DAE / IA - 2019

MATH - 113 APPLIED MATHEMATICS - I

PAPER 'B' PART - A (OBJECTIVE)

Time: 30 Minutes Marks: 15

Q.1: *Encircle the correct answer.*

1. Magnitude of the vector $2\mathbf{i}-2\mathbf{j}-\mathbf{k}$ is:

[a] 4 [b] 3 [c] 2 [d] 1

2. If θ is the angle between the vector $\vec{a}$ and $\vec{b}$, then Cosθ is:

[a] $\vec{a}.\vec{b}$ [b] $\frac{\vec{a}.\vec{b}}{|a||b|}$

[c] $\frac{\vec{a}.\vec{b}}{|\vec{a}|}$ [d] $\frac{\vec{a}.\vec{b}}{|\vec{b}|}$

3. If $\vec{a}=3\mathbf{i}+4\mathbf{j}-\mathbf{k}$, $\vec{b}=-2\mathbf{i}+3\mathbf{j}+\mathbf{k}$ then $\vec{a}.\vec{b}$

[a] 6 [b] -6 [c] -5 [d] 5

4. The order of the matrix [1 2 3] is:

[a] 1×3 [b] 3×1 [c] 3×3 [d] 2×3

5. If $A=\begin{bmatrix}2&3&4\\0&1&-1\\2&0&1\end{bmatrix}$ is a matrix, then Co-factor of 4 is:

[a] -2 [b] 2 [c] -3 [d] 4

6. Which matrix can be rectangular matrix?

[a] Diagonal [b] Identify

[c] Scalar [d] None of these

7. If a = 4cm, b = 2cm are adjacent sides of a triangle and θ = 30º is the included angle then area is:

[a] 2 sq.cm [b] 4 sq.cm

[c] 8 sq.cm [d] 12 sq.cm

8. Area of rectangle having sides 8cm and 5cm is:

[a] 13 sq.cm [b] 40 sq.cm

[c] 45 sq.cm [d] 30 sq.cm

9. Area of regular hexagon circumscribed about a circle of radius 2cm is:

[a] $\frac{24}{\sqrt{3}}$ [b] $\frac{20}{\sqrt{3}}$ [c] $\frac{36}{\sqrt{3}}$ [d] $\frac{2}{\sqrt{3}}$

10. Circumference of a circle whose radius is cm is $\frac{1}{2}$ equal to:

[a] π [b] $2\pi r$ [c] $\frac{\pi r}{2}$ [d] $\frac{r}{2}$

11. Exact area of an irregular figure is calculated by:

[a] Mid-ordinates rule

[b] Simpson's

[c] Trapezoidal rule

[d] None of these

12. Volume of hexagonal prism with height 'h' and side 'a' is:

[a] $\frac{3\sqrt{3}}{2}a^2h$ [b] $\frac{1}{2}a^2h$

[c] a^2h [d] $3\sqrt{3}a^2h$

13. Volume of circular cylinder of height 'h' and radius 'r' is:

[a] πr^2h [b] $2\pi rh$

[c] $2\pi rh^2$ [d] $2\pi r^2h$

14. Lateral surface area of regular pyramid if perimeter of base is P and slant height 'ℓ' is:

[a] $P\ell$ [b] $\frac{1}{3}P\ell$ [c] $\frac{1}{2}P\ell$ [d] $\frac{1}{6}P\ell$

15. Volume of hemi-sphere is:

[a] $\frac{2}{3}\pi r^3$ [b] $\frac{4}{3}\pi r^3$ [c] $\frac{1}{2}\pi r^3$ [d] $3\pi r^2$

Answer Key

1	b	2	b	3	d	4	a	5	a
6	d	7	a	8	b	9	a	10	a
11	b	12	a	13	a	14	c	15	a

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DAE / IA - 2019

MATH - 113 APPLIED MATHEMATICS - I

PAPER 'B' PART - B (SUBJECTIVE)

Time: 2:30 Hrs Marks: 60

Section - I

Q.1. Write short answers to any Eighteen (18) questions.

1. What are parallel vectors?

Sol. Two vectors $\vec{a}$ and $\vec{b}$ are parallel if there exist a non-zero $k \in R$, such that $\vec{a} = k\vec{b}$

2. Given the vectors: $\vec{a} = 3i - 2j + k$, $\vec{b} = 2i - 4j - 3k$, $\vec{c} = -i + 2j + 2k$. Find $\vec{a} + \vec{b} + \vec{c}$

Sol. $\vec{a} + \vec{b} + \vec{c}$

$= 3i - 2j + k + 2i - 4j - 3k - i + 2j + 2k$

$= \boxed{4i - 4j + 0k}$

3. If $\vec{a} = 2i + 3j + 4k$ & $\vec{b} = i - j + k$. Find $|\vec{a} \times \vec{b}|$

Sol. $\vec{a} \times \vec{b} = \begin{vmatrix} i & j & k \\ 2 & 3 & 4 \\ 1 & -1 & 1 \end{vmatrix}$

$= i\begin{vmatrix} 3 & 4 \\ -1 & 1 \end{vmatrix} - j\begin{vmatrix} 2 & 4 \\ 1 & 1 \end{vmatrix} + k\begin{vmatrix} 2 & 3 \\ 1 & -1 \end{vmatrix}$

$= i(3+4) - j(2-4) + k(-2-3)$

$= i(7) - j(-2) + k(-5)$

$= 7i + 2j - 5k$

$|\bar{a} \times \bar{b}| = \sqrt{49 + 4 + 25} = \boxed{\sqrt{78}}$

4. Prove that $\vec{a}$ and $\vec{b}$ are perpendicular to each other if $\vec{a} = i + 3j - 2k$ & $\vec{b} = i - j - k$

Sol. $\vec{a} \cdot \vec{b} = (i + 3j - 2k) \cdot (i - j - k)$

$= (1)(1) + (3)(-1) + (-2)(-1)$

$= 1 - 3 + 2 = \boxed{0}$

Hence $\vec{a}$ and $\vec{b}$ are perpendicular.

5. If the vectors $3i + j - k$ and $\lambda i - 4j + 4k$ are parallel, find value of λ.

Sol. As $\vec{a}$ and $\vec{b}$ are parallel, so

$\vec{a} \times \vec{b} = 0$

$\begin{vmatrix} i & j & k \\ 3 & 1 & -1 \\ \lambda & -4 & 4 \end{vmatrix} = 0$

$i\begin{vmatrix} 1 & -1 \\ -4 & 4 \end{vmatrix} - j\begin{vmatrix} 3 & -1 \\ \lambda & 4 \end{vmatrix} + k\begin{vmatrix} 3 & 1 \\ \lambda & -4 \end{vmatrix} = 0$

$i(4-4) - j(12+\lambda) + k(-12-\lambda) = 0$

$i(0) - (12+\lambda)j + (-12-\lambda)k = 0$

comparing coefficients of 'j', we get:

$12 + \lambda = 0 \Rightarrow \boxed{\lambda = -12}$

6. Define identity matrix.

Sol. A diagonal matrix in which all diagonal elements are 1 is called identity matrix.

7. Show that $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & -3 \\ 3 & -3 & 6 \end{bmatrix}$ is symmetric.

Sol. $A^t = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & -3 \\ 3 & -3 & 6 \end{bmatrix}^t$

$= \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & -3 \\ 3 & -3 & 6 \end{bmatrix} = A$

As, $A^t = A$ so, A is **symmetric**.

Proved.

8. Find x and y if

$$\begin{bmatrix} 2 & 1 \\ -3 & 2 \end{bmatrix} = \begin{bmatrix} x+3 & 1 \\ -3 & 3y-4 \end{bmatrix}$$

Sol. $\begin{bmatrix} 2 & 1 \\ -3 & 2 \end{bmatrix} = \begin{bmatrix} x+3 & 1 \\ -3 & 3y-4 \end{bmatrix}$

Comparing corresponding elements of both matrices :

$$x+3=2 \quad x=2-3 \quad \boxed{x=-1}$$

$$3y-4=2 \quad 3y=2+4 \quad 3y=6 \quad y=\frac{6}{3} \quad \boxed{y=2}$$

9. **Find the inverse of $\begin{bmatrix} 2 & 1 \\ 6 & 3 \end{bmatrix}$**

Sol. Let $A=\begin{bmatrix} 2 & 1 \\ 6 & 3 \end{bmatrix}$

$$|A| = \begin{vmatrix} 2 & 1 \\ 6 & 3 \end{vmatrix}$$

$$|A| = (2)(3)-(6)(1)$$

$$|A| = 6-6$$

$$|A| = 0$$

As, $|A|=0$ so inverse of A does not exits.

10. **Find A^{-1} if $A=\begin{bmatrix} 5 & 3 \\ 1 & 1 \end{bmatrix}$**

Sol. $A=\begin{bmatrix} 5 & 3 \\ 1 & 1 \end{bmatrix}$

$$|A| = \begin{vmatrix} 5 & 3 \\ 1 & 1 \end{vmatrix} = 5-3=2$$

$$\text{Adj } A = \begin{bmatrix} 1 & -3 \\ -1 & 5 \end{bmatrix}$$

$$A^{-1} = \frac{\text{Adj}(A)}{|A|}$$

$$A^{-1} = \frac{\begin{bmatrix} 1 & -3 \\ -1 & 5 \end{bmatrix}}{2}$$

$$A^{-1} = \boxed{\begin{bmatrix} \frac{1}{2} & -\frac{3}{2} \\ -\frac{1}{2} & \frac{5}{2} \end{bmatrix}}$$

11. **Define plane figure.**

Sol. The figure which occupy an area with only two dimensions is called plane figure.

12. **A triangular blank of equal sides is to punch in a copper plate, the area of the blank should be 24 sq.cm find the side.**

Sol. Let each side of triangular blank = a cm

Area of triangle of equal sides = 24

$$\frac{\sqrt{3}}{4}a^2 = 24$$

$$a^2 = 24 \times \frac{4}{\sqrt{3}}$$

$$a^2 = 50.40$$

$$\sqrt{a^2} = \sqrt{50.40} \Rightarrow \boxed{a = 7.4\text{cm}}$$

13. **If the perimeter of a square is 40cm. Find the area of the square.**

Sol. Let, length of one side of square = a = ?

As, Perimeter of square = 40cm

$$4a = 40$$

$$a = 40/4 = 10\text{cm}$$

$$\text{Area of square} = a^2$$

$$\text{Area} = (10)^2 = \boxed{100\,\text{sq. cm}}$$

14. **The diagonals of a rhombus are 40cm and 30cm, find its area.**

Sol. Here $d_1 = 40m, \& \ d_2 = 30m$

$$\text{Area of Rhombus} = \frac{d_1 \times d_2}{2}$$

$$\text{Area} = \frac{40 \times 30}{2} = \boxed{600\,\text{sq.m}}$$

15. **Define a polygon.**

Sol. A plane figure bounded by a finite number of straight lines is called polygon.

16. **The perimeter of a regular hexagon is 12cm, find its area.**

Sol. Perimeter of hexagon = 12 cm

$$6a = 12$$

$$\Rightarrow \quad a = 12/6 = 2\text{cm}$$

$$\text{Area} = \frac{na^2}{4}\cot\left(\frac{180°}{n}\right)$$

$$A = \frac{6(2)^2}{4}\cot\left(\frac{180°}{6}\right)$$

$$A = 6\cot 30° = \frac{6}{\tan 30°}$$

$$A = \boxed{10.39\,\text{sq.cm}}$$

17. **Find the radius of a circle the area of which is 9.3129 sq.cm.**

Sol. As, Area of circle = 9.3129 sq.cm

$$\pi r^2 = 9.3129$$

$$r^2 = \frac{9.3129}{\pi}$$

$$r^2 = 2.96 \Rightarrow \boxed{r = 1.72\,\text{cm}}$$

18. **Write the area of the segment in terms of height and length of the chord of the segment.**

Sol. $\text{Area of segment} = \frac{h}{6c}\left(3h^2 + 4c^2\right)$

19. **Three cubes of metal whose edges are 3, 4 and 5cm respectively, are melted without any loss of metal into a single – cube. Find**
(i) edge of the new cube
(ii) surface area of the new cube.

Sol. **(i)** Here : Edges of melted cubes are 3cm, 4cm, 5cm

Let, edge of new cube = a cm

(ii) As, volume of new cube = volume of melted cubes

$$a^3 = 3^3 + 4^3 + 5^3$$

$$a^3 = 216$$

$$\left(a^3\right)^{1/3} = (216)^{1/3} \Rightarrow \boxed{a = 6\text{cm}}$$

$$\text{Surface area of new cube} = 6a^2$$

$$\text{S.A.} = 6 \times (6)^2 = 6 \times 36 = \boxed{216\,\text{cm}^2}$$

20. **The dimension of a marriage hall are 100m, 50m and 18m respectively, find volume of the hall.**

Sol. Here: ℓ = 100m, b = 50m & h = 18m

$$\text{Volume} = \ell bh$$

$$V = 100 \times 50 \times 18 = \boxed{90000\,\text{m}^3}$$

21. **If base of a field is 50m and number of ordinates are 11, then find breadth of strip.**

Sol. Length of base = 50m

As, No. of ordinates = 11

so, No. of strips = 10

S = Width of each strip

$$S = \frac{\text{Length of base}}{\text{No. of strips}}$$

$$S = \frac{50}{10} = \boxed{5\text{m}}$$

22. **In a hollow cylinder, the circles of cross-section are concentric. If the**

internal diameter of these circles be 2.2cm and 3.8cm respectively and the height be 6.5cm, find the volume of hollow interior.

Sol. Here : $D = 3.8\text{cm}, d = 2.2\text{cm}$ & $h = 6.5\text{cm}$

A = Area of base (Annulus)

$$A = \frac{\pi}{4}\left(D^2 - d^2\right)$$

$$A = \frac{\pi}{4}\left((3.8)^2 - (2.2)^2\right)$$

$$A = \frac{\pi}{4}(9.6)$$

$$A = \boxed{7.54\,\text{cm}^2}$$

Volume of hollow interior $= A \times h$

$$= 7.54 \times 6.5 = \boxed{49\text{cm}^3}$$

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23. Find the diameter of the cylinder if its volume is 704cm³ and height is 14cm.

Sol. Here : $d = ?, V = 704\text{cm}^3$ & $h = 14\text{cm}$

As, Volume of Cylinder $= 704\text{cm}^3$

$$\Rightarrow \quad \pi r^2 h = 704$$

$$\Rightarrow \quad r^2 = \frac{704}{\pi h}$$

$$\Rightarrow \quad r^2 = \frac{704}{\pi(14)}$$

$$\Rightarrow \quad r^2 = 16$$

$$\Rightarrow \quad r = 4\,\text{cm}$$

Diameter $= d = 2r = 2(4) = \boxed{8\,\text{cm}}$

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24. Define pyramid.

Sol. A pyramid is a solid, whose base is a plane polygon and sides being triangles that meet in a common vertex.

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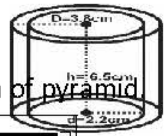
25. ~~**Let A_1 be the area of the base, and A_2 be the area of the top, a is the side of the base and b is the side of the top ℓ is the slant height and h is the height of the frustum of a pyramid, then find**~~

~~**(i) Volume of the frustum of a pyramid.**~~

~~**(ii) Lateral surface area.**~~



Sol.

(i) Volume of the frustum of pyramid

$$\boxed{V = \frac{h}{3}\left[A_1 + A_2 + \sqrt{A_1 A_2}\right]}$$

(ii) Leteral Surface area of the frustum of a Pyramid (L.S.A.) $= \frac{1}{2} \times$ sum of perimeter of the base and top $\times$ slant height

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26. Define sphere.

Sol. A sphere is a solid bounded by a closed surface, every point of which is equidistance from a fixed point called the Centre.

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27. Find the volume of a segment of a sphere whose height is $4\frac{1}{2}$ cm and diameter for whose base is 8cm.

Sol. Here : $h = 4\frac{1}{2} = \frac{9}{2} = 4.5\text{cm}$,

& $d = 8\text{cm} \Rightarrow r = 4\text{cm}$

Volume of segment of shpere

$$V = \frac{\pi h}{6}\left[h^2 + 3r^2\right]$$

$$V = \frac{\pi(4.5)}{6}\left[(4.5)^2 + 3(4)^2\right]$$

$$V = \boxed{160.81\,\text{cm}^3}$$

Section - II

Note : Attemp any three (3) questions $3 \times 8 = 24$

Q.2.(a) Show that the vectors $4i - 6j + 9k$ and $-6i + 9j - \frac{27}{2}k$ are parallel.

Sol. See Q.7 of Ex # 8.1 (Page # 373)

(b) Find $\left|(\vec{a} \times \vec{b}) \times \vec{c}\right|$ if $\vec{a} = i - 2j - 3k$, $\vec{b} = 2i + j - k$, $\vec{c} = i + 3j - 2k$.

Sol. See Q.18 of Ex # 8.2 (Page # 388)

Q.3.(a) If $A = \begin{bmatrix} 2 & -2\sqrt{2} \\ \sqrt{2} & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 2\sqrt{2} \\ -\sqrt{2} & 2 \end{bmatrix}$, show that A and B commute.

Sol. See Q.9 of Ex # 9.1 (Page # 413)

(b) Find the inverse if it exists, of the matrix. $A = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 4 \\ 0 & 2 & 2 \end{bmatrix}$

Sol. See Q.4(iii) of Ex # 9.3 (Page # 440)

Q.4.(a) A track round the inside of a rectangular grassy plot 40m by 30m occupies 600 sq.m show that the width of the track is 5m.

Sol. See Q.2 of Ex # 11 (Page # 474)

(b) The distance between the corners of a hexagonal nut is 2.28 cm. Find the distance between the jaws of the wrench needed to fit this nut.

Sol. See Q.6 of Ex # 12 (Page # 487)

Q.5.(a) Find area of an irregular figure by Simpson's Rule if the ordinates are 9, 11, 13, 12, 10, 13, 15, 17, 14, 12, 7 meters and base is 73 meters.

Sol. See Q.6 of Ex # 14 (Page # 509)

(b) The length, width and height of a rectangular prism are 6, 4 and 3 meters respectively. Find the volume, the surface area and the length of the diagonal.

Sol. See Q.1 of Ex # 15 (Page # 518)

Q.6.(a) ~~Find the cost of canvas, at the rate of Rs.5 per square meter, required to make a tent in the form of a frustum of a square pyramid. The sides of the base and top are 6m and 4m respectively and the height is 8m, taking no account of waste.~~

Sol. See Q.5 of Ex # 17[B] (Page # 556)

(b) ~~Find the curved and total surface area and the volume of the frustum of a cone whose top and bottom diameters are 6m and 10m and the height is 12m.~~

Sol. See Q.1 of Ex # 18[B] (Page # 573)

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