DAE / IA - 2018

MATH-123 APPLIED MATHEMATICS-I

PAPER 'B' PART-A (OBJECTIVE)

Time: 30 Minutes Marks: 15

Q.1: *Encircle the correct answer.*

1. **The additive inverse of $a + ib$ is:**
[a] $-a+ib$ [b] $a-ib$
[c] $-a-ib$ [d] $a+ib$

2. **Modulus of $3+4i$ is:**
[a] 47 [b] 16
[c] 5 [d] 3

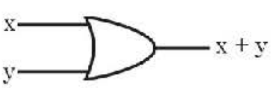
3. **$(1+2i)(3-5i)$ is equal to:**
[a] $13+i$ [b] $-2-i$
[c] $-4+3i$ [d] $3i$

4. **If the degree of numerator is less than the degree of denominator, then the fraction is:**
[a] Proper [b] Improper
[c] Neither proper non-improper
[d] Both proper and improper

5. **The number of partial fraction of $\frac{6x+27}{6x^3-9x}$ are:**
[a] 2 [b] 3
[c] 4 [d] None of these

6. **$(25)_{10}$ when converted to octal is:**
[a] $(31)_8$ [b] $(2.5)_8$
[c] $(13)_8$ [d] None of these

7. **In Boolean Algebra $X+\overline{X}Y$ is equal to:**
[a] X [b] $\overline{X}$
[c] $X+Y$ [d] $\overline{X}+Y$

8. 

is the symbol for the logic gate:
[a] OR gate [b] NOR gate
[c] NAND gate
[d] AND gate

9. **$y=2$ is a line parallel to:**
[a] x – axis [b] y – axis
[c] $y=x$ [d] $x=3$

10. **Point $(-4, -5)$ lies in the quadrant:**
[a] 1^{st} [b] 2^{nd}
[c] 3^{rd} [d] 4^{th}

11. ~~**Ratio formula for y-coordinate is:**~~
[a] $\frac{x_1r_2+x_2r_1}{r_1+r_2}$ [b] $\frac{y_1r_2+y_2r_1}{r_1+r_2}$
[c] $\frac{x-y}{2}$ [d] None of these

12. **Slope of the line through (x_1, y_1) and (x_2, y_2) is:**
[a] $\frac{x_1+x_2}{y_1+y_2}$ [b] $\frac{y_2+y_1}{x_2+y_1}$
[c] $\frac{y_2-y_1}{x_2-x_1}$ [d] None of these

13. **Give three points are collinear if their slopes are:**
[a] Equal [b] Unequal
[c] $m_1m_2=-1$ [d] None of these

14. **Standard form of the equation of the circle is:**
[a] $x^2+y^2+2gh+2fy+c=0$
[b] $(x-h)^2+(y-k)^2=r^2$
[c] $x^2+y^2+x+y+1=0$
[d] None of these

15. **Radius of the circle $x^2+y^2-2x-4y=8$ is:**
[a] 8 [b] $\sqrt{8}$
[c] $\sqrt{12}$ [d] None of these

Answer Key

1	c	2	c	3	a	4	b	5	b
6	a	7	c	8	a	9	a	10	c
11	b	12	c	13	a	14	b	15	d

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DAE / IA - 2018

MATH-123 APPLIED MATHEMATICS - I

PAPER 'B' PART - B (SUBJECTIVE)

Time: 2:30 Hrs Marks: 60

Section - I

Q.1. Write short answers to any Eighteen (18) questions.

1. Write the conjugate and Modulus of $-2+i$.

Sol. Let $z=-2+i$

Conjugate $=\bar{z}=\overline{-2+i}=\boxed{-2-i}$

As, $a=-2$, $b=1$

Modulus $=|z|=\sqrt{a^2+b^2}$

$|z|=\sqrt{(-2)^2+(1)^2}$

$|z|=\sqrt{4+1}=\boxed{\sqrt{5}}$

2. Find the conjugate and modulus of $\dfrac{1+i}{1-i}$

Sol. Let $z=\dfrac{1+i}{1-i}$

$z=\dfrac{1+i}{1-i}\times\dfrac{1+i}{1+i}$

$z=\dfrac{1+i+i+i^2}{(1)^2-(i)^2}$

$z=\dfrac{1+2i-1}{1+1}=\dfrac{2i}{2}=i$

Conjugate $=\bar{z}=\bar{i}=\boxed{-i}$

Here $a=0$ & $b=1$

Modulus $=|z|=\sqrt{a^2+b^2}$

$|z|=\sqrt{(0)^2+(1)^2}$

$|z|=\sqrt{0+1}=\sqrt{1}=\boxed{1}$

3. Show that $\left|\dfrac{1+2i}{2-i}\right|=1$

Sol. L.H.S. $=\left|\dfrac{1+2i}{2-i}\right|$

$=\dfrac{\sqrt{(1)^2+(2)^2}}{\sqrt{(2)^2+(-1)^2}}$

$=\dfrac{\sqrt{1+4}}{\sqrt{4+1}}=\dfrac{\sqrt{5}}{\sqrt{5}}=1=$ R.H.S. **Proved.**

4. Factorize $9a^2+64b^2$

Sol. $9a^2+64b^2$

$=9a^2-64b^2i^2$

$=(3a)^2-(8bi)^2$

$=\boxed{(3a-8bi)(3a+8bi)}$

5. ~~Express $|z|=6$ and $\arg z=\dfrac{3\pi}{4}$ in the form $x+yi$~~ :

Sol. $z=r\ \text{cis}\ \theta=6\,\text{cis}\left(\dfrac{3\pi}{4}\right)$

$z=2\ \text{cis}135°$

$z=6[\cos 135°+i\sin 135°]$

$z=6\left[-\dfrac{\sqrt{2}}{2}+i\dfrac{\sqrt{2}}{2}\right]=6\left[\dfrac{-\sqrt{2}+\sqrt{2}i}{2}\right]$

$z=3(-\sqrt{2}+\sqrt{2}i)=\boxed{-3\sqrt{2}+3\sqrt{2}i}$

6. Define proper fraction and give example.

Sol. A fraction in which the degree of the numerator is less than the degree of the denominator is called proper fraction.

Example: $\dfrac{2x}{(x-2)(x+5)}$

7. **Resolve** $\dfrac{x^2+1}{(x+1)(x-1)}$ **into partial fractions.**

Sol. $\dfrac{x^2+1}{(x+1)(x-1)}$ $\left\{\begin{matrix}\text{Improper}\\ \text{Fraction}\end{matrix}\right\}$

$$=1+\frac{2}{(x+1)(x-1)}$$

$$\begin{array}{r|l} & \quad 1 \\ x^2-1 & x^2+1 \\ & \underline{\pm x^2 \mp 1} \\ & \qquad 2 \end{array}$$

Take $\dfrac{2}{(x+1)(x-1)}=\dfrac{A}{x+1}+\dfrac{B}{x-1}\rightarrow(i)$

$$2=A(x-1)+B(x+1)\rightarrow(ii)$$

Put $x=-1$ in eq.(ii)

$2=A(-1-1)+B(-1+1)$

$2=A(-2)+B(0)$

$2=-2A+0 \Rightarrow \boxed{A=-1}$

Put $x=1$ in eq.(ii)

$2=A(1-1)+B(1+1)$

$2=A(0)+B(2)$

$2=0+2B \Rightarrow \boxed{B=1}$

Put values of A & B in eq.(i),

we get : $\boxed{1-\dfrac{1}{x+1}+\dfrac{1}{x-1}}$

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8. **Write an identity equation of** $\dfrac{(x-1)(x-2)(x-3)}{(x-4)(x-5)(x-6)}$

Sol. $\dfrac{(x-1)(x-2)(x-3)}{(x-4)(x-5)(x-6)}\rightarrow(i)$ $\left\{\begin{matrix}\text{Improper}\\ \text{Fraction}\end{matrix}\right\}$

$$\begin{aligned}&(x-1)(x-2)(x-3)\\ &=(x-1)(x^2-3x-2x+6)\\ &=(x-1)(x^2-5x+6)\\ &=x^3-5x^2+6x-x^2+5x-6\\ &=x^3-6x^2+11x-6\end{aligned}$$

$$\begin{aligned}&(x-4)(x-5)(x-6)\\ &=(x-4)(x^2-6x-5x+30)\\ &=(x-4)(x^2-11x+30)\\ &=x^3-11x^2+30x-4x^2+44x-120\\ &=x^3-15x^2+74x-120\end{aligned}$$

$$\begin{array}{r|l} & \qquad\qquad 1 \\ x^3-15x^2+74x-120 & x^3-6x^2+11x-6 \\ & \underline{{}_-x^3 \mp 15x^2 \pm 74x \mp 120} \\ & \qquad 9x^2-63x+114 \end{array}$$

So eq.(i) becomes: $=1+\dfrac{9x^2-63x+114}{(x-4)(x-5)(x-6)}$

$$=\boxed{1+\frac{A}{x-4}+\frac{B}{x-5}+\frac{C}{x-6}}$$

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9. **Form of partial fraction of** $\dfrac{1}{(x+1)^2(x-2)}$ **is ________**

Sol.

$$\frac{1}{(x+1)^2(x-2)}=\boxed{\frac{A}{(x+1)}+\frac{B}{(x+1)^2}+\frac{C}{(x-2)}}$$

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10. **Convert binary number 10101_2 to decimal number.**

Sol. 10101_2

$$=1\times2^4+0\times2^3+1\times2^2+0\times2^1+1\times2^0$$

$$=16+0+4+0+1=\boxed{21}$$

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11. **Multiply the binary numbers**

$111_2\times101_2$

Sol.

$$\begin{array}{r} 111 \\ \underline{101} \\ 111 \\ 000\times \\ \underline{111\times\times} \\ 100011 \end{array}$$

$$111_2\times101_2=\boxed{(100011)_2}$$

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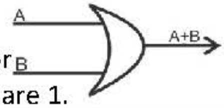
12. **Define AND Gate and draw logic circuit diagram.**

Sol. The OR gate is an electronic circuit that gives a high output (1) if one or more of its inputs are 1.



13. **Prove by Boolean Algebra Rules.**

$$AB + AC + ABC = AB + AC$$

Sol. $\text{L.H.S.} = AB + AC + ABC$

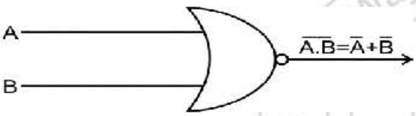
$= AB + AC(1 + B)$

$= AB + AC(1) \quad \because 1 + B = 1$

$= AB + AC = \text{R.H.S.}$ **Proved.**

14. **Construct a logic diagram for expression $\overline{AB} = \overline{A} + \overline{B}$**

Sol.



15. **Write distance formula between two points and give one example.**

Sol. Let $A(x_1, y_1)$ & $B(x_2, y_2)$ be two different points, then,

$$\text{Distance} = |AB| = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

Example: Let A(0, 0), B(1, 1) be two points, then

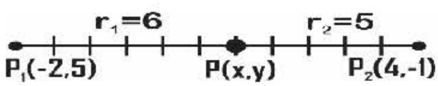
$$|\overline{AB}| = \sqrt{(0-1)^2 + (0-1)^2}$$

$$|\overline{AB}| = \sqrt{1+1} = \sqrt{2}$$

16. ~~**Find the coordinates of the point $P(x, y)$ which divide internally the segment through $P_1(-2, 5)$ and $P_2(4, -1)$ of the ratio of $\frac{r_1}{r_2} = \frac{6}{5}$.**~~

Sol. Here : $r_1 = 6, r_2 = 5, (x_1, y_1) = (-2, 5)$

& $(x_2, y_2) = (4, -1)$



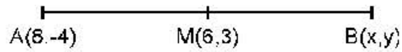
$$P(x, y) = \left(\frac{r_1x_2 + r_2x_1}{r_1 + r_2}, \frac{r_1y_2 + r_2y_1}{r_1 + r_2}\right)$$

$$= \left(\frac{6(4)+5(-2)}{6+5}, \frac{6(-1)+5(5)}{6+5}\right)$$

$$= \left(\frac{24-10}{11}, \frac{-6+25}{11}\right) = \boxed{\left(\frac{14}{11}, \frac{19}{11}\right)}$$

17. **If the mid-point of a segment is (6, 3) and one end point is (8, – 4), what are the coordinates of the other end point.**

Sol. Let B(x, y) be require end point.



$$\text{As, Mid-point} = (6, 3)$$

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = (6, 3)$$

$$\left(\frac{8+x}{2}, \frac{-4+y}{2}\right) = (6, 3)$$

Comparing both order pairs, we have :

$$\frac{8+x}{2} = 6 \text{ and } \frac{-4+y}{2} = 3$$

$8 + x = 12$ | $-4 + y = 6$

$x = 12 - 8 = 4$ | $y = 6 + 4 = 10$

Hence other end point = $\boxed{(4, 10)}$

18. ~~**Find the angle between the lines having slopes -3 and 2.**~~

Sol. $\text{Let, } m_1 = -3 \text{ and } m_2 = 2$

$$\theta = \tan^{-1}\left(\frac{m_2 - m_1}{1 + m_2m_1}\right) = \tan^{-1}\left(\frac{2-(-3)}{1+(2)(-3)}\right)$$

$$\theta = \tan^{-1}\left(\frac{2+3}{1-6}\right) = \tan^{-1}\left(\frac{5}{-5}\right) = \boxed{135^\circ}$$

19. ~~**Find the equation of a line through the points $(-1, 2)$ and $(3, 4)$.**~~

Sol. $\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-2}{3-(-1)} = \frac{2}{4} = \frac{1}{2}$

Equation of line in point - slope form:

$$y-y_1=m(x-x_1)$$

$$y-2=\frac{1}{2}(x-(-1))$$

$$2(y-2)=1(x+1)$$

$$2y-4=x+1 \Rightarrow 2y-4-x-1=0$$

$$-x+2y-5=0 \Rightarrow \boxed{x-2y+5=0}$$

20. ~~**Find an equation on the line with the following intercepts: a = 2, b = −5**~~

Sol. Equation of line in intercept form:

$$\frac{x}{a}+\frac{y}{b}=1 \Rightarrow \frac{x}{2}+\frac{y}{-5}=1$$

$$\frac{5x-2y}{10}=1 \Rightarrow 5x-2y=10$$

$$\Rightarrow \boxed{5x-2y-10=0}$$

21. **Reduce the equation $3x+4y-2=0$ into intercept form.**

Sol. $3x+4y-2=0$

$$3x+4y=2$$

Dividing both sides by 2, we have:

$$\frac{3x}{2}+\frac{4y}{2}=\frac{2}{2} \Rightarrow \boxed{\frac{x}{2/3}+\frac{y}{1/2}=1}$$

22. ~~**Find the equation of the line passing through the point $(1, -2)$ making an angle of 135° with the X-axis.**~~

Sol. Let, $\theta=135^\circ$

Slope $= m=\tan\theta=\tan 135^\circ=-1$

Equation of line in point - slope form:

$$y-y_1=m(x-x_1)$$

$$y-(-2)=-1(x-1)$$

$$y+2=-x+1$$

$$y+2+x-1=0 \Rightarrow \boxed{x+y+1=0}$$

23. **Find the points of intersection of the lines $x+2y-3=0$, $2x-3y+8=0$**

Sol. Let $\begin{cases}\ell_1: x+2y-3=0 \rightarrow (i)\\ \ell_2: 2x-3y+8=0 \rightarrow (ii)\end{cases}$

Multiplying eq. (i) by 2 & subtracting eq.(ii) from it:

$$2x+4y-6=0$$
$$\pm 2x \mp 3y \pm 8=0$$
$$7y-14=0$$

$$7y=14 \Rightarrow y=\frac{14}{7}=2$$

Put $y=2$ in eq.(i), we have:

$$x+2(2)-3=0$$
$$x+4-3=0$$
$$x+1=0 \Rightarrow x=-1$$

Point of intersection is $\boxed{(-1,2)}$

24. **Find the equation of circle with center (-3, 4) and radius 4.**

Sol. Standard form of eq. of circle:

$$(x-h)^2+(y-k)^2=r^2$$

Put $h=-3,\ k=4\ \&\ r=4$

$$(x+3)^2+(y-4)^2=(4)^2$$

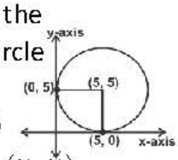
$$(x)^2+2(x)(3)+(3)^2+(y)^2-2(y)(4)+(4)^2=16$$

$$x^2+6x+9+y^2-8y+\cancel{16}-\cancel{16}=0$$

$$\boxed{x^2+y^2+6x-8y+9=0}$$

25. **Find the equation of the circle which is tangent to the positive x and y-axis and radius 5 units.**

Sol. As circle is tangent to the x-axis and y-axis, so circle lie in I-Quadrant.



As, Radius $= r=5$

so, centre $=(h, k)=(5, 5)$

Standard form of eq. of circle:

$(x-h)^2+(y-k)^2=r^2$

Put $h=5,\ k=5\ \&\ r=5$

$(x-5)^2+(y-5)^2=(5)^2$

$(x)^2-2(x)(5)+(5)^2+(y)^2-2(y)(5)+(5)^2=25$

$x^2-10x+25+y^2-10y+25-25=0$

$\boxed{x^2+y^2-10x-10y+25=0}$

26. **Find the equation of circle with center (3,0) and tangent to y-axis.**

Sol. As circle is tangent to y-axis, so, r = 3

Standard form of equation of circle:

$(x-h)^2+(y-k)^2=r^2$

$(x-3)^2+(y-0)^2=(3)^2$

$(x)^2-2(x)(3)+(3)^2+(y)^2=9$

$x^2-6x+\not{9}+y^2-\not{9}=0 \Rightarrow \boxed{x^2+y^2-6x=0}$

27. **Find the equation of circle with center on origin and radius is ½.**

Sol. Standard form of eq. of circle:

$(x-h)^2+(y-k)^2=r^2$

Put $h=0,\ k=0\ \&\ r=\frac{1}{2}$

$(x-0)^2+(y-0)^2=\left(\frac{1}{2}\right)^2 \Rightarrow \boxed{x^2+y^2-\frac{1}{4}=0}$

Section - II

Note: Attemp any three (3) questions $3\times 8=24$

Q.2.(a) Simplify $\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)^{-3}$

Sol. See Q.4(viii) of Ex # 8.1 (Page # 305)

(b) ~~Perform the indicated operation in~~ $[3(\cos 22^\circ+i\sin 22^\circ)][2(\cos 8^\circ+i\sin 8^\circ)]$ ~~and express the results in the form of~~ $a+bi$.

Sol. See Q.1(i) of Ex # 8.3 (Page # 329)

Q.3.(a) Resolve $\frac{9x^2-9x+6}{(x-1)(2x-1)(x+2)}$ into partial fractions.

Sol. See Q.9 of Ex # 9.1 (Page # 354)

(b) Resolve $\frac{x^5}{x^4-1}$ into partial fractions.

Sol. See Q.8 of Ex # 9.3 (Page # 376)

Q.4.(a) Convert 962.84 decimal number to binary equivalent.

Sol. See Q.2[c] of Ex # 10 (Page # 405)

(b) Prepare a truth table for $ABC+A\overline{B}\,\overline{C}+\overline{A}\,\overline{B}\,\overline{C}$

Sol. See Q.1(ii) of Ex # 11 (Page # 425)

Q.5.(a) If one end of a line whose length is 13 units is the point $(4, 8)$ and the ordinate of the other end is 3. What is its abscissa?

Sol. See Q.6 of Ex # 12.1 (Page # 451)

(b) ~~Find the point which is~~ $\frac{7}{10}$ ~~of the way from the point~~ $(4, 5)$ ~~to the point~~ $(-6, 10)$.

Sol. See Q.4 of Ex # 12.2 (Page # 458)

Q.6.(a) Show that the circles $x^2 + y^2 + 2x - 2y - 7 = 0$ and $x^2 + y^2 - 6x + 4y + 9 = 0$ touch externally.

Sol. See Q.11 of Ex # 13 (Page # 544)

(b) Find the equation of the circle having $(-2, 5)$ and $(3, 4)$ as the end point of its diameter. Find also its center and radius.

Sol. See Q.9[a] of Ex # 13 (Page # 541)

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