DAE / IA - 2018

MATH-113 APPLIED MATHEMATICS-I

PAPER 'B' PART - A (OBJECTIVE)

Time: 30 Minutes Marks: 15

Q.1: *Encircle the correct answer.*

1. Unit vector $\mathbf{i}+\mathbf{j}+\mathbf{k}$ is;

[a] $\mathrm{i}+\mathrm{j}+\mathrm{k}$ [b] $\frac{1}{3}(\mathrm{i}+\mathrm{j}+\mathrm{k})$

[c] $\frac{1}{\sqrt{3}}(\mathrm{i}+\mathrm{j}+\mathrm{k})$ [d] $\frac{1}{2}(\mathrm{i}+\mathrm{j}+\mathrm{k})$

2. $\vec{a}.\vec{b}$ is a;

[a] Vector quantity
[b] Scalar quantity
[c] Unity [d] None of these

3. If $\vec{a}=-\mathbf{i}-\mathbf{j}-\mathbf{k}$, $\vec{b}=2\mathbf{i}+\mathbf{j}$, then, $\vec{a}.\vec{b}$

[a] 3 [b] -3
[c] -2 [d] -1

4. The matrix is $\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$ called:

[a] Identity [b] Scalar
[c] Diagonal [d] Null

5. ~~If all the elements of a row or a column are zero, then value of the determinant is:~~

[a] 1 [b] 2
[c] zero [d] None of these

6. If A and B are symmetric, then AB =

[a] BA [b] $\mathrm{A^tB^t}$
[c] $\mathrm{B^tA^t}$ [d] Both a and b

7. A right triangle has one angle of:

[a] 30º [b] 90º
[c] 60º [d] 45º

8. Area of a square having side 4cm is equal to:

[a] 30 sq.cm [b] 16 sq.cm
[c] 8 sq.cm [d] 20 sq.cm

9. The perimeter of a regular hexagon is 12cm, its area is:

[a] 10.932 sq.cm [b] 10.392cm^2
[c] 10.239sq^2 [d] 10.329cm^2

10. Area of a circle whose radius is 'a' cm is:

[a] πr^2 [b] πa^2
[c] πa [d] $\frac{\pi}{2}a^2$

11. In Simpson's Rule, D stands for:

[a] Sum of 1st + last ordinates
[b] Sum of odd ordinates
[c] Sum of even ordinates
[d] None of these

12. Total surface area of the cube is:

[a] a^2 [b] $3a^2$
[c] $6a^2$ [d] $4a^2$

13. If 20 and 10 are the major and minor axis respectively, then volume of an elliptic cylinder of height 5m

[a] 1000π [b] 250π
[c] 200π [d] 100π

14. If the base of pyramid is square, the pyramid is called:

[a] Square pyramid
[b] Hexagonal pyramid
[c] Triangular pyramid
[d] Rectangular pyramid

15. Volume of cone of radius of base 3 cm and heights 12cm is:

[a] 108πcu.cm [b] 36πcu.cm
[c] 12πcu.cm [d] 54cu.cm

Answer Key

1	c	2	b	3	b	4	d	5	c
6	d	7	b	8	b	9	b	10	b
11	b	12	c	13	b	14	a	15	a

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DAE / IA - 2018

MATH - 113 APPLIED MATHEMATICS - I

PAPER 'B' PART - B (SUBJECTIVE)

Time : 2 : 30 Hrs Marks : 60

Section - I

Q.1. Write short answers to any Eighteen (18) questions.

1. **Find unit vector along the vector $4\mathbf{i} - 3\mathbf{j} - 5\mathbf{k}$**

Sol. Let $\vec{a} = 4i - 3j - 5k$

$|\vec{a}| = \sqrt{(4)^2 + (-3)^2 + (-5)^2}$

$|\vec{a}| = \sqrt{16 + 9 + 25} = \sqrt{50}$

$|\vec{a}| = \sqrt{25 \times 2} = 5\sqrt{2}$

Unit Vector $= \hat{a} = \dfrac{\vec{a}}{|\vec{a}|} = \boxed{\dfrac{4i - 3j - 5k}{5\sqrt{2}}}$

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2. **Given the vectors $\vec{a} = 3\mathbf{i} + \mathbf{j} - \mathbf{k}$ and $\vec{b} = 2\mathbf{i} + \mathbf{j} - \mathbf{k} = 2i + j - k$, find magnitude of $3\vec{a} - \vec{b}$.**

Sol. $3\vec{a} - \vec{b} = 3(3i + j - k) - (2i + j - k)$

$3\vec{a} - \vec{b} = 9i + 3j - 3k - 2i - j + k$

$3\vec{a} - \vec{b} = 7i + 2j - 2k$

$|3\vec{a} - \vec{b}|$

$= \sqrt{(7)^2 + (2)^2 + (-2)^2}$

$= \sqrt{49 + 4 + 4}$

$= \boxed{\sqrt{57}}$

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3. **Define Vector product.**

Sol. The vector product of two vectors $\vec{a}$ & $\vec{b}$ is denoted by $\vec{a} \times \vec{b}$ and is defined as $\vec{a} \times \vec{b} = |\vec{a}||\vec{b}|\sin\theta\hat{n}$.

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4. **Find scalar x and y such that $x(i + 2j) + y(3i + 4j) = 7i + 9j$**

Sol. $x(i + 2j) + y(3i + 4j) = 7i + 9j$

$xi + 2xj + 3yi + 4yj = 7i + 9j$

$(x + 3y)i + (2x + 4y)j = 7i + 9j$

Comparing coefficients of i & j, we have :

$x + 3y = 7 \rightarrow (i) \;|\; 2x + 4y = 9 \rightarrow (ii)$

Multiplying eq.(i) by 2 :

$2x + 6y = 14 \rightarrow (iii)$

Subtracting eq.(iii) & eq.(ii)

$2x + 6y = 14$

$-2x \pm 4y = -9$

$2y = 5$

$\Rightarrow \boxed{y = 5/2}$

Put $y = 5/2$ in eq.(i)

$x + 3\left(\dfrac{5}{2}\right) = 7$

$x = 7 - \dfrac{15}{2}$

$x = \dfrac{14 - 15}{2}$

$x = \boxed{-\dfrac{1}{2}}$

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5. **Find the real numbers x, y and z such that $(x + 4)\hat{i} + (y - 5)\hat{j} + (z - 1)\hat{k} = 0$**

Sol. $(x + 4)\hat{i} + (y - 5)\hat{j} + (z - 1)\hat{k} = 0$

Comparing coefficients of i, j & k, we have :

$x + 4 = 0 \;|\; y - 5 = 0 \;|\; z - 1 = 0$

$\boxed{x = -4} \;|\; \boxed{y = 5} \;|\; \boxed{z = 1}$

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6. **Define Diagonal matrix.**

Sol. A square matrix in which all elements except diagonal elements are zero is called diagonal matrix.

7. Evaluate $\begin{vmatrix} 1 & 2 & -2 \\ -1 & 1 & -3 \\ 2 & 4 & -1 \end{vmatrix}$

Sol. $\begin{vmatrix} 1 & 2 & -2 \\ -1 & 1 & -3 \\ 2 & 4 & -1 \end{vmatrix}$

$= 1\begin{vmatrix} 1 & -3 \\ 4 & -1 \end{vmatrix} - 2\begin{vmatrix} -1 & -3 \\ 2 & -1 \end{vmatrix} + (-2)\begin{vmatrix} -1 & 1 \\ 2 & 4 \end{vmatrix}$

$= 1(-1+12) - 2(1+6) - 2(-4-2)$

$= 11 - 14 + 12 = \boxed{9}$

8. If $\begin{bmatrix} 1 & -1 & 2 \\ 3 & 2 & 5 \\ -1 & 0 & 4 \end{bmatrix}$ & $B = \begin{bmatrix} 2 & 1 & -1 \\ 1 & 3 & 4 \\ -1 & 2 & 1 \end{bmatrix}$

find $A - B$

Sol. $A - B$

$= \begin{bmatrix} 1 & -1 & 2 \\ 3 & 2 & 5 \\ -1 & 0 & 4 \end{bmatrix} - \begin{bmatrix} 2 & 1 & -1 \\ 1 & 3 & 4 \\ -1 & 2 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1-2 & -1-1 & 2+1 \\ 3-1 & 2-3 & 5-4 \\ -1+1 & 0-2 & 4-1 \end{bmatrix}$

$= \boxed{\begin{bmatrix} -1 & -2 & 3 \\ 2 & -1 & 1 \\ 0 & -2 & 3 \end{bmatrix}}$

9. **Define the minor of an element of a matrix.**

Sol. If $A = [a_{ij}]$ is a square matrix of order n, then minor of an element a_{ij} of A is denoted by M_{ij} is the determinant of order $(n-1, n-1)$ and which is obtained by deleting the ith row and jth column of A.

10. **What is the cofactor of 4 in matrix** $\begin{bmatrix} 3 & 1 & -4 \\ 2 & 5 & 4 \\ 1 & 4 & 8 \end{bmatrix}$

Sol. Here: $4 = a_{32}$, so

Minor of $4 = M_{32}$

$= \begin{vmatrix} 3 & -4 \\ 2 & 6 \end{vmatrix} = 18 + 8 = \boxed{26}$

Cofactor of $4 = C_{32}$

$= (-1)^{3+2} M_{32} = (-1)^5 (26) = \boxed{-26}$

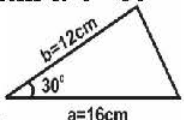
11. **Define isosceles triangle.**

Sol. A triangle whose two sides are equal and third side is different is called Isosceles triangle.

12. **Find the area of a triangle whose two adjacent sides are 16cm and 12cm and their included angle is 30º.**

Sol. Let, $a = 16\text{cm}, b = 12\text{cm}$ & $\theta = 30°$

$\text{Area} = \frac{1}{2}ab\sin\theta$



$\text{Area} = \frac{1}{2}(16)(12)\sin 30°$

$\text{Area} = \boxed{48 \text{ sq.cm}}$

13. **Find the base of a parallelogram whose area is 256sq.cm and height 32cm.**

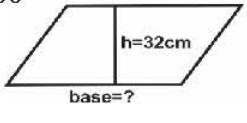
Sol. Here: base = ? & height = 32cm

Area of parallelogram $= 256$

$\text{Base} \times \text{Height} = 256$

$\text{Base} \times 32 = 256$

$\text{Base} = \frac{256}{32}$



$\boxed{\text{Base} = 8\text{cm}}$

14. **The diagonals of a Rhombus are 6 and 8 cm respectively. Find the length of the side of Rhombus.**

Sol. Let length of one side of a Rhombus = a and

$$d_1 = \overline{AC} = 6\text{cm} \;\&\; d_2 = \overline{BD} = 8\text{cm}$$

$$\left|\overline{OA}\right| = \frac{d_1}{2} = \frac{6}{2} = 3\text{cm} \quad \&$$

$$\left|\overline{OB}\right| = \frac{d_2}{2} = \frac{8}{2} = 4\text{cm}$$

In right ΔAOB

By Pythagoras theorem

$$\left|\overline{AB}\right|^2 = \left|\overline{OA}\right|^2 + \left|\overline{OB}\right|^2$$

$$(a)^2 = (3)^2 + (4)^2$$

$$a^2 = 9 + 16$$

$$a^2 = 25$$

$$\boxed{a = 5\text{cm}}$$

15. **Define inscribed polygon and circumscribed circle.**

Sol. If a circle passes through the corners of a polygon, then this polygon is called inscribed polygon.

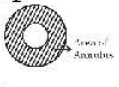
16. **Write the area of regular polygon of 'n' sides when the radius of the circumscribed circle 'R' is given.**

Sol. $$\text{Area} = \boxed{\frac{nR^2}{2}\sin\left(\frac{360^\circ}{n}\right)\ \text{sq.unit}}$$

17. **Define area of the Annulus (Ring).**

Sol. Let, Radius of inner circle = r

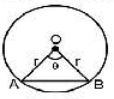
and, Radius of outer circle = R



Area of Annulus (Ring) $= \pi\left(R^2 - r^2\right)$ sq.unit

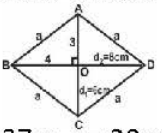
18. **Write the formula of Areas of the minor segment and major segment when angle is 'θ' and radius 'r' are given.**

Sol.



$$\text{Area of} \begin{cases} \text{minor Segment} = \frac{1}{2}r^2(\theta - \sin\theta) \\ \text{major Segment} = \frac{1}{2}r^2(2\pi - \theta + \sin\theta) \end{cases}$$

19. **Find the area of the whole surface of a right triangular prism whose height is 36cm and sides of whose base are 51, 37 and 20cm, respectively.**



Sol. h = 36m

Sides of Δ are a=15m, b=37m, c = 20m

$$s = \frac{a+b+c}{2} = \frac{51+37+20}{2} = \frac{108}{2} = 54$$

$$\text{Area of base} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$A = \sqrt{54(54-51)(54-37)(54-20)}$$

$$A = \sqrt{54 \times 3 \times 17 \times 34}$$

$$A = \sqrt{93636}$$

$$A = 306\ \text{sq.m}$$

L.S.A. = Perimeter of base × Height

$$\text{L.S.A.} = (51+37+20) \times 36$$

$$\text{L.S.A.} = 108 \times 36 = 3888\,\text{m}^2$$

$$\text{T.S.A.} = 2(\text{Area of base}) + \text{L.S.A.}$$

$$\text{T.S.A.} = 2(306) + 3888$$

$$\text{T.S.A.} = \boxed{4500\,\text{m}^2}$$

20. **Write the formula which is used to find volume and total surface area of polygon prism.**

Sol. $$\text{Volume} = \frac{na^2}{4}\cot\frac{180}{n} \times \text{height}$$

$$\text{T.S.A.} = S = 2(\ell b + bh + \ell h)$$

21. **Write the Simpsons Rule.**

Sol. $\text{Area} = \frac{S}{3}[A + 2D + 4E]$ sq.unit

A = Sum of 1st + last ordinate
D = Sum of all odd ordinates
E = Sum of all even ordinates other than 1st and last ordinate
S = Width of each strip

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22. Write the formula of volume of cylinder if radius is given.

Sol.

$\boxed{\text{Volume of cylinder} = \pi r^2 h \text{ cu. unit}}$

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23. Find the height of the cylinder if volume is 528cm³ and diameter is 4cm.

Sol.

Here : $h = ?\ V = 528\text{cm}^3$ & $d = 4\text{cm}$

As, $r = \frac{d}{2} = \frac{4}{2} = 2\text{cm}$

As, Volume of Cylinder $= 528\text{cm}^3$

$\Rightarrow \quad \pi r^2 h = 528$

$\Rightarrow \quad h = \frac{528}{\pi(2)^2}$

$\Rightarrow \quad \boxed{h = 42\text{cm}}$

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24. Find the volume of a pyramid with a square base of side 10cm and height 15cm.

Sol. Here : $V = ?,\ a = 10\text{cm}$ & $h = 15\text{cm}$

Area of base (square) $= a^2$

$= (10)^2 = 100\text{cm}^2$

$\text{Volume} = \frac{1}{3} \times \text{Area of base} \times \text{height}$

$V = \frac{1}{3} \times 100 \times 15$

$V = \boxed{500\text{cm}^3}$

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25. If 'a' is the side of the base of polygon, h is the height and ℓ is the slant height of a regular pyramid, the find:

(i) Lateral surface area

(ii) Total surface area

Sol.

Lateral surface area (L.S.A.)

$= \frac{1}{2} \times \text{Perimeter of base} \times \text{Slant height}$

Total surface area (T.S.A.)

$= \text{L.S.A.} + \text{Area of base}$

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26. Define cone.

Sol. A cone is a solid figure generated by a line, one end of which is fixed and the other end describes a closed curve in a plane.

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27. The area of cross-section of a prism is 52sq.m. What is the weight of the frustum of the prism of the smallest length is 10cm and the greatest length is 24.3 cm? Density of material 0.29 Lb/cu.cm.

Sol. Here: $h_1 = 10\text{cm}$, $h_2 = 24.3\text{cm}$

Volume of frustum

$= \frac{h_1 + h_2}{2} \times \text{Area of base}$

$V = \frac{10 + 24.3}{2} \times 52$

$V = 891.8\text{cm}^3$

Weight = Volume × Density

Weight $= 891.8 \times 0.29$

Weight $= \boxed{258.62\text{lbs}}$

Section - II

Note : Attemp any three (3) questions $3 \times 8 = 24$

Q.2.(a) If the position vectors of $\vec{A}$ and $\vec{B}$ are $5\hat{i} - 2\hat{j} + 4\hat{k}$ and

$\hat{i}+3\hat{j}+7\hat{k}$ respectively, find the magnitude and direction cosines of $\overline{AB}$

Sol. See Q.10 of Ex# 8.1 (Page # 375)

(b) Using cross product, find the area of triangle whose vertices are (0, 0, 0), (1, 1, 1), (0, 0, 3).

Sol. See Q.21(i) of Ex# 8.2 (Page # 390)

Q.3.(a) Prove that :

$$\begin{vmatrix} \sin\alpha & \cos\alpha & 0 \\ -\sin\beta & \cos\beta & \sin\gamma \\ \cos\beta & \sin\beta & \cos\gamma \end{vmatrix} = \sin(\alpha+\beta+\gamma)$$

Sol. See Q.6(iii) of Ex# 9.2 (Page # 427)

(b) ~~By use of Determinant's properties Verify that:~~

$$\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = (b-c)(c-a)(a-b)$$

Sol. See example # 07 of Ch# 09

Q.4.(a) In a quadrilateral the diagonal is 125cm and the two perpendicular on it from the other two angles are 19cm and 25cm respectively, find the area.

Sol. See Q.9 of Ex# 11 (Page # 478)

(b) A road 10m wide is to be made around a circular plot of 75m diameter. Find the cost of the ground needed for the road at Rs.40.00 per square meter.

Sol. See Q.2 of Ex# 13 (Page # 496)

Q.5.(a) Find the area of the field, whose ordinates are 0, 20, 22.5, 33.5, 45, 42, 33.5, 25.5 and 0 meter respectively. The width of each strip is 14m. find the approximate cost of purchasing the field at a cost of Rs. 5,000/per sq.m.

Sol. See example # 03 of Ch# 14

(b) The diameter of right circular cylinder is 38cm and its length is 28cm. Find its volume, lateral surface and total surface.

Sol. See Q.1 of Ex# 16 (Page # 532)

Q.6.(a) Find the slant surface of a right pyramid whose height is 65m and whose base is a regular hexagon of side $48\sqrt{3}\,m$.

Sol. See Q.8 of Ex# 17[A] (Page # 548)

(b) The slant height of a cone is 25cm, and the area of its curved surface is 550sq.cm. Find its volume.

Sol. See Q.3 of Ex# 18[A] (Page # 566)

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