DAE / IA - 2019

MATH - 123 APPLIED MATHEMATICS - I

PAPER 'B' PART - A(OBJECTIVE)

Time : 30Minutes Marks : 15

Q.1: *Encircle the correct answer.*

1. **Conjugate of $(2+3i)+(1-i)$ is:**

[a] $3-2i$ [b] $3+4i$
[c] $3-4i$ [d] $3+2i$

2. **Product of $2+3i$ and $2-3i$ is:**

[a] $\sqrt{13}$ [b] 13
[c] $\sqrt{2}$ [d] $\sqrt{-5}$

3. **If $Z=a+bi$ then, $\bar{Z}$ is equal to:**

[a] $a+bi$ [b] $a-bi$
[c] $a+b$ [d] $a-b$

4. **If the degree of numerator N(x) is equal or greater than the degree of denominator D(x), then the fraction is:**

[a] Proper [b] Improper
[c] Neither proper nor-improper
[d] Both proper and improper

5. **The number of partial fractions of $\dfrac{x^3-3x^2+1}{(x-1)(x+1)(x^2-1)}$ are:**

[a] 2 [b] 3 [c] 4 [d] 5

6. **$(11)_8 \times (7)_8$ is equal to:**

[a] $(105)_8$ [b] $(77)_8$
[c] $(43)_8$ [d] None of these

7. **According to Boolean Algebra is $X+\bar{X}$ equal to:**

[a] X [b] $\bar{X}$ [c] 0 [d] 1

8. **$X+XZ$ is equal to:**

[a] Z [b] X
[c] $X+Z$ [d] XZ

9. **Slope of the $\dfrac{x}{a}+\dfrac{y}{b}=1$ is:**

[a] $\dfrac{a}{b}$ [b] $\dfrac{b}{a}$ [c] $-\dfrac{b}{a}$ [d] $-\dfrac{a}{b}$

10. **Equation of the line in slope intercept form is:**

[a] $\dfrac{x}{y}+\dfrac{y}{b}=1$ [b] $y=mx+c$
[c] $y-y_1=m(x-x_1)$
[d] None of these

11. **When two lines are perpendicular:**

[a] $m_1=m_2$ [b] $m_1m_2=-1$
[c] $m_1=-m_2$ [d] None of these

12. **Slope of the line through (x_1, y_1) and (x_2, y_2):**

[a] $\dfrac{x_1+x_2}{y_1+y_2}$ [b] $\dfrac{y_2+y_1}{x_2+y_1}$
[c] $\dfrac{y_2-y_1}{x_2-x_1}$ [d] None of these

13. **$y-y_1=m(x-x_1)$ is the equation of straight line is:**

[a] Slope intercept form
[b] Intercept form
[c] Point slope form
[d] None of these

14. ~~**General form of the equation of circle is:**~~

[a] $x^2+y^2+2gx+2fy+c=0$
[b] $(x-h)^2+(y-k)^2=r^2$
[c] $x^2+y^2+x+y+1=0$
[d] None of these

15. **Radius of the circle $x^2+y^2+2gx+2fy+c=0$ is:**

[a] c [b] c^2
[c] $\sqrt{g^2+f^2-c}$ [d] None of these

Answer Key

1	c	2	a	3	a	4	c	5	b
6	a	7	b	8	a	9	b	10	c
11	b	12	c	13	c	14	a	15	a

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DAE / IA - 2019

MATH - 123 APPLIED MATHEMATICS - I

PAPER 'B' PART - B (SUBJECTIVE)

Time : 2 : 30 Hrs Marks : 60

Section - I

Q.1. Write short answers to any Eighteen (18) questions.

1. Simplify the complex number $\frac{-9+4i}{8-3i}$

Sol. $\frac{-9+4i}{8-3i} = \frac{-9+4i}{8-3i} \times \frac{8+3i}{8+3i}$

$= \frac{-72-27i+32i+12i^2}{(8)^2-(3i)^2}$

$= \frac{-72+5i-12}{64+9}$

$= \frac{-84+5i}{73} = \boxed{-\frac{84}{73}+\frac{5}{73}i}$

2. Find the values of x & y from the equation:

$(2x-y-1)-i(x-3y)=(y-x)-i(2-2y)$

Sol. $(2x-y-1)-i(x-3y)=(y-x)-i(2-2y)$

Comparing real & imaginary parts:

$2x-y-1=y-x$	$x-3y=2-2y$
$2x-y-y+x=1$	$x-3y+2y=2$
$3x-2y=1 \rightarrow (i)$	$x-y=2 \rightarrow (ii)$

Multiply eq. (ii) by 2 & subtracting eq. (i)

$2x-2y=4$

$-3x \mp 2y = -1$

$-x \quad = 3 \Rightarrow \boxed{x=-3}$

Put $x=-3$ in eq. (i), we have:

$-3-y=2$

$-y=2+3$

$-y=5 \Rightarrow \boxed{y=-5}$

3. If $Z=2+3i$, prove that $Z\overline{Z}=13$

Sol. As, $Z=2-3i$ then $\overline{Z}=2-3i$

$\text{L.H.S.} = Z\overline{Z} = (2+3i)(2-3i)$

$=(2)^2-(3i)^2$

$=4+9=13=\text{R.H.S.}$ **Proved.**

4. Factorize $36a^2+100b^2$

Sol. $36a^2+100b^2$

$=36a^2-100b^2i^2$

$=(6a)^2-(10bi)^2$

$=\boxed{(6a-10bi)(6a+10bi)}$

5. ~~**Express When $|z|=2, \arg z=\frac{\pi}{3}$ in the form $x+yi$.**~~

Sol. $z = r \text{ cis } \theta = 2\text{cis}\left(\frac{\pi}{3}\right) = 2 \text{ cis } 60°$

$z=2[\cos(60°)+i\sin(60°)]$

$z=2\left[\frac{1}{2}+i\frac{\sqrt{3}}{2}\right]=2\left[\frac{1+\sqrt{3}i}{2}\right]$

$\boxed{z=1+\sqrt{3}i}$

6. What is partial fractions.

Sol. The process, which convert a single rational fraction, into the sum of two or more single rational fractions is called partial fractions.

7. Resolve $\frac{7x+25}{(x+3)(x+4)}$ into partial fractions.

Sol. $\frac{7x+25}{(x+3)(x+4)} = \frac{A}{x+3}+\frac{B}{x+4} \rightarrow (i)$

$7x+25 = A(x+4)+B(x+3) \rightarrow (ii)$

Put $x=-3$ in eq.(ii)

$$7(-3)+25=A(-3+4)+B(-3+3)$$
$$-21+25=A(1)+B(0)$$
$$4=A+0 \Rightarrow \boxed{A=4}$$

Put $x=-4$ in eq.(ii)

$$7(-4)+25=A(-4+4)+B(-4+3)$$
$$-28+25=A(0)+B(-1)$$
$$-3=0-B \Rightarrow \boxed{B=3}$$

Put values of A, & B in eq. (i),

we get: $\boxed{\dfrac{4}{x+3}+\dfrac{3}{x+4}}$

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8. Write an identity equation of $\dfrac{6x^3+5x^2-7}{3x^2-2x-1}$

Sol. $\dfrac{6x^3+5x^2-7}{3x^2-2x-1}$ {Improper Fraction}

$$\begin{array}{r|l} & 2x+3 \\ 3x^2-2x-1 & 6x^3+5x^2-7 \\ & \underline{-6x^3 \mp 4x^2 \mp 2x} \\ & 9x^2+2x-7 \\ & \underline{-9x^2 \mp 6x \mp 3} \\ & 8x-4 \end{array}$$

$3x^2-2x-1$
$=3x^2-3x+x-1$
$=3x(x-1)+1(x-1)$
$=(x-1)(3x+1)$

$$=2x+3+\frac{8x-4}{3x^2-2x-1}$$
$$=2x+3+\frac{8x-4}{(x-1)(3x+1)}$$
$$=\boxed{2x+3+\frac{A}{x-1}+\frac{B}{3x+1}}$$

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9. From of partial fraction of $\dfrac{1}{(x^2+1)(x-4)^2}$ is____________.

Sol. $\dfrac{1}{(x^2+1)(x-4)^2}=\boxed{\dfrac{Ax+B}{(x^2+1)}+\dfrac{C}{(x-4)}+\dfrac{D}{(x-4)^2}}$

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10. Define 'Decimal number'.

Sol. The Decimal number system is a number system of base equal to 10.

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11. Convert $(110011.11)_2$ to decimal numbers.

Sol. $(110011.11)_2$

$$=1\times2^5+1\times2^4+0\times2^3+0\times2^2$$
$$+1\times2^1+1\times2^0++1\times2^{-1}+1\times2^{-2}$$
$$=32+16+0+0+2+1+\frac{1}{2}+\frac{1}{4}$$
$$=51+0.5+0.25=\boxed{51.75}$$

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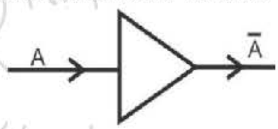
12. Define NOT Gates and draw logic circuit diagram.

Sol. The NOT gate is an electronic circuit that produces an inverted version of the input at its output.



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13. Prove $XY+YZ+\overline{Y}Z=XY+Z$ by Boolean algebra rules:

Sol. $L.H.S.=XY+YZ+\overline{Y}Z$

$$=XY+Z(Y+\overline{Y})$$
$$=XY+Z(1) \quad \because \; Y+\overline{Y}=1$$
$$=XY+Z=R.H.S.$$ **Proved.**

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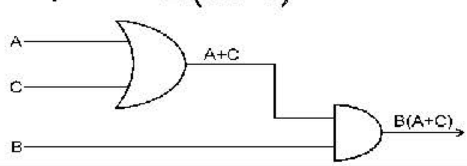
14. Construct a logic Diagram for expression $B(A+C)$.

Sol.



15. **Find the distance between the points $(-3\ ,1)$ and $(3\ ,-2)$.**

Sol. Distance between $(-3\ ,1)\ \&\ (3\ ,-2)$.

$$=\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$$

$$=\sqrt{(-3-3)^2+(1-(-2))^2}$$

$$=\sqrt{(-6)^2+(3)^2}=\sqrt{36+9}$$

$$=\sqrt{45}=\sqrt{9\times5}=\boxed{3\sqrt{5}}$$

16. **Show that the points A(-1, -1), B(4, 1) and C(12, 4) lies on a straight line.**

Sol.
$$\begin{vmatrix}x_1 & y_1 & 1\\ x_2 & y_2 & 1\\ x_3 & y_3 & 1\end{vmatrix}=\begin{vmatrix}-1 & -1 & 1\\ 4 & 1 & 1\\ 12 & 4 & 1\end{vmatrix}$$

$$=-1\begin{vmatrix}1 & 1\\ 4 & 1\end{vmatrix}-(-1)\begin{vmatrix}4 & 1\\ 12 & 1\end{vmatrix}+1\begin{vmatrix}4 & 1\\ 12 & 4\end{vmatrix}$$

$$=-1(1-4)+1(4-12)+1(16-12)$$

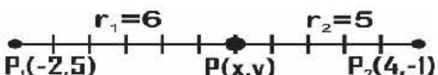
$$=-1(-3)+1(-8)+1(4)$$

$$=3-8+4=-1\neq0$$

Hence given points are **not collinear**.

17. ~~**Find the coordinates of the point $P(x,y)$ which divide internally the segment through $P_1(-2,5)$ and $P_2(4,-1)$ of the ratio of $\frac{r_1}{r_2}=\frac{6}{5}$.**~~

Sol. Here : $r_1=6,\ r_2=5,\ (x_1,y_1)=(-2,5)$
$\&\ \ (x_2,y_2)=(4,-1)$



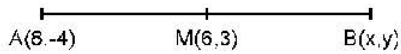
$$P(x\ ,\ y)=\left(\frac{r_1x_2+r_2x_1}{r_1+r_2},\frac{r_1y_2+r_2y_1}{r_1+r_2}\right)$$

$$=\left(\frac{6(4)+5(-2)}{6+5},\frac{6(-1)+5(5)}{6+5}\right)$$

$$=\left(\frac{24-10}{11},\frac{-6+25}{11}\right)=\boxed{\left(\frac{14}{11},\frac{19}{11}\right)}$$

18. **If the mid-point of a segment is (6, 3) and one end point is (8, – 4), what are the coordinates of the other end point.**

Sol. Let B(x, y) be require end point.



As, Mid – point $=(6,\ 3)$

$$\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)=(6,\ 3)$$

$$\left(\frac{8+x}{2},\frac{-4+y}{2}\right)=(6,\ 3)$$

Comparing both order pairs, we have :

$$\frac{8+x}{2}=6\ \text{and}\ \frac{-4+y}{2}=3$$

$8+x=12$ | $-4+y=6$
$x=12-8=4$ | $y=6+4=10$

Hence other end point $=\boxed{(4,10)}$

19. **Find the slope of a line which is perpendicular to the line joining $P_1(2,\ 4)\ ,\ P_2(-2,\ 1)$.**

Sol. Slope of line joining given point :

$$=m_1=\frac{y_2-y_1}{x_2-x_1}=\frac{1-4}{-2-2}=\frac{-3}{-4}=\frac{3}{4}$$

Slope of require line $=m_2=?$

As, both lines are perpendicular,

$$\text{So,}\ \ m_1m_2=-1\Rightarrow\left(\frac{3}{4}\right)m_2=-1$$

$$\Rightarrow m_2=-1\times\frac{4}{3}\ \ \Rightarrow\boxed{m_2=-\frac{4}{3}}$$

20. ~~Find the equation of a line through the points $(-1, 2)$ and $(3, 4)$.~~

Sol. Slope $= \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{4-2}{3-(-1)} = \dfrac{2}{4} = \dfrac{1}{2}$

Equation of line in point - slope form:

$$y - y_1 = m(x - x_1)$$

$$y - 2 = \frac{1}{2}(x - (-1))$$

$$2(y-2) = 1(x+1)$$

$$2y - 4 = x + 1 \Rightarrow 2y - 4 - x - 1 = 0$$

$$-x + 2y - 5 = 0 \Rightarrow \boxed{x - 2y + 5 = 0}$$

21. ~~Find the equation of line having x-intercept -2 and y intercept 3.~~

Sol. Let, x − intercept = $a = -2$
& y − intercept = $b = 3$

Equation of line in intercept form: $\dfrac{x}{a} + \dfrac{y}{b} = 1 \Rightarrow \dfrac{x}{-2} + \dfrac{y}{3} = 1$

$$\frac{-3x + 2y}{6} = 1 \Rightarrow -3x + 2y = 6$$

$$-3x + 2y - 6 = 0 \Rightarrow \boxed{3x - 2y + 6 = 0}$$

22. **Reduce the equation $3x + 4y - 2 = 0$ into intercept form.**

Sol. $3x + 4y - 2 = 0$

$3x + 4y = 2$

Dividing both sides by 2, we have:

$$\frac{3x}{2} + \frac{4y}{2} = \frac{2}{2}$$

$$\frac{x}{2/3} + \frac{y}{2/4} = 1 \Rightarrow \boxed{\frac{x}{2/3} + \frac{y}{1/2} = 1}$$

23. ~~Find the distance from the point $(-2, 1)$ to the line $3x + 4y - 12 = 0$~~

Sol. Distance between point & line

$$D = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

$$D = \frac{|3(-2) + 4(1) - 12|}{\sqrt{(3)^2 + (4)^2}}$$

$$D = \frac{|-6 + 4 - 12|}{\sqrt{9 + 16}} = \frac{|-14|}{\sqrt{25}} = \boxed{\frac{14}{5}}$$

24. ~~Write the general form of the circle, also represent the center and radius of this form.~~

Sol. $x^2 + y^2 + 2gx + 2fy + c = 0$

Center $= \boxed{(-g, -f)}$ & Radius $= \boxed{\sqrt{g^2 + f^2 - c}}$

25. **Find the equation of the circle with center $(-\sqrt{2}, -2)$ & $r = \sqrt{6}$.**

Sol. Center $= (h, k) = (-\sqrt{2}, -2)$

& Radius $= r = \sqrt{6}$

Standard form of eq. of circle:

$$(x-h)^2 + (y-k)^2 = r^2$$

Put, $h = -\sqrt{2}$, $k = -2$ & $r = \sqrt{6}$

$$(x + \sqrt{2})^2 + (y+2)^2 = (\sqrt{6})^2$$

$$(x)^2 + 2(x)(\sqrt{2}) + (\sqrt{2})^2 + (y)^2 + 2(y)(2) + (2)^2 = 6$$

$$x^2 + 2\sqrt{2}x + 2 + y^2 + 4y + 4 - 6 = 0$$

$$\boxed{x^2 + y^2 + 2\sqrt{2}x + 4y = 0}$$

26. **Reduce the equation into standard form: $x^2 + y^2 - 10y = 0$**

Sol. $x^2 + y^2 - 10y = 0$

Adding the square of one half of the coefficient of y i.e., $(5)^2$ on both sides:

$$x^2+y^2-10y+(5)^2=0+(5)^2$$

$$(x)^2+(y-5)^2=(5)^2$$

$$\boxed{(x-0)^2+(y-5)^2=(5)^2}$$

27. **What type of circle is represented by $x^2+y^2+2x-4y+8=0$**

Sol. $x^2+y^2+2x-4y+8=0$

Comparing this equation with general form of equation of circle:

$$x^2+y^2+2gx+2fy+c=0$$

$2g=2$	$2f=-4$	
$g=\frac{2}{2}$	$f=-\frac{4}{2}$	$c=8$
$g=1$	$f=-2$	

$$\text{Radius}=r=\sqrt{g^2+f^2-c}$$

$$r=\sqrt{(-1)^2+(2)^2-8}$$

$$r=\sqrt{1+4-8}=\sqrt{-3}=\boxed{\sqrt{3}i}$$

So, it is an **Imaginary circle**.

Section - II

Note: Attemp any three (3) questions $3\times 8=24$

Q.2.(a) ~~**Perform the indicated operation in $(1+i)(1-\sqrt{3}i)$ and give the result in polar form.**~~

Sol. See Q.2(i) of Ex# 8.3 (Page # 331)

(b) **Find the value of 'x' and 'y' in $(2x-3y)+i(x-y)6=2-i(2x-y+3)$**

Sol. See Q.3(v) of Ex# 8.1 (Page # 303)

Q.3.(a) **Resolve $\frac{6x+27}{4x^3-9x}$ into partial fractions.**

Sol. See Q.8 of Ex# 9.1 (Page # 353)

(b) **Resolve $\frac{4x^3}{(x+1)^2(x^2-1)}$ into partial fractions.**

Sol. See Q.3 of Ex# 9.2 (Page # 360)

Q.4.(a) **Convert $(10111.101)_2$ binary number to its octal equivalent.**

Sol. See Q.6[d] of Ex # 10 (Page # 410)

(b) **Minimize the expression**

$$X=\left(A\overline{B}C+\overline{ABC}\right)C$$

Sol. See Q.3[d] of Ex # 11 (Page # 430)

Q.5.(a) ~~**Find the ratio in which the line joining (−2, 2) and (4, 5) is cut by the axis of y.**~~

Sol. See example 09 of Chapter. # 12

(b) ~~**Find the distance between the parallel lines $3x-4y+11=0$ and $3x-4y-9=0$.**~~

Sol. See Q.7 of Ex # 12.6 (Page # 505)

Q.6. **(a)** **Find the equation of a circle having $(-3, 7)$ and $(2, -1)$ as the end point of its diameter.**

Sol. See Q.9[b] of Ex # 13 (Page # 542)

(b) **Find the equation of the circle through $(2, -1)$ and $(-2, 0)$ with center on $2x-y-1=0$.**

Sol. See Q.3[d] of Ex # 13 (Page # 528)

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