DAE / IA - 2019

MATH - 123 APPLIED MATHEMATICS - I

PAPER 'A' PART - A(OBJECTIVE)

Time : 30 Minutes Marks : 15

Q.1: *Encircle the correct answer.*

1. The factors of $x^2 - 7x + 12 = 0$ are:
[a] (x-4)(x+3) **[b]** (x-4)(x-3)
[c] (x+4)(x+3) **[d]** (x+4)(x-3)

2. If the discriminant $b^2 - 4ac$ is negative, the roots are:
[a] Real [b] Rational
[c] Irrational [d] Imaginary

3. ~~If 2 and -5 are the roots of the equation, then the equation is:~~
[a] $x^2 + 3x + 10 = 0$
[b] $x^2 - 3x - 10 = 0$
[c] $x^2 + 3x - 10 = 0$
[d] $2x^2 - 5x + 1 = 0$

4. Third term of $(x+y)^4$ is:
[a] $4x^2y^2$ **[b]** $4x^3y$
[c] $6x^2y^2$ **[d]** $6x^3y$

5. $\begin{pmatrix}3\\0\end{pmatrix}$ will have the value:
[a] 0 **[b]** 1 **[c]** 2 **[d]** 3

6. In the expansion of $(1+x)^n$ the co-efficient of 3rd term is:
[a] $\begin{bmatrix}n\\0\end{bmatrix}$ **[b]** $\begin{bmatrix}n\\1\end{bmatrix}$ **[c]** $\begin{bmatrix}n\\2\end{bmatrix}$ **[d]** $\begin{bmatrix}n\\3\end{bmatrix}$

7. ~~One degree is equal to:~~
[a] πrad **[b]** $\frac{\pi}{180}$rad
[c] $\frac{180}{\pi}$rad **[d]** $\frac{\pi}{360}$rad

8. ~~An angle subtends at the center of a circle by an arc equal to the radius of the circle is called:~~
[a] Right angle **[b]** Degree
[c] Radian **[d]** Acute angle

9. $\sin(90° - \theta)$ is equal to:
[a] $-\sin\theta$ **[b]** $\sin\theta$
[c] $-\cos\theta$ **[d]** $\cos\theta$

10. $\cos A - \cos B$ is equal to:
[a] $2\cos\frac{A+B}{2}\cos\frac{A-B}{2}$
[b] $-2\sin\frac{A+B}{2}\sin\frac{A-B}{2}$
[c] $2\sin\frac{A+B}{2}\cos\frac{A-B}{2}$
[d] $2\cos\frac{A+B}{2}\sin\frac{A-B}{2}$

11. In a triangle ABC $\angle A = 70º$, $\angle B = 60º$ then $\angle C$ is:
[a] 30° **[b]** 40° **[c]** 50° **[d]** 60°

12. ~~If C = 90º, b = 1, c = $\sqrt{2}$, then angle A is:~~
[a] 15° **[b]** 30° **[c]** 45° **[d]** 60°

13. $\vec{a}.\vec{b} = 0$ implies that $\vec{a}$ and $\vec{b}$ are:
[a] Perpendicular **[b]** Parallel
[c] Non-parallel **[d]** Oblique

14. $|\vec{a} \times \vec{b}|$ is area of the figure called:
[a] Triangle [b] Rectangle
[c] Parallelogram [d] Sector

15. ~~The trigonometric form of~~ $Z = a + jb$ is:
[a] $|Z|(\cos\theta - j\sin\theta)$
[b] $|Z|(\cos\theta + j\sin\theta)$
[c] $(\cos\theta + j\sin\theta)$
[d] $(\cos\theta - j\sin\theta)$

Answer Key

1	b	2	d	3	c	4	c	5	b
6	c	7	b	8	c	9	d	10	b
11	c	12	c	13	a	14	c	15	b

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DAE / IA - 2019

MATH-123 APPLIED MATHEMATICS-I

PAPER 'A' PART-B (SUBJECTIVE)

Time: 2:30 Hrs Marks: 60

Section - I

Q.1. Write short answers to any Eighteen (18) questions.

1. Solve the quadratic equation $x(x+7)=(2x-1)(x+4)$ by factorization.

Sol. $x(x+7)=(2x-1)(x+4)$

$x^2+7x=2x^2+8x-x-4$

$x^2+7x-2x^2-8x+x+4=0$

$-x^2+4=0$

$x^2-4=0$

$(x)^2-(2)^2=0$

$(x-2)(x+2)=0$

Either	OR
$x-2=0$	$x+2=0$
$x=2$	$x=-2$

$S.S.=\{-2,2\}$

2. Solve the equation $x^2-6x+8=0$ by completing the square.

Sol. $x^2-6x+8=0$

$x^2-6x=-8$

Adding the square of one half of the coefficient of x i.e., $(3)^2$ on both sides

$x^2-6x+(3)^2=-8+(3)^2$

$(x-3)^2=-8+9$

$(x-3)^2=1$

Taking square root on both sides

$\sqrt{(x-3)^2}=\pm\sqrt{1}$

$x-3=\pm1$

$x=3\pm1$

Either	OR
$x=3+1$	$x=3-1$
$x=4$	$x=2$

$S.S.=\{2,4\}$

3. Discuss the nature of the roots of the equation $x^2+x+1=0$

Sol. Here: $a=1,\ b=1,\ c=1$

$\text{Disc.}=b^2-4ac$

$=(1)^2-4(1)(1)=1-4=-3$

∴ The roots are **Imaginary.**

4. For what value of k the roots of the equation $kx^2+4x+3=0$ are equal.

Sol. Here: $a=k,\ b=4,\ c=3$

As Roots are equal, So

$\text{Disc}=b^2-4ac=0$

$\Rightarrow (4)^2-4(k)(3)=0$

$\Rightarrow 16-12k=0$

$\Rightarrow -12k=-16$

$\Rightarrow k=\dfrac{-16}{-12} \Rightarrow \boxed{k=\dfrac{4}{3}}$

5. ~~Find the sum and product of the roots of the equation~~ $9x^2+6x+1=0$

Sol. Here: $a=9,\ b=6,\ c=1$

Sum of Roots	Product of Roots
$S=-\dfrac{b}{a}$	$P=\dfrac{c}{a}$
$S=-\dfrac{6}{9}=\boxed{-\dfrac{2}{3}}$	$P=\boxed{\dfrac{1}{9}}$

6. **Expand $\left(\frac{x}{2}-\frac{2}{y}\right)^4$ by Binomial theorem.**

Sol. $\left(\frac{x}{2}-\frac{2}{y}\right)^4$

$$=\binom{4}{0}\left(\frac{x}{2}\right)^4\left(\frac{2}{y}\right)^0-\binom{4}{1}\left(\frac{x}{2}\right)^3\left(\frac{2}{y}\right)^1+\binom{4}{2}\left(\frac{x}{2}\right)^2\left(\frac{2}{y}\right)^2$$
$$-\binom{4}{3}\left(\frac{x}{2}\right)^1\left(\frac{2}{y}\right)^3+\binom{4}{4}\left(\frac{x}{2}\right)^0\left(\frac{2}{y}\right)^4$$
$$=(1)\left(\frac{x^4}{16}\right)(1)-4\left(\frac{x^3}{8}\right)\left(\frac{2}{y}\right)+6\left(\frac{x^2}{4}\right)\left(\frac{4}{y^2}\right)$$
$$-4\left(\frac{x}{2}\right)\left(\frac{8}{y^3}\right)+(1)(1)\left(\frac{16}{y^4}\right)$$
$$=\boxed{\frac{x^4}{16}-\frac{x^3}{y}+6\frac{x^2}{y^2}-16\frac{x}{y^3}+\frac{16}{y^4}}$$

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7. **Calculate $(1.04)^5$ by binomial theorem up to two decimal places.**

Sol. $(1.04)^5=(1+0.04)^5$

$$=\binom{5}{0}(1)^5(0.04)^0+\binom{5}{1}(1)^4(0.04)^1+\binom{5}{2}(1)^3(0.04)^2+\dots$$
$$=(1)(1)(1)+5(1)(0.04)+10(1)(0.0016)+\dots$$
$$=1+0.2+0.016+\dots=1.2160=\boxed{1.22}$$

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8. **Find the 8th term in the expansion of $\left(2x^2-\frac{1}{x^2}\right)^{12}$**

Sol. Here : $a=2x^2,\ b=-\frac{1}{x^2},\ n=12\ \&\ r=7$

$$T_{r+1}=\binom{n}{r}a^{n-r}b^r$$
$$T_{7+1}=\binom{12}{7}\left(2x^2\right)^{12-7}\left(-\frac{1}{x^2}\right)^7$$
$$T_8=(792)\left(2x^2\right)^5\left(-\frac{1}{x^{14}}\right)$$
$$T_8=(792)\left(32x^{10}\right)\left(-\frac{1}{x^{14}}\right)$$
$$\boxed{T_8=-\frac{25344}{x^4}}$$

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9. ~~**Expand $(1+2x)^{-2}$ to three terms.**~~

Sol. $(1+2x)^{-2}$

$$=1+(-2)(2x)+\frac{(-2)(-2-1)}{2!}(2x)^2+\dots$$
$$=1-4x+\frac{(-2)(-3)}{2}\left(4x^2\right)+\dots$$
$$=\boxed{1-4x+12x^2+\dots}$$

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9. **Which will be the middle term/terms in the expansion of $\left(x+\frac{3}{x}\right)^{15}$**

Sol. As $n = 15$ (Odd), so

$$\text{Middle terms}=\left(\frac{n+1}{2}\right)^{th}+\left(\frac{n+3}{2}\right)^{th}\ \text{terms.}$$
$$\text{Middle terms}=\left(\frac{15+1}{2}\right)^{th}+\left(\frac{15+3}{2}\right)^{th}\ \text{terms.}$$
$$\text{Middle terms}=8^{th}+9^{th}\ \text{terms.}$$

$\boxed{\text{Hence } T_8\ \&\ T_9 \text{ are two middle terms.}}$

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11. ~~**Convert $42^\circ\,36'\,12''$ into radians measure.**~~

Sol. $42^\circ36'12''=\left(42+\frac{36}{60}+\frac{12}{3600}\right)^\circ$

$$=(42+0.6+0.0033)^\circ$$
$$=42.6033^\circ$$
$$=42.6033\times\frac{\pi}{180}=\boxed{0.74\text{rad}}$$

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13. **Find x, if**

$\tan^2 45^\circ-\cos^2 60^\circ=x\sin 45^\circ\cos 45^\circ\tan 60^\circ$

Sol. $\tan^2 45^\circ-\cos^2 60^\circ=x\sin 45^\circ\cos 45^\circ\tan 60^\circ$

$$(1)^2 - \left(\frac{1}{2}\right)^2 = x\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)\left(\sqrt{3}\right)$$

$$1 - \frac{1}{4} = x\left(\frac{2\sqrt{3}}{4}\right)$$

$$\frac{4-1}{4} \times \frac{4}{2\sqrt{3}} = x$$

$$\frac{3}{2\sqrt{3}} = x \Rightarrow \boxed{x = \frac{\sqrt{3}}{2}}$$

13. Prove that:

$$\tan^2 30° + \tan^2 45° + \tan^2 60° = \frac{13}{3}$$

Sol. $\text{L.H.S.} = \tan^2 30° + \tan^2 45° + \tan^2 60°$

$$= \left(\frac{1}{\sqrt{3}}\right)^2 + (1)^2 + \left(\sqrt{3}\right)^2 = \frac{1}{3} + 1 + 3$$

$$= \frac{1+3+9}{3} = \frac{13}{3} = \text{R.H.S.}$$ **Proved.**

14. Show that:

$$\cot^4\theta + \cot^2\theta = \operatorname{cosec}^4\theta - \operatorname{cosec}^2\theta$$

Sol. $\text{L.H.S.} = \cot^4\theta + \cot^2\theta$

$$= \cot^2\theta\left(\cot^2\theta + 1\right)$$

$$= \cot^2\theta\left(\operatorname{cosec}^2\theta\right) \because \cot^2\theta + 1 = \operatorname{cosec}^2\theta$$

$$= \left(\operatorname{cosec}^2\theta - 1\right)\left(\operatorname{cosec}^2\theta\right)$$

$$= \operatorname{cosec}^4\theta - \operatorname{cosec}^2\theta = \text{R.H.S.}$$ **Proved.**

15. Prove that: $\cos(-\beta) = \cos\beta$

Sol. As, $\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

Put $\alpha = 0$, we have:

$$\cos(0-\beta) = \cos(0)\cos\beta + \sin(0)\sin\beta$$

$$\cos(-\beta) = 1.\cos\beta + 0.\sin\beta$$

$$\cos(-\beta) = \cos\beta + 0$$

$$\boxed{\cos(-\beta) = \cos\beta}$$ **Proved.**

16. Show that

$$\sin(\alpha+\beta) + \sin(\alpha-\beta) = 2\sin\alpha\cos\beta$$

Sol. $\text{L.H.S.} = \sin(\alpha+\beta) + \sin(\alpha-\beta)$

$$= \sin\alpha\cos\beta + \cancel{\cos\alpha\sin\beta} + \sin\alpha\cos\beta - \cancel{\cos\alpha\sin\beta}$$

$$= 2\sin\alpha\cos\beta = \text{R.H.S.}$$ **Proved.**

17. ~~If $\sin\theta = \frac{4}{5}$ and the terminal side of θ lies in 1st quadrant, find $\cos\frac{\theta}{2}$.~~

Sol. As, $\cos\theta = \sqrt{1-\sin^2\theta}$

$$= \sqrt{1-\left(\frac{4}{5}\right)^2} = \sqrt{\frac{25-16}{25}} = \sqrt{\frac{9}{25}} = 3/5$$

$$\cos\frac{\theta}{2} = \sqrt{\frac{1+\cos\theta}{2}} = \sqrt{\frac{1+\frac{3}{5}}{2}} = \sqrt{\frac{\frac{5+3}{5}}{2}}$$

$$\cos\frac{\theta}{2} = \sqrt{\frac{8}{10}} = \sqrt{\frac{4}{5}} = \boxed{\frac{2}{\sqrt{5}}}$$

18. ~~Express cosθ – cos4θ as product.~~

Sol. $\cos\theta - \sin 4\theta$

$$= -2\sin\left(\frac{\theta+4\theta}{2}\right)\sin\left(\frac{\theta-4\theta}{2}\right)$$

$$= -2\sin\left(\frac{5\theta}{2}\right)\sin\left(-\frac{3\theta}{2}\right)$$

$$= \boxed{2\sin\left(\frac{5\theta}{2}\right)\sin\left(\frac{3\theta}{2}\right)}$$

19. ~~In right triangle ABC, γ = 90°, a = 5, c = 13 then find value of angle α.~~

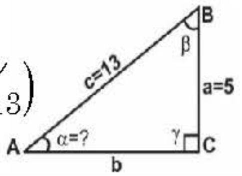
Sol. We know that, from figure:

$$\sin\alpha = \frac{a}{c}$$

$$\sin\alpha = \frac{5}{13}$$

$$\alpha = \sin^{-1}\left(\frac{5}{13}\right)$$

$$\boxed{\alpha = 22^\circ 37'}$$



20. **The sides of a triangle are 16, 20 and 33 meters respectively. Find its greatest angle.**

Sol. Let a = 16, b = 20, c = 33

As side 'c' is greatest so, we will find angle 'γ'.

Using law of cosines:

$$\cos\gamma = \frac{a^2 + b^2 - c^2}{2ab}$$

$$\cos\gamma = \frac{(16)^2 + (20)^2 - (33)^2}{2(16)(20)}$$

$$\cos\gamma = -0.6765$$

$$\gamma = \cos^{-1}(-0.6765)$$

$$\boxed{\gamma = 132^\circ 34'}$$

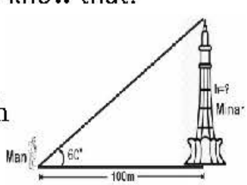
21. ~~**A minaret stands on a horizontal ground. A man on the ground 100m from the minaret, the angle of elevation of the top of the minaret to be 60º. Find its height.**~~

Sol. From figure, we know that:

$$\tan 60^\circ = \frac{h}{100}$$

$$100\tan 60^\circ = h$$

$$\boxed{h = 173.20\text{m}}$$



22. **In any triangle ABC if A = 16, b = 17, γ = 25º, Find c.**

Sol. By using law of cosines:

$$c^2 = a^2 + b^2 - 2ab\cos\gamma$$

$$c^2 = (16)^2 + (17)^2 - 2(16)(17)\cos 25^\circ$$

$$c^2 = 256 + 289 - 493.03$$

$$c^2 = 51.97$$

$$\sqrt{c^2} = \sqrt{51.97} \Rightarrow \boxed{c = 7.2}$$

23. **What are parallel vectors?**

Sol. Two vectors $\vec{a}$ and $\vec{b}$ are parallel if there exist a non-zero $k \varepsilon R$, such that $\vec{a} = k\vec{b}$.

24. **Given the vectors: $\vec{a} = 3i - 2j + k$, $\vec{b} = 2i - 4j - 3k$, $\vec{c} = -i + 2j + 2k$ Find $\vec{a} + \vec{b} + \vec{c}$**

Sol. $\vec{a} + \vec{b} + \vec{c}$

$$= 3i - 2j + k + 2i - 4j - 3k + -i + 2j + 2k$$

$$= \boxed{4i - 4j + 0k}$$

25. **Find the area of parallelogram with adjacent sides,**

$\vec{a} = 7i - j + k$ & $\vec{b} = 2j - 3k$

Sol.
$$\vec{a} \times \vec{b} = \begin{vmatrix} i & j & k \\ 7 & -1 & 1 \\ 0 & 2 & -3 \end{vmatrix}$$

$$= i\begin{vmatrix} -1 & 1 \\ 2 & -3 \end{vmatrix} - j\begin{vmatrix} 7 & 1 \\ 0 & -3 \end{vmatrix} + k\begin{vmatrix} 7 & -1 \\ 0 & 2 \end{vmatrix}$$

$$= i(3-2) - j(-21-0) + k(14+0)$$

$$= i + 21j + 14k$$

$$|\vec{a} \times \vec{b}| = \sqrt{(1)^2 + (21)^2 + (14)^2}$$

$$|\vec{a} \times \vec{b}| = \sqrt{1 + 441 + 196} = \sqrt{638}$$

Area of parallelogram

$$= |\vec{a} \times \vec{b}| = \boxed{\sqrt{638}\ \text{sq. unit}}$$

26. ~~Write the phasor (vector)~~ $z = a + jb$ ~~in trigonometric and exponential forms.~~

Sol. Trigonometric OR Polar Form

$$\boxed{z = r(\cos\theta + j\sin\theta) = r\angle\theta}$$

Exponential form:

$$\boxed{z = re^{j\theta}}$$

27. ~~Given that~~ $Z_1 = 4\angle 60^\circ$, $Z_2 = 2\angle 30^\circ$ ~~find~~ $\frac{Z_1}{Z_2}$.

Sol. $\frac{Z_1}{Z_2}$

$$= \frac{4\angle 60^\circ}{2\angle 30^\circ}$$

$$= 2\angle(60^\circ - 30^\circ)$$

$$= \boxed{2\angle 30^\circ}$$

Section - II

Note: Attemp any three (3) questions $3 \times 8 = 24$

Q.2.(a) Solve the equation

$$x^2 - 3\left(x + \frac{25}{4}\right) = 9x - \frac{25}{2}$$

By using quadratic formula.

Sol. See Q.3(iv) of Ex # 1.1 (Page # 21)

(b) Show that the roots of the equation

$x^2 - 2\left(m + \frac{1}{m}\right)x + 3 = 0$ are real.

Sol. See Q.5(i) of Ex # 1.2 (Page # 35)

Q.3.(a) Find the 5th term in the expansion of $\left(2x^2 - \frac{3}{x}\right)^{10}$

Sol. See Q.5(i) of Ex # 2.1 (Page # 83)

(b) ~~Expand~~ $(4 + x)^{1/2}$ ~~up to four terms.~~

Sol. See Q.1(v) of Ex # 2.2 (Page # 100)

Q.4.(a) Prove that

$$\sqrt{\frac{1 - \sin\theta}{1 + \sin\theta}} = \sec\theta - \tan\theta$$

Sol. See Q.9 of Ex # 3.3 (Page # 134)

(b) Prove that:

$$\frac{\tan\theta}{1 - \cot\theta} + \frac{\cot\theta}{1 - \tan\theta} = \sec\theta\,\mathrm{cosec}\,\theta + 1$$

Sol. See Q.12 of Ex # 3.3 (Page # 135)

Q.5.(a) ~~Prove that:~~

$$\sin 20^\circ \sin 40^\circ \sin 60^\circ \sin 80^\circ = \frac{3}{16}$$

Sol. See Q.14 of Ex # 4.3 (Page # 192)

(b) In any triangle ABC if $a = 211.3$, $\beta = 48^\circ 16'$, $\gamma = 71^\circ 38'$, find b.

Sol. See Q.6 of Ex # 5.3 (Page # 224)

Q.6.(a) If $\vec{a} = 3i - j - 4k$,

$\vec{b} = -2i + 4j - 3k$ and $\vec{c} = i + 2j - k$

Find unit vector parallel to $3\vec{a} - 2\vec{b} + 4\vec{c}$

Sol. See Q.1 of Ex # 6.1 (Page # 247)

(b) Find the cosine of the angle between the vectors: $\bar{a} = 2i - 8j + 3k$, $\bar{b} = 4j + 3k$

Sol. See Q.3(i) of Ex # 6.2 (Page # 256)

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