=2, r_2=1, (x_1, y_1)=(5,1)$

& $(x_2, y_2)=(-2,9)$

Using the Ratio formula:

$$P(x, y)=\left(\frac{r_1x_2+r_2x_1}{r_1+r_2}, \frac{r_1y_2+r_2y_1}{r_1+r_2}\right)$$

$$=\left(\frac{2(-2)+1(5)}{2+1}, \frac{2(9)+1(1)}{2+1}\right)$$

$$=\left(\frac{-4+5}{3}, \frac{18+1}{3}\right)=\boxed{\left(\frac{1}{3}, \frac{19}{3}\right)}$$

17. **If a line ℓ_1 contains points $(2, 6)$ and $(0, y)$. Find 'y' if ℓ_1 is parallel to ℓ_2 and the slope of $\ell_2 = \frac{3}{4}$.**

Sol. Slope of line

$$\ell_1 = m_1 = \frac{y_2-y_1}{x_2-x_1} = \frac{y-6}{0-2} = \frac{y-6}{-2}$$

& slope of line $\ell_2 = m_2 = \frac{3}{4}$

As, both lines are parallel,

So, $m_1 = m_2$

$$\frac{y-6}{-2} = \frac{3}{4}$$

$$4(y-6) = 3(-2)$$

$$4y-24 = -6$$

$$4y = -6+24$$

$$4y = 18 \Rightarrow y = \frac{18}{4} \Rightarrow \boxed{y = \frac{9}{2}}$$

18. ~~**Find the equation of a line through the points $(-1, 2)$ and $(3, 4)$.**~~

Sol. Slope $= \frac{y_2-y_1}{x_2-x_1} = \frac{4-2}{3-(-1)} = \frac{2}{4} = \frac{1}{2}$

Equation of line in point - slope form:

$$y-y_1 = m(x-x_1)$$

$$y-2 = \frac{1}{2}(x-(-1))$$

$$2(y-2) = 1(x+1)$$

$$2y-4 = x+1$$

$$2y-4-x-1 = 0$$

$$-x+2y-5 = 0 \Rightarrow \boxed{x-2y+5=0}$$

19. ~~**Write an equation of line parallel to $2x-7y=8$ and containing the origin.**~~

Sol. As given line: $2x-7y=8$

$$-7y = -2x+8$$

Dividing each term on '-7', we get:

$$y = \frac{2}{7}x - \frac{8}{7}$$

Comparing it with slope intercept form: $y = mx + c$

Slope of given line $= m = \frac{2}{7}$

As given line and require line are **Parallel**

So, Slope of require line $= m = \frac{2}{7}$

Equation of line in point - slope form:

$$y-y_1 = m(x-x_1)$$

$$y-0 = \frac{2}{7}(x-0)$$

$$7y = 2x \Rightarrow \boxed{2x-7y=0}$$

20. **Find 'k' so that $x+y+1=0$, $kx-y+3=0$, $4x-5y+k=0$ will be concurrent.**

Sol.
$$\begin{vmatrix} 1 & 1 & 1 \\ k & -1 & 3 \\ 4 & -5 & k \end{vmatrix} = 0$$

$$1\begin{vmatrix} -1 & 3 \\ -5 & k \end{vmatrix} - 1\begin{vmatrix} k & 3 \\ 4 & k \end{vmatrix} + 1\begin{vmatrix} k & -1 \\ 4 & -5 \end{vmatrix} = 0$$

$$1(-k+15) - 1(k^2-12) + 1(-5k+4) = 0$$

$$-k+15-k^2+12-5k+4 = 0$$

$$-k^2-6k+31 = 0$$

$k^2 + 6k - 31 = 0$

Here: $a = 1,\ b = 6,\ c = -31$

By us Quadratic formula:

$$k = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$k = \frac{-6 \pm \sqrt{(6)^2 - 4(1)(-31)}}{2(1)}$$

$$k = \frac{-6 \pm \sqrt{36 + 124}}{2} = \frac{-6 \pm 4\sqrt{160}}{2}$$

$$k = \frac{2\left(-3 \pm 2\sqrt{10}\right)}{2} \Rightarrow \boxed{k = -3 \pm 2\sqrt{10}}$$

21. ~~**Find the angle between the lines having slopes −3 and 2.**~~

Sol. Let, $m_1 = -3$ and $m_2 = 2$

$$\theta = \tan^{-1}\left(\frac{m_2 - m_1}{1 + m_2 m_1}\right)$$

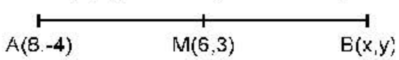
$$\theta = \tan^{-1}\left(\frac{2 - (-3)}{1 + (2)(-3)}\right)$$

$$\theta = \tan^{-1}\left(\frac{2+3}{1-6}\right) = \tan^{-1}\left(\frac{5}{-5}\right)$$

$$\theta = \tan^{-1}(-1) = \boxed{135^\circ}$$

22. **If the mid-point of a segment is (6, 3) and one end point is (8, – 4), what are the coordinates of the other end point.**

Sol. Let B(x, y) be require end point.



As, Mid – point $= (6,\ 3)$

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = (6,\ 3)$$

$$\left(\frac{8 + x}{2}, \frac{-4 + y}{2}\right) = (6,\ 3)$$

Comparing both order pairs, we have:

$$\frac{8+x}{2} = 6 \text{ and } \frac{-4+y}{2} = 3$$

$8 + x = 12$	$-4 + y = 6$
$x = 12 - 8 = 4$	$y = 6 + 4 = 10$

Hence other end point $= \boxed{(4,\ 10)}$

23. ~~**Find the distance from the point (−2, 1) to the line 3x + 4y − 12 = 0**~~

Sol. Distance between point & line

$$D = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}} = \frac{|3(-2) + 4(1) - 12|}{\sqrt{(3)^2 + (4)^2}}$$

$$D = \frac{|-6 + 4 - 12|}{\sqrt{9 + 16}} = \frac{|-14|}{\sqrt{25}} = \boxed{\frac{14}{5}}$$

24. **Find the equation of circle with center (1, -3) and r = 3.**

Sol. Here: Center $= (h,\ k) = (1, -3)$

& Radius $= r = 3$

Standard form of equation of circle:

$$(x - h)^2 + (y - k)^2 = r^2$$

Put $h = 1,\quad k = -3\quad \&\quad r = 3$

$$(x - 1)^2 + (y + 3)^2 = (3)^2$$

$$(x)^2 - 2(x)(1) + (1)^2 + (y)^2 + 2(y)(3) + (3)^2 = 9$$

$$x^2 - 2x + 1 + y^2 + 6y + 9 - 9 = 0$$

$$\boxed{x^2 + y^2 - 2x + 6y + 1 = 0}$$

25. **Find the center and radius of the circle $6x^2 + 6y^2 - 18y = 0$**

Sol. $6x^2 + 6y^2 - 18y = 0$

Dividing each term by 6, we get:

$$x^2 + y^2 - 3y = 0$$

Comparing with general form:

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

$2g = 0$	$2f = -3$	$c = 0$
$g = 0$	$f = -3/2$	

$$\text{Center} = (-g, -f)$$

$$\text{Center} = \left(0, -\left(-\frac{3}{2}\right)\right) = \boxed{\left(0, \tfrac{3}{2}\right)}$$

$$\text{Radius} = r = \sqrt{g^2 + f^2 - c}$$

$$r = \sqrt{(0)^2 + \left(-\frac{3}{2}\right)^2 - 0} = \sqrt{\frac{9}{2}} = \boxed{\frac{3}{2}}$$

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26. **Reduce the equation into standard form: $x^2 + y^2 - 10y = 0$**

Sol. Adding the square of one half of the coefficient of y i.e., $(5)^2$ on both sides:

$$x^2 + y^2 - 10y + (5)^2 = 0 + (5)^2$$

$$(x)^2 + (y-5)^2 = (5)^2$$

$$\boxed{(x-0)^2 + (y-5)^2 = (5)^2}$$

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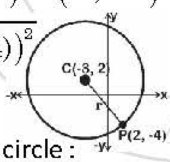
27. **Find the equation of circle. Central at $(-3, 2)$ and passes through the point $(2, -4)$.**

Sol. Radius = r =

Distance between $(-3, 2)$ & $(2, -4)$

$$r = \sqrt{(-3-2)^2 + (2-(-4))^2}$$

$$r = \sqrt{25+36} = \boxed{\sqrt{61}}$$



Standard form of eq. of circle:

$$(x-h)^2 + (y-k)^2 = r^2$$

Put, $h = -3,\ k = 2\ \&\ r = \sqrt{61}$

$$(x)^2 + 2(x)(3) + (3)^2 + (y)^2 - 2(y)(2) + (2)^2 = 61$$

$$x^2 + 6x + 9 + y^2 - 4y + 4 - 61 = 0$$

$$\boxed{x^2 + y^2 + 6x - 4y - 48 = 0}$$

Section - II

Note : Attemp any three (3) questions $3 \times 8 = 24$

Q.2.(a) **Simplify the expression $\left(-1 + i\sqrt{3}\right)^3$**

Sol. See Q.4(vii) of Ex# 8.1 (Page # 305)

(b) **Reduce $\dfrac{(3+4i)(1-2i)}{1+i}$ into the form $a + bi$.**

Sol. See Q.4(xi) of Ex# 8.1 (Page # 306)

Q.3. **Resolve into partial fractions. $\dfrac{1}{x^4(x+1)}$.**

Sol. See example 07 of Chapter 09

Q.4.(a) **Convert the octal number $(217.436)_8$ to binary number.**

Sol. See Q.5[e] of Ex# 10 (Page # 409)

(b) **Prove by use of Boolean Rules $AB + \overline{A}C + BC = AB + \overline{A}C$**

Sol. See example Q.5[c] of chapter 11

Q.5.(a) **Show that the points $(0, 6)$, $(9, -6)$ and $(-3, 0)$ are the vertices of a right triangle.**

Sol. See Q.4[a] of Ex# 12.3 (Page # 468)

(b) ~~Find the equation of the line passing through $(-1, 7)$ and perpendicular to the line through the points $(2, 3)$ and $(0, -4)$.~~

Sol. See Q.12 of Ex# 12.4 (Page # 481)

Q.6. **Find the equation of circle passing through the points $(1, 2)$, $(0, -1)$ and $(-1, 1)$.**

Sol. See Q.3[a] of Ex# 13 (Page # 524)

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