DAE / IA - 2018

MATH-113 APPLIED MATHEMATICS - I

PAPER 'A' PART - A (OBJECTIVE)

Time: 30 Minutes Marks: 15

Q.1: *Encircle the correct answer.*

1. **The factors of $x^2 - 7x + 12 = 0$ are:**
[a] $(x-4)(x+3)$ [b] $(x-4)(x-3)$
[c] $(x+4)(x+3)$ [d] $(x+4)(x-3)$

2. **If the discriminant $b^2 - 4ac$ is a perfect square, its roots will be:**
[a] Imaginary [b] Rational
[c] Equal [d] Irrational

3. **The 7^{th} term of an A.P 1,4,7 ……. is:**
[a] 17 [b] 19
[c] 21 [d] 23

4. **If x, y are in G.P then**
[a] 2y = x + y [b] 2y = xz
[c] $y^2 = xz$ [d] $z^2 = xy$

5. **The sum of infinite terms of a G.P. a, ar, ar^2, . . . if $|r|<1$ is:**
[a] $\frac{a}{1-r}$ [b] $\frac{a(1-r^n)}{1-r}$
[c] ar^{n-1} [d] None of these

6. **Third term of $(x+y)^4$ is:**
[a] $4x^2y^2$ [b] $4x^3y$
[c] $6x^2y^2$ [d] $6x^3y$

7. **The value of $\binom{n}{n}$ is equal to:**
[a] Zero [b] 1
[c] n [d] −n

8. **The fraction $\frac{2x+5}{x^2+5x+6}$ is known as:**
[a] Proper [b] Improper
[c] Both proper and improper
[d] None of these

9. **One radian is equal to:**
[a] 90° [b] $\left(\frac{90}{\pi}\right)^\circ$
[c] 180° [d] $\left(\frac{180}{\pi}\right)^\circ$

10. **The terminal side of θ lies in 4^{th} quadrant, sign of the sign θ will be:**
[a] Positive [b] Negative
[c] Both positive and negative
[d] None of these

11. **If sin θ is +ve and tan θ is −ve then the terminal side of the angle lies in:**
[a] 1^{st} quad [b] 2^{nd} quad
[c] 3^{rd} quad [d] 4^{th} quad

12. **$\tan(45^\circ - x)$ is equal to:**
[a] $\frac{\cos x + \sin x}{\cos x - \sin x}$ [b] $\frac{1+\tan x}{1-\tan x}$
[c] $\frac{1+\cot x}{1-\cot x}$ [d] $\frac{\cos x - \sin x}{\cos x + \sin x}$

13. **$\sin 2\alpha$ is equal to:**
[a] $1-\sin^2\alpha$ [b] $\cos 2\alpha$
[c] $1-\cos^2\alpha$ [d] $2\sin\alpha\cos\alpha$

14. **Law of sines is:**
[a] $2\cos\frac{A+B}{2}\cos\frac{A-B}{2}$
[b] $-2\sin\frac{A+B}{2}\sin\frac{A-B}{2}$
[c] $2\sin\frac{A+B}{2}\cos\frac{A-B}{2}$
[d] $2\cos\frac{A+B}{2}\sin\frac{A-B}{2}$

15. **If c = 90º, a = 1, c = 2, then angle α is:**
[a] 90° [b] 60°
[c] 45° [d] 30°

Answer Key

1	b	2	c	3	b	4	c	5	a
6	c	7	b	8	a	9	d	10	b
11	b	12	d	13	d	14	b	15	d

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DAE / IA - 2018

MATH - 113 APPLIED MATHEMATICS - I

PAPER 'A' PART - B (SUBJECTIVE)

Time: 2:30 Hrs Marks: 60

Section - I

Q.1. Write short answers to any Eighteen (18) questions.

1. Solve the equation $x^2 - 2x - 899 = 0$ by completing the square.

Sol. $x^2 - 2x - 889 = 0$

$x^2 - 2x = 889$

Adding the square of one half of the coefficient of x i.e., $(1)^2$ on both sides

$x^2 - 2x + (1)^2 = 889 + (1)^2$

$(x-1)^2 = 889 + 1 = 900$

$\sqrt{(x-1)^2} = \pm\sqrt{900}$

$x - 1 = \pm 30 \Rightarrow x = \pm 30 + 1$

Either

$x = 30 + 1$

$x = 31$

OR

$x = -30 + 1$

$x = -29$

$S.S. = \{-29, 30\}$

2. Discuss the nature of the roots of the equation $9x^2 + 6x + 1 = 0$

Sol. Here: $a = 9,\ b = 6,\ c = 1$

$\text{Disc.} = b^2 - 4ac$

$= (6)^2 - 4(9)(1) = 36 - 36 = 0$

∴ The roots are **Equal and Real.**

3. ~~For what value of k, the sum of roots of~~ $3x^2 + kx + 5 = 0$ ~~may be equal to the product of roots.~~

Sol. Here: $a = 3,\ b = k,\ c = 5$

As, Sum of roots = Product of roots

$-\frac{b}{a} = \frac{c}{a} \Rightarrow -\frac{k}{3} = \frac{5}{3}$

$-k = 5 \Rightarrow \boxed{k = -5}$

4. Define a sequence.

Sol. A set of numbers arranged in order by some fixed rule is called a sequence. For examples:
(i) $2, 4, 6, \ldots$ **(ii)** $3, 9, 27, \ldots$

5. If $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P, show that $b = \frac{2ac}{a+c}$

Sol. $d = \frac{1}{b} - \frac{1}{a}, \rightarrow (i)$ & $d = \frac{1}{c} - \frac{1}{b} \rightarrow (ii)$

Put eq.(i) in eq.(ii)

$\frac{1}{b} - \frac{1}{a} = \frac{1}{c} - \frac{1}{b} \Rightarrow \frac{1}{b} + \frac{1}{b} = \frac{1}{c} + \frac{1}{a}$

$\frac{1+1}{b} = \frac{a+c}{ac} \Rightarrow \frac{2}{b} = \frac{a+c}{ac}$

$\frac{b}{2} = \frac{ac}{a+c} \Rightarrow \boxed{b = \frac{2ac}{a+c}}$ **Proved.**

6. Write the nth term of a Geometric progression.

Sol. $\boxed{a_n = ar^{n-1}}$ Where $a = 1^{st}$ term,
r = common Ratio and
n = No. of terms.

7. Define geometric mean.

Sol. If a, G, b are three consecutive terms in a G.P., then G is called G.M. of a and b and $\boxed{G = \pm\sqrt{ab}}$

8. ~~Find the sum of the series~~ $1 + \frac{1}{3} + \frac{1}{9} + \ldots$ ~~to 6 terms.~~

Sol. Here: $a_1 = 1,\ r = \frac{1}{3} \div 1 = \frac{1}{3},\ n = 6$ & $S_6 = ?$

$$S_n = \frac{a(1-r^n)}{1-r} \Rightarrow S_6 = \frac{1\left(1-\left(\frac{1}{3}\right)^6\right)}{1-\frac{1}{3}} = \frac{1-\frac{1}{729}}{\frac{3-1}{3}}$$

$$S_6 = \frac{\frac{729-1}{729}}{2/3} = \frac{728}{729} \times \frac{3}{2} = \boxed{\frac{364}{243}}$$

9. ~~Is the series~~ $1+4+16+64+...$ ~~divergent or convergent.~~

Sol. Here: $a=1$ & $r=4\div 1=4$

$$S_n=\frac{a\left(r^n-1\right)}{r-1}=\frac{1\left(4^n-1\right)}{4-1}=\frac{4^n-1}{3}$$

Put $n=\infty$

$$S_\infty=\frac{4^\infty-1}{3}=\frac{\infty-1}{3}=\boxed{\infty}$$ **Divergent.**

10. **Expand $(2x-3y)^4$ by Binomial theorem.**

Sol. $(2x-3y)^4$

$$=\binom{4}{0}(2x)^4(3y)^0-\binom{4}{1}(2x)^3(3y)^1+\binom{4}{2}(2x)^2(3y)^2-\binom{4}{3}(2x)^1(3y)^3+\binom{4}{4}(2x)^0(3y)^4$$

$$=(1)(16x^4)(1)-4(8x^3)(3y)+6(4x^2)(9y^2)-4(2x)(27y^3)+(1)(1)(81y^4)$$

$$=\boxed{16x^4-96x^3y+216x^2y^2-216xy^3+81y^4}$$

11. **Calculate $(1.04)^5$ by binomial theorem up to two decimal places.**

Sol. $(1.04)^5=(1+0.04)^5$

$$=\binom{5}{0}(1)^5(0.04)^0+\binom{5}{1}(1)^4(0.04)^1+\binom{5}{2}(1)^3(0.04)^2+...$$

$$=(1)(1)(1)+5(1)(0.04)+10(1)(0.0016)+...$$

$$=1+0.2+0.016+...=1.2160=\boxed{1.22}$$

12. ~~Using Binomial series calculate~~ $\sqrt[3]{65}$ ~~to the nearest hundredth.~~

Sol. $$\sqrt[3]{65}=(64+1)^{1/3}=\left[64\left(1+\frac{1}{64}\right)\right]^{1/3}$$

$$=4\left(1+\frac{1}{64}\right)^{1/3}=4\left[1+\left(\frac{1}{3}\right)\left(\frac{1}{64}\right)+.....\right]$$

$$=4\left[1+\frac{1}{192}+...\right]=4(1.0052)=\boxed{4.02}$$

13. ~~Write the first three terms in the expansion of~~ $(2+x)^{-3}$.

Sol. $$(2+x)^{-3}=\left[2\left(1+\frac{x}{2}\right)\right]^{-3}$$

Put $b=\frac{x}{2}$ & $n=-3$ in Binomial series Formula, we have

$$=2^{-3}\left[1+(-3)\left(\frac{x}{2}\right)+\frac{(-3)(-3-1)}{2!}\left(\frac{x}{2}\right)^2+...\right]$$

$$=\frac{1}{2^3}\left[1-\frac{3x}{2}+\frac{(-3)(-4)}{2}\left(\frac{x^2}{4}\right)+...\right]$$

$$=\frac{1}{8}\left[1-\frac{3x}{8}+\frac{3x^2}{2}+...\right]$$

14. **Resolve $\frac{2x}{(x-2)(x+5)}$ into partial fractions.**

Sol. $$\frac{2x}{(x-2)(x+5)}=\frac{A}{x-2}+\frac{B}{x+5}\rightarrow(i)$$

$$2x=A(x+5)+B(x-2)\rightarrow(ii)$$

Put $x=2$ in eq. (ii)

$$2(2)=A(2+5)+B(2-2)$$

$$4=A(7)+B(0)$$

$$4=7A+0\Rightarrow\boxed{A=4/7}$$

Put $x=-5$ in eq. (ii)

$$2(-5)=A(-5+5)+B(-5-2)$$

$$-10=A(0)+B(-7)$$

$$-10=0-7B\Rightarrow\boxed{B=10/7}$$

Put values of A, & B in eq. (i), we get: $$\boxed{\frac{4}{7(x-2)}+\frac{10}{7(x+5)}}$$

15. **Form of partial fractions of $\frac{1}{(x^2+1)(x-4)^2}$ is ________.**

Sol. $$\frac{1}{(x^2+1)(x-4)^2}=\frac{Ax+B}{(x^2+1)}+\frac{C}{(x-4)}+\frac{C}{(x-4)^2}$$

16. ~~Define radian measure.~~

Sol. Radian is the measure of the angle subtended at the center of the circle by an arc, whose length is equal to the radius of the circle.

17. ~~Find the length of arc cut off on a circle of radius 3cm by a central angle of 2 radians.~~

Sol. Here ℓ = ?, r = 3cm, θ = 2 rad

By using formula: $\ell = r\theta$

$\ell = r\theta = (3)(2) = \boxed{6\text{cm}}$

18. **Prove that:**

$$\tan^2 30° + \tan^2 45° + \tan^2 60° = \frac{13}{3}$$

Sol. $\text{L.H.S.} = \tan^2 30° + \tan^2 45° + \tan^2 60°$

$$= \left(\frac{1}{\sqrt{3}}\right)^2 + (1)^2 + \left(\sqrt{3}\right)^2 = \frac{1}{3} + 1 + 3$$

$$= \frac{1+3+9}{3} = \frac{13}{3} = \text{R.H.S.}$$ **Proved.**

19. **Prove that:**

$$\frac{1}{1+\sin\theta} + \frac{1}{1-\sin\theta} = 2\sec^2\theta$$

Sol. $\text{L.H.S.} = \frac{1}{1+\sin\theta} + \frac{1}{1-\sin\theta}$

$$= \frac{1(1-\sin\theta)+1(1+\sin\theta)}{(1+\sin\theta)(1-\sin\theta)}$$

$$= \frac{1-\sin\theta+1+\sin\theta}{(1)^2-(\sin\theta)^2}$$

$$= \frac{2}{1-\sin^2\theta} = \frac{2}{\cos^2\theta}$$

$= 2\sec^2\theta = \text{R.H.S.}$ **Proved.**

20. **Prove that:** $\cos(-\beta) = \cos\beta$

Sol. As, $\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

Put $\alpha = 0$, we have:

$\cos(0-\beta) = \cos(0)\cos\beta + \sin(0)\sin\beta$

$\cos(-\beta) = 1.\cos\beta + 0.\sin\beta$

$\cos(-\beta) = \cos\beta + 0$

$\boxed{\cos(-\beta) = \cos\beta}$ **Proved.**

21. **Express** $\sin x\cos 2x - \sin 2x\cos x$ **as single term.**

Sol. $\sin x\cos 2x - \sin 2x\cos x$

$= \sin x\cos 2x - \cos x\sin 2x$

$= \sin(x-2x) \quad \because \left\{\begin{array}{l}\sin(\alpha-\beta)\\ =\sin\alpha\cos\beta-\cos\alpha\sin\beta\end{array}\right\}$

$= \sin(-x) = \boxed{-\sin x}$

22. **Prove that:** $\sin^2\alpha = \frac{1-\cos 2\alpha}{2}$

Sol. Take $\cos 2\alpha = \cos(\alpha+\alpha)$

$\cos 2\alpha = \cos\alpha\cos\alpha - \sin\alpha\sin\alpha$

$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$

$\cos 2\alpha = 1 - \sin^2\alpha - \sin^2\alpha$

$\cos 2\alpha = 1 - 2\sin^2\alpha$

$\Rightarrow 2\sin^2\alpha = 1 - \cos 2\alpha$

$\Rightarrow \sin^2\alpha = \frac{1-\cos 2\alpha}{2}$ **Proved.**

23. ~~Express~~ $\cos\theta - \cos 4\theta$ ~~as product.~~

Sol. $\cos\theta - \cos 4\theta$

$$= -2\sin\left(\frac{\theta+4\theta}{2}\right)\sin\left(\frac{\theta-4\theta}{2}\right)$$

$$= -2\sin\frac{5\theta}{2}\sin\left(\frac{-3\theta}{2}\right) = \boxed{2\sin\frac{5\theta}{2}\sin\frac{3\theta}{2}}$$

24. **Define the law of Sine.**

Sol. In any triangle ABC, with usual notations.

$$\boxed{\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}}$$

25. **Given that α = 30º, γ = 135º and c = 10 find 'a'.**

Sol. By using law of sines:

$$\frac{a}{\sin\alpha}=\frac{b}{\sin\beta}=\frac{c}{\sin\gamma}$$

$$\text{We take}: \frac{a}{\sin\alpha}=\frac{c}{\sin\gamma}$$

$$\frac{a}{\sin 30^\circ}=\frac{10}{\sin 135^\circ}$$

$$a=\frac{10\sin 30^\circ}{\sin 135^\circ}$$

$$\boxed{a=7.07}$$

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26. **In any triangle ABC in which b = 45, c = 34, ∝ = 52º, find a.**

Sol. By using Law of cosines:

$$a^2=b^2+c^2-2bc\cos 52^\circ$$

$$a^2=(45)^2+(34)^2-2(45)(34)\cos 52^\circ$$

$$a^2=2025+1156-1883.92$$

$$a^2=1297.07$$

$$\boxed{a=36.01}$$

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27. **Define angle of depression.**

Sol. **Angle of Depression:**
If the line of sight is downward from the horizontal, the angle is called angle of Depression.

Section - II

Note: Attemp any three (3) questions $3\times 8=24$

Q.2.(a) Solve the equation

$$\frac{1}{a+b+x}=\frac{1}{a}+\frac{1}{b}+\frac{1}{x}$$

By factorization.

Sol. See $Q.1(vi)$ of Ex # 1.1 (Page # 8)

(b) ~~If the difference of the roots of $x^2-7x+k-4=0$ is 5, find the value of k and the roots.~~

Sol. See $Q.3(i)$ of Ex # 1.3 (Page # 42)

Q.3.(a) ~~Sum the series .3+.33+.333+....to n terms.~~

Sol. See $Q.3(i)$ of Ex # 2.6 (Page # 120)

(b) **Insert 6 G.M's between 2 and 256.**

Sol. See example # 20 of Ch # 02

Q.4.(a) Find the term involving x^9 in the expansion of $\left(x^3+\frac{1}{x}\right)^7$.

Sol. See $Q.7(iii)$ of Ex # 3.1 (Page # 154)

(b) Resolve $\frac{2x+1}{(x+3)(x-1)(x+2)^2}$ into partial fractions.

Sol. See Q.4 of Ex # 4.2 (Page # 196)

Q.5.(a) ~~A space man land on the moon and observes that the Earth's diameter subtends an angle of $1^\circ 54'$ at his place of landing. If the Earth's radius is 6400km, find the distance between the Earth and the moon.~~

Sol. See Q.5 of Ex # 5.1 (Page # 239)

(b) Prove that:

$$(\operatorname{cosec}\theta-\cot\theta)^2=\frac{1-\cos\theta}{1+\cos\theta}$$

Sol. See Q.6 of Ex # 5.3 (Page # 255)

Q.6.(a) ~~Prove that:~~

$$\sin 20^\circ\sin 40^\circ\sin 60^\circ\sin 80^\circ=\frac{3}{16}$$

Sol. See Q.14 of Ex # 6.3 (Page # 314)

(b) **A town B is 15km due North of a town A. The road from A to B runs North 27º East to G, then North 34º West to B. Find the distance by road from the town A to B.**

Sol. See example # 11 of Ch # 07

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