DAE / IA - 2019

MATH-113 APPLIED MATHEMATICS-I

PAPER 'A' PART-A (OBJECTIVE)

Time: 30 Minutes Marks: 15

Q.1: *Encircle the correct answer.*

1. Factors of $x^3 - 1$ are:
[a] $(x-1)(x^2-x-1)$
[b] $(x-1)(x^2+x+1)$
[c] $(x-1)(x^2+x-1)$
[d] $(x-1)(x^2-x+1)$

2. Roots of the equation $x^2 + x - 1 = 0$ are:
[a] Equal [b] Irrational
[c] Imaginary [d] Rational

3. If a, b, c are in A.P then:
[a] $b-a=c-b$ [b] $\frac{b}{a}=\frac{c}{b}$
[c] $a+b=b+c$ [d] $\frac{a}{c}=\frac{b}{a}$

4. The 6th term of G.P $1, \sqrt{2}, \sqrt{4}$ is:
[a] $4\sqrt{2}$ [b] 4 [c] $\sqrt{2}$ [d] 2

5. The sum of infinite geometric series $1+\frac{1}{3}+\frac{1}{9}+\ldots$ is:
[a] $\frac{2}{3}$ [b] $-\frac{2}{3}$ [c] $\frac{3}{2}$ [d] $-\frac{3}{2}$

6. The value of $\begin{bmatrix} n \\ r \end{bmatrix}$ is:
[a] $\frac{n!}{r!(n-r)!}$ [b] $\frac{n}{r(n-r)}$
[c] $\frac{n!}{r!(n-r)}$ [d] $\frac{n!}{(n-r)!}$

7. The middle term of $\left[\frac{x}{y}-\frac{y}{x}\right]^4$ is:
[a] $\frac{4x^2}{y^2}$ [b] 6 [c] 8 [d] $\frac{4x}{y}$

8. The equivalent partial fraction of $\frac{x^4}{(x^2+1)(x^2+3)}$ is:
[a] $\frac{Ax+B}{x^2+1}+\frac{Cx+D}{x^2+3}$ [b] $\frac{Ax+B}{x^2+1}+\frac{Cx}{x^2+3}$
[c] $1+\frac{Ax+B}{x^2+1}+\frac{Cx+D}{x^2+3}$ [d] $\frac{Ax}{x^2+1}+\frac{Bx}{x^2+3}$

9. One degree is equal to:
[a] π rad [b] $\frac{\pi}{180}$ rad
[c] $\frac{180}{\pi}$ rad [d] $\frac{\pi}{360}$ rad

10. The terminal side of θ lies in 4th quadrant, both sin θ are:
[a] $\sin\theta>0, \tan\theta>0$
[b] $\sin\theta>0, \tan\theta<0$
[c] $\sin\theta<0, \tan\theta<0$
[d] $\sin\theta<0, \tan\theta>0$

11. If $\sin x = \frac{\sqrt{3}}{2}$ and the terminal ray of x lies in 1st quadrant, then cos x is equal to:
[a] $\frac{1}{\sqrt{2}}$ [b] $-\frac{1}{2}$ [c] $\frac{1}{2}$ [d] $-\frac{1}{\sqrt{2}}$

12. $\tan\left(\frac{\pi}{2}+\theta\right)$ is equal to:
[a] $\tan\theta$ [b] $\cot\theta$ [c] $-\cot\theta$ [d] $-\tan\theta$

13. sin (A + B) – sin (A – B) is equal to:
[a] $2\sin A\cos B$ [b] $2\cos A\sin B$
[c] $-2\sin A\sin B$ [d] $2\cos A\cos B$

14. If c = 90º, b = 1, $c=\sqrt{2}$ then side a is:
[a] 1 [b] 2 [c] $\sqrt{2}$ [d] 3

15. In triangle ABC, the law of cosine is:
[a] $\cos A=\frac{b^2+c^2+a^2}{2bc}$
[b] $\cos A=\frac{b^2+c^2-a^2}{2bc}$
[c] $\cos A=\frac{b^2+c^2+a^2}{2ab}$
[d] $\cos A=\frac{b^2+c^2-a^2}{2ac}$

Answer Key

1	b	2	c	3	a	4	a	5	c
6	a	7	b	8	c	9	b	10	c
11	c	12	d	13	d	14	a	15	b

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DAE / IA - 2019

MATH - 113 APPLIED MATHEMATICS - I

PAPER 'A' PART - B (SUBJECTIVE)

Time: 2:30 Hrs Marks: 60

Section - I

Q.1. Write short answers to any Eighteen (18) questions.

1. Solve the quadratic equation $\mathbf{mx^2+(1+m)x+1=0}$ by factorization.

Sol. $mx^2+(1+m)x+1=0$

$mx^2+x+mx+1=0$

$x(mx+1)+1(mx+1)=0$

$(mx+1)(x+1)=0$

Either

$mx+1=0$

$mx=-1 \Rightarrow x=-\frac{1}{m}$

OR

$x+1=0$

$x=-1$

$$\text{S.S.}=\boxed{\left\{-1,-\frac{1}{m}\right\}}$$

2. Discuss the nature of the roots of the equation $\mathbf{x^2-2\sqrt{2}x+2=0}$

Sol. Here: $a=1,\ b=-2\sqrt{2},\ c=2$

$\text{Disc.}=b^2-4ac$

$=\left(-2\sqrt{2}\right)^2-4(2)(1)=8-8=0$

∴ The roots are **Equal and Real.**

3. ~~Form the quadratic equation whose roots are~~ $-2+\sqrt{3},\ -2-\sqrt{3}$

Sol. $S=-2+\sqrt{3}+\left(-2-\sqrt{3}\right)$

$S=-2+\sqrt{3}-2-\sqrt{3}=-4$

$P=\left(-2+\sqrt{3}\right)\left(-2-\sqrt{3}\right)$

$P=(-2)^2-\left(\sqrt{3}\right)^2=4-(3)=1$

$x^2-Sx+P=0$

$x^2-(-4)x+1=0\Rightarrow\boxed{x^2+4x+1=0}$

4. **Define infinite sequence.**

Sol. A sequence is called infinite sequence, if it has infinite terms.

Example: 4, 6, 8, 10, . . .

5. ~~**Write the formula to find the sum of 'n' terms of an arithmetic sequence.**~~

Sol. $\boxed{S_n=\frac{n}{2}\left[2a_1+(n-1)d\right]}$

6. **Find the A.M between $\sqrt{5}-4$ and $\sqrt{5}+4$**

Sol. Let $a=\sqrt{5}-4$ and $b=\sqrt{5}+4$

$\text{A.M.}=A=\frac{a+b}{2}$

$A=\frac{\sqrt{5}-4+\sqrt{5}+4}{2}=\frac{2\sqrt{5}}{2}=\boxed{\sqrt{5}}$

7. **If $\frac{1}{a},\frac{1}{b},\frac{1}{c}$ are G.P, show that the common ratio is $\pm\sqrt{\frac{a}{c}}$**

Sol. As, $\frac{1}{a},\ \frac{1}{b},\ \frac{1}{c}$ are in G.P.

Here: $a_1=\frac{1}{a},\quad a_3=\frac{1}{c}\quad \&\quad r=?$

As, $a_3=\frac{1}{c}$

$a_1r^2=\frac{1}{c}\qquad \because a_3=a_1r^2$

$\left(\frac{1}{a}\right)r^2=\frac{1}{c}$

$r^2=\frac{a}{c}\ \Rightarrow\ \boxed{r=\pm\sqrt{a/c}}$ **Proved.**

8. ~~**Sum to 5 term the series**~~ $\mathbf{1+3+5+\ ...}$

Sol. Here: $a = 1,\ r = \frac{3}{1} = 3\ \ \&\ n = 5$

$$S_n = \frac{a(r^n - 1)}{r - 1} = \frac{1((3)^5 - 1)}{3 - 1}$$

$$S_5 = \frac{243 - 1}{2} = \frac{242}{2} \Rightarrow \boxed{S_5 = 121}$$

9. ~~Is the series~~ $6 + 3 + \frac{3}{2} + \frac{3}{4} + \frac{3}{8} + \ldots$ ~~divergent or convergent.~~

Sol. Since $r = \frac{3}{6} = \frac{1}{2} < 1$ so sum is possible. So, the given series is **convergent.**

10. Expand the Bi-nomial theorem $\left(\frac{x}{y} + \frac{y}{x}\right)^4$.

Sol. $\left(\frac{x}{y} + \frac{y}{x}\right)^4$

$$= \binom{4}{0}\left(\frac{x}{y}\right)^4\left(\frac{y}{x}\right)^0 + \binom{4}{1}\left(\frac{x}{y}\right)^3\left(\frac{y}{x}\right)^1 + \binom{4}{2}\left(\frac{x}{y}\right)^2\left(\frac{y}{x}\right)^2 + \binom{4}{3}\left(\frac{x}{y}\right)^1\left(\frac{y}{x}\right)^3 + \binom{4}{4}\left(\frac{x}{y}\right)^0\left(\frac{y}{x}\right)^4$$

$$= (1)\left(\frac{x^4}{y^4}\right)(1) + 4\left(\frac{x^3}{y^3}\right)\left(\frac{y}{x}\right) + 6\left(\frac{x^2}{y^2}\right)\left(\frac{y^2}{x^2}\right) + 4\left(\frac{x}{y}\right)\left(\frac{y^3}{x^3}\right) + 1(1)\left(\frac{y^4}{x^4}\right)$$

$$= \boxed{\frac{x^4}{y^4} + 4\frac{x^2}{y^2} + 6 + 4\frac{y^2}{x^2} + \frac{y^4}{x^4}}$$

11. State Binomial Theorem for positive integer n.

Sol. The rule for expansion of $(a + b)^n$, where 'n' is any positive integral power, is called binomial theorem, and defined as:

$$(a + b)^n = \binom{n}{0}a^n b^0 + \binom{n}{1}a^{n-1}b^1 + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{n}a^0 b^n$$

12. ~~Expand~~ $(1 + 2x)^{-2}$ ~~to three terms.~~

Sol. $(1 + 2x)^{-2}$

$$= 1 + (-2)(2x) + \frac{(-2)(-2-1)}{2!}(2x)^2 + \ldots$$

$$= 1 - 4x + \frac{(-2)(-3)}{2}(4x^2) + \ldots$$

$$= \boxed{1 - 4x + 12x^2 + \ldots}$$

13. Which term is the middle term or terms in the Binomial expansion of $(a + b)^n$, When 'n' is odd.

Sol. When n is odd

Then, there are two middle terms:

$$\text{Middle term} = \left(\frac{n+1}{2}\right)^{th} + \left(\frac{n+3}{2}\right)^{th} \text{ terms.}$$

14. Define proper fraction and give example.

Sol. A fraction in which the degree of the numerator is less than the degree of the denominator is called proper fraction.

Example: $\frac{2x}{(x-2)(x+5)}$

15. Form of partial fractions of $\frac{1}{(x+1)(x-2)}$ is ______.

Sol. $\frac{1}{(x+1)(x-2)} = \boxed{\frac{A}{(x+1)} + \frac{B}{(x-2)}}$

16. ~~Convert~~ $42^\circ\ 36'\ 12''$ ~~into radian measure.~~

Sol. $42^\circ 36' 12''$

$$= \left(42 + \frac{36}{60} + \frac{12}{3600}\right)^\circ$$

$$= (42 + 0.6 + 0.0033)^\circ$$

$$= 42.6033^\circ$$

$$= 42.6033 \times \frac{\pi}{180} = \boxed{0.74\ \text{rad}}$$

17. ~~Find the radius of the circle, when $\ell = 8.4m, \theta = 2.8$ rad.~~

Sol. We know that: $\ell = r\theta$

$$\Rightarrow r = \frac{\ell}{\theta} = \frac{8.4}{2.8} = \boxed{3m}$$

18. Prove that: $\dfrac{2\tan\frac{\pi}{6}}{1-\tan^2\frac{\pi}{6}} = \sqrt{3}$

Sol. $\text{L.H.S.} = \dfrac{2\tan\frac{\pi}{6}}{1-\tan^2\frac{\pi}{6}} = \dfrac{2\tan 30^\circ}{1-\tan^2 30^\circ}$

$$= \frac{2\left(\frac{1}{\sqrt{3}}\right)}{1-\left(\frac{1}{\sqrt{3}}\right)^2} = \frac{\frac{2}{\sqrt{3}}}{1-\frac{1}{3}} = \frac{\frac{2}{\sqrt{3}}}{\frac{3-1}{3}} = \frac{\frac{2}{\sqrt{3}}}{\frac{2}{3}}$$

$$= \frac{2}{\sqrt{3}} \times \frac{3}{2} = 3^{1-\frac{1}{2}} = 3^{1/2} = \sqrt{3} = \text{R.H.S.}$$

Proved.

19. Show that:

$$\cot^4\theta + \cot^2\theta = \operatorname{cosec}^4\theta - \operatorname{cosec}^2\theta$$

Sol. $\text{L.H.S.} = \cot^4\theta + \cot^2\theta$

$= \cot^2\theta(\cot^2\theta + 1)$

$= \cot^2\theta(\operatorname{cosec}^2\theta) \because \cot^2\theta + 1 = \operatorname{cosec}^2\theta$

$= (\operatorname{cosec}^2\theta - 1)(\operatorname{cosec}^2\theta)$

$= \operatorname{cosec}^4\theta - \operatorname{cosec}^2\theta = \text{R.H.S.}$ **Proved.**

20. Prove that: $\sin(-\theta) = -\sin\theta$

Sol. We know that

$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

Put $\alpha = 0$ & $\beta = \theta$ we have:

$\sin(0-\theta) = \sin(0)\cos\theta - \cos(0)\sin\theta$

$\sin(-\theta) = 0.\cos\beta - 1.\sin\theta$

$\sin(-\theta) = 0 - \sin\theta$

$\boxed{\sin(-\theta) = -\sin\theta}$ **Proved.**

21. Show that:

$$\cos(\alpha+\beta) - \cos(\alpha-\beta) = -2\sin\alpha\sin\beta$$

Sol. $\text{L.H.S.} = \cos(\alpha+\beta) - \cos(\alpha-\beta)$

$= [\cos\alpha\cos\beta - \sin\alpha\sin\beta] - [\cos\alpha\cos\beta + \sin\alpha\sin\beta]$

$= \cancel{\cos\alpha\cos\beta} - \sin\alpha\sin\beta - \cancel{\cos\alpha\cos\beta} - \sin\alpha\sin\beta$

$= -2\sin\alpha\sin\beta = \text{R.H.S.}$ **Proved.**

22. ~~If $\sin\theta = \frac{4}{5}$ and the terminal side of θ lies in 1st quadrant, find $\cos\frac{\theta}{2}$.~~

Sol. As, $\cos\theta = \sqrt{1-\sin^2\theta}$

$$= \sqrt{1-\left(\frac{4}{5}\right)^2} = \sqrt{\frac{25-16}{25}} = \sqrt{\frac{9}{25}} = 3/5$$

$$\cos\frac{\theta}{2} = \sqrt{\frac{1+\cos\theta}{2}} = \sqrt{\frac{1+\frac{3}{5}}{2}} = \sqrt{\frac{\frac{5+3}{5}}{2}}$$

$$\cos\frac{\theta}{2} = \sqrt{\frac{8}{10}} = \sqrt{\frac{4}{5}} = \boxed{\frac{2}{\sqrt{5}}}$$

23. ~~Express~~ $\cos 3\theta\cos\theta$ ~~as sum or difference.~~

Sol. $\cos 3\theta\cos\theta$

$= \frac{1}{2}[2\cos 3\theta\cos\theta]$

$= \frac{1}{2}[\cos(3\theta+\theta) + \cos(3\theta-\theta)]$

$= \boxed{\frac{1}{2}[\cos 4\theta + \cos 2\theta]}$

24. Define the laws of cosines.

Sol.
i. $a^2 = b^2 + c^2 - 2bc\cos\alpha$
ii. $b^2 = c^2 + a^2 - 2ca\cos\beta$
iii. $c^2 = a^2 + b^2 - 2ab\cos\gamma$

25. In any triangle ABC, if a = 9, b = 5, $\gamma = 32^\circ$ find c.

Sol. By using law of cosines.

$$c^2 = a^2 + b^2 - 2ab\cos\gamma$$

$$c^2 = (9)^2 + (5)^2 - 2(9)(5)\cos 32^\circ$$

$$c^2 = 81 + 25 - 76.32$$

$$c^2 = 29.67$$

$$\sqrt{c^2} = \sqrt{29.67} \Rightarrow \boxed{c = 5.45}$$

26. **In any triangle ABC if**

a = 3, b = 7, β = 85º find α.

Sol. By using law of sines:

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$$

$$\text{We take: } \frac{a}{\sin\alpha} = \frac{b}{\sin\beta}$$

$$\sin\alpha = \frac{a\sin\beta}{b} \Rightarrow \sin\alpha = \frac{3\sin 85^\circ}{7}$$

$$\sin\alpha = 0.4269$$

$$\alpha = \sin^{-1}(0.4269) \Rightarrow \boxed{\alpha = 25^\circ 16'}$$

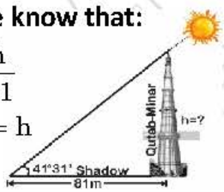
27. ~~**The shadow of Qutab-Minar is 81m long when the measure of the angle of elevation of the sun is 41º31′. Find the height of the Qutab-Minar.**~~

Sol. **From figure, we know that:**

$$\tan 41^\circ 31' = \frac{h}{81}$$

$$81\tan 41^\circ 31' = h$$

$$\boxed{h = 71.70\,\text{m}}$$



Section - II

Note : Attemp any three (3) questions $3 \times 8 = 24$

Q.2.(a) **Solve the equation** $x^2 - 3\left(x + \frac{25}{4}\right) = 9x - \frac{25}{2}$ **by using quadratic formula.**

Sol. See Q.3(iv) of Ex # 1.1 (Page # 21)

(b) **Show that the roots of the equation** $(a+2b)x^2 + 2(a+b+c)x + (a+2c) = 0$ **are rational.**

Sol. See Q.6(i) of Ex # 1.2 (Page # 36)

Q.3.(a) **The sum of three numbers in A.P is 24 and their product is 440. Find the numbers.**

Sol. See Q.10 of Ex # 2.3 (Page # 95)

(b) **If the second term of a G.P is 2 and the 11th term is** $\frac{1}{256}$**, what is the first term and the nth term?**

Sol. See Q.6 of Ex # 2.4 (Page # 106)

Q.4.(a) **If x is nearly equal to unity, prove that**

$$\frac{mx^n - nx^m}{x^n - x^m} = \frac{1}{1-x}$$

Sol. See Q.4 of Ex # 3.2 (Page # 169)

(b) **Resolve** $\frac{6x+27}{4x^3 - 9x}$ **into partial fractions.**

Sol. See Q.8 of Ex # 4.1 (Page # 188)

Q.5.(a) **If** $\sin\theta = \frac{4}{5}$, and $\frac{\pi}{2} < \theta < \pi$ **find trigonometric ratios of θ.**

Sol. See Q.8 of Ex # 5.2 (Page # 248)

(b) **Prove that:**

$$(1-\tan\theta)^2 + (1-\cot\theta)^2 = (\sec\theta - \operatorname{cosec}\theta)^2.$$

Sol. See Q.15 of Ex # 5.3 (Page # 259)

Q.6.(a) ~~**Prove that:**~~

$$\cos 20^\circ \cos 40^\circ \cos 60^\circ \cos 80^\circ = \frac{1}{16}$$

Sol. See Q.17 of Ex # 6.3 (Page # 318)

(b) **Solve the triangle ABC with given data** $c = 4$, $\alpha = 70^\circ$, $\gamma = 42^\circ$

Sol. See Q.1 of Ex # 7.5 (Page # 354)

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