DAE / IA - 2018

MATH-123 APPLIED MATHEMATICS - I

PAPER 'A' PART - A(OBJECTIVE)

Time : 30 Minutes Marks : 15

Q.1: *Encircle the correct answer.*

1. To make $x^2 - 5x$ a complete square we should add:

[a] 25 [b] $\frac{25}{4}$ [c] $\frac{25}{9}$ [d] $\frac{25}{16}$

2. A second degree equation is known as:

[a] Linear [b] Quadratic
[c] Cubic [d] None of these

3. ~~Sum of roots of~~ $ax^2 - bx + c = 0$ ~~is:~~

[a] $-\frac{c}{a}$ [b] $\frac{c}{a}$ [c] $-\frac{b}{a}$ [d] $\frac{b}{a}$

4. The value of $\binom{n}{r}$ is:

[a] $\frac{n!}{r!(n-r)!}$ [b] $\frac{n}{r(n-r)}$

[c] $\frac{n!}{r!(n-r)}$ [d] $\frac{n!}{(n-r)!}$

5. In the expansion of $(a+b)^n$ the term $\binom{n}{r}a^{n-r}b^r$ will be:

[a] nth term [b] rth term
[c] $(r+1)$th term
[d] $(r-1)$th term

6. The value of $\binom{n}{n}$ is equal to:

[a] 0 [b] 1 [c] n [d] $-n$

7. ~~One radian is equal to:~~

[a] $90°$ [b] $\left(\frac{90}{\pi}\right)°$
[c] $180°$ [d] $\left(\frac{180}{\pi}\right)°$

8. Then terminal side of θ lies in 4th quadrant, both sin θ and tan θ are:

[a] $\sin\theta > 0, \tan\theta > 0$
[b] $\sin\theta > 0, \tan\theta < 0$
[c] $\sin\theta < 0, \tan\theta < 0$
[d] $\sin\theta < 0, \tan\theta > 0$

9. $\tan(45° - x)$ is equal to:

[a] $\frac{\cos x + \sin x}{\cos x - \sin x}$ [b] $\frac{1 + \tan x}{1 - \tan x}$
[c] $\frac{1 + \cot x}{1 - \cos x}$ [d] $\frac{\cos x - \sin x}{\cos x + \sin x}$

10. $2\cos^2\frac{\theta}{2}$ is equal to:

[a] $1 + \cos\theta$ [b] $1 - \cos\theta$
[c] $1 + \sin\theta$ [d] $1 - \sin\theta$

11. If a = 2, b = 2, A = 30º, then Bº is:

[a] $45°$ [b] $30°$ [c] $60°$ [d] $90°$

12. ~~In a right triangle if one angle is~~ $30°$ ~~, then the other will be:~~

[a] $45°$ [b] $50°$ [c] $60°$ [d] $75°$

13. Magnitude of the vector $2i - 2j - k$ is:

[a] 4 [b] 3 [c] 2 [d] 1

14. If ℓ, m and n are direction cosines of a vector, then:

[a] $\ell^2 - m^2 - n^2 = 1$
[b] $\ell^2 - m^2 + n^2 = 1$
[c] $\ell^2 + m^2 - n^2 = 1$
[d] $\ell^2 + m^2 + n^2 = 1$

15. ~~If~~ $z = 1 - j$, ~~then~~ $\arg(z)$ ~~is:~~

[a] $45°$ [b] $135°$ [c] $225°$ [d] $315°$

Answer Key

1	b	2	b	3	d	4	a	5	c
6	b	7	d	8	c	9	d	10	a
11	b	12	c	13	b	14	d	15	b

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DAE / IA - 2018

MATH-123 APPLIED MATHEMATICS-I

PAPER 'A' PART-B (SUBJECTIVE)

Time: 2:30 Hrs Marks: 60

Section-I

Q.1. Write short answers to any Eighteen (18) questions.

1. Solve the Quadratic equation $6x^2 - 5x = 4$ by factorization.

Sol. $6x^2 - 5x = 4$

$6x^2 - 5x - 4 = 0$

$6x^2 - 8x + 3x - 4 = 0$

$2x(3x-4) + 1(3x-4) = 0$

$(3x-4)(2x+1) = 0$

Either	OR
$3x - 4 = 0$	$2x + 1 = 0$
$3x = 4$	$2x = -1$
$x = \frac{4}{3}$	$x = -\frac{1}{2}$

$S.S. = \left\{-\frac{1}{2}, \frac{4}{3}\right\}$

2. Solve the Quadratic equation $2x^2 + 12x - 110 = 0$ by completing the square.

Sol. $2x^2 + 12x - 110 = 0$

$2x^2 + 12x = 110$

Dividing both sides by '2', we get:

$x^2 + 6x = 55$

Adding the square of one half of the coefficient of x i.e., $(3)^2$ on both sides

$x^2 + 6x + (3)^2 = 55 + (3)^2$

$(x+3)^2 = 55 + 9$

$(x+3)^2 = 64$

$\sqrt{(x+3)^2} = \pm\sqrt{64}$

$x + 3 = \pm 8$

$x = \pm 8 - 3$

Either	OR
$x = 8 - 3$	$x = -8 - 3$
$x = 5$	$x = -11$

$S.S. = \{-11, 5\}$

3. Discuss the nature of the roots of the equation $x^2 - 2\sqrt{2}x + 2 = 0$

Sol. Here: $a = 1,\ b = -2\sqrt{2},\ c = 2$

$Disc. = b^2 - 4ac$

$= (-2\sqrt{2})^2 - 4(2)(1) = 8 - 8 = 0$

$\therefore$ The roots are **Equal and Real.**

4. ~~If the sum of the roots of~~ $4x^2 + kx - 7 = 0$ ~~is 3, Find the value of 'k'.~~

Sol. Here: $a = 4,\ b = k,\ c = -7$

As, Sum of the roots $= 3$

$\Rightarrow -\frac{b}{a} = 3$

$\Rightarrow -\frac{k}{4} = 3$

$\Rightarrow -k = 12 \Rightarrow \boxed{k = -12}$

5. ~~Form the quadratic equation whose roots are~~ $i\sqrt{3}$ and $-i\sqrt{3}$

Sol.

$S = i\sqrt{3} + (-i\sqrt{3})$	$P = (i\sqrt{3})(-i\sqrt{3})$
$S = i\sqrt{3} - i\sqrt{3}$	$P = -(i)^2(\sqrt{3})^2$
$S = 0$	$P = -(-1)(3) = 3$

$x^2 - Sx + P = 0$

$x^2 - 0x + 3 = 0 \Rightarrow \boxed{x^2 + 3 = 0}$

6. **Expand $(x+y)^4$ by Binomial Theorem.**

Sol. $(x+y)^4$

$$=\binom{4}{0}(x)^4(y)^0+\binom{4}{1}(x)^3(y)^1+\binom{4}{2}(x)^2(y)^2+\binom{4}{3}(x)^1(y)^3+\binom{4}{4}(x)^0(y)^4$$

$$=(1)(x^4)(1)+(4)(x^3)(y)+(6)(x^2)(y^2)+(4)(x)(y^3)+(1)(1)(y^4)$$

$$=\boxed{x^4+4x^3y+6x^2y^2+4xy^3+y^4}$$

7. **Calculate $(0.98)^6$ by Binomial Theorem up to two decimal places.**

Sol. $(0.98)^6=(1-0.02)^6$

$$=\binom{6}{0}(1)^6(0.02)^0-\binom{6}{1}(1)^5(0.02)^1+\binom{6}{2}(1)^4(0.02)^2+\ldots$$

$$=(1)(1)(1)-6(1)(0.02)+15(1)(0.0004)+\ldots$$

$$=1-0.12+0.006+\ldots$$

$$=0.886=\boxed{0.89}$$

8. **Find the 7th term in the expansion of $\left(x-\frac{1}{x}\right)^9$**

Sol. Here: $a=x,\ b=-\frac{1}{x},\ n=9\ \&\ r=6$

Using general term formula:

$$T_{r+1}=\binom{n}{r}a^{n-r}b^r$$

$$T_{6+1}=\binom{9}{6}(x)^{9-6}\left(-\frac{1}{x}\right)^6$$

$$T_7=84x^3\left(\frac{1}{x^6}\right)=\frac{84}{x^3}$$

9. ~~**Expand $(4-3x)^{1/2}$ to three terms.**~~

Sol. $(4-3x)^{1/2}=\left[4\left(1-\frac{3x}{4}\right)\right]^{1/2}=2\left(1-\frac{3x}{4}\right)^{1/2}$

Put $b=-\frac{3x}{4}$ & $n=\frac{1}{2}$ in Binomial series Formula, we have

$$=2\left[1+\left(\frac{1}{2}\right)\left(-\frac{3x}{4}\right)+\frac{\left(\frac{1}{2}\right)\left(\frac{1}{2}-1\right)}{2!}\left(-\frac{3x}{4}\right)^2+\ldots\right]$$

$$=2\left[1-\frac{3x}{8}+\frac{1}{2}\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(\frac{9x^2}{16}\right)+\ldots\right]$$

$$=2\left[1-\frac{3x}{8}-\frac{9x^2}{128}+\ldots\right]=\boxed{2-\frac{3x}{4}-\frac{9x^2}{64}+\ldots}$$

10. **Which term is the middle term or terms in the Binomial expansion of $(a+b)^n$.**

(i) When 'n' is even (ii) When 'n' is odd

Sol. **(i)** **When n is even**

Then, Middle term = $\left(\frac{n+2}{2}\right)^{th}$ term

Sol. **(ii)** **When n is odd**

Then, there are two middle terms:

Middle term $=\left(\frac{n+1}{2}\right)^{th}+\left(\frac{n+3}{2}\right)^{th}$ terms.

11. ~~**Convert 120º into radians measure.**~~

Sol.

$$120^\circ=120\times\frac{\pi}{180}=\frac{2\pi}{3}=\boxed{2.09\,\text{rad}}$$

12. ~~**If a minute hand of a clock is 10cm long, how far does the trip of the hand move in 30 minutes?**~~

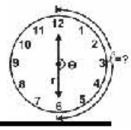
Sol. Here: r = 10cm, ℓ =?

θ = hand moves in 30 minutes

$$\theta = 180^\circ = 180 \times \frac{\pi}{180} = \pi \text{ rad}$$

By using formula: $\ell = r\theta$

$$\ell = r\theta = (10)\pi = \boxed{31.4\,\text{cm}}$$



13. **Prove that:** $\dfrac{2\tan\frac{\pi}{6}}{1-\tan^2\frac{\pi}{6}} = \sqrt{3}$

Sol. $$\text{L.H.S.} = \frac{2\tan\frac{\pi}{6}}{1-\tan^2\frac{\pi}{6}} = \frac{2\tan 30^\circ}{1-\tan^2 30^\circ}$$

$$= \frac{2\left(\frac{1}{\sqrt{3}}\right)}{1-\left(\frac{1}{\sqrt{3}}\right)^2} = \frac{\frac{2}{\sqrt{3}}}{1-\frac{1}{3}} = \frac{\frac{2}{\sqrt{3}}}{\frac{3-1}{3}} = \frac{\frac{2}{\sqrt{3}}}{\frac{2}{3}}$$

$$= \frac{2}{\sqrt{3}} \times \frac{3}{2} = 3^{1-\frac{1}{2}} = 3^{1/2} = \sqrt{3} = \text{R.H.S.}$$

Proved.

14. **Prove that:** $1-2\sin^2\theta = 2\cos^2\theta - 1$

Sol. $$\text{L.H.S.} = 1-2\sin^2\theta$$

$$= 1-2\left(1-\cos^2\theta\right)$$

$$= 1-2+2\cos^2\theta$$

$$= 2\cos^2\theta - 1 = \text{R.H.S.}$$ **Proved.**

15. **Prove that:** $\sin(-\theta) = -\sin\theta$

Sol. We know that

$$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$

Put $\alpha = 0$ & $\beta = \theta$ we have:

$$\sin(0-\theta) = \sin(0)\cos\theta - \cos(0)\sin\theta$$

$$\sin(-\theta) = 0.\cos\beta - 1.\sin\theta$$

$$\sin(-\theta) = 0 - \sin\theta$$

$$\boxed{\sin(-\theta) = -\sin\theta}$$ **Proved.**

16. **Express** $\sin x\cos 2x - \sin 2x\cos x$ **as single term.**

Sol. $$\sin x\cos 2x - \sin 2x\cos x$$

$$= \sin x\cos 2x - \cos x\sin 2x$$

$$= \sin(x-2x) \quad \because \left\{\begin{array}{l}\sin(\alpha-\beta)\\ =\sin\alpha\cos\beta-\cos\alpha\sin\beta\end{array}\right\}$$

$$= \sin(-x) = \boxed{-\sin x}$$

17. **Prove that:** $\cos^2\alpha = \dfrac{1+\cos 2\alpha}{2}$

Sol. Take $\cos 2\alpha = \cos(\alpha+\alpha)$

$$= \cos\alpha\cos\alpha - \sin\alpha\sin\alpha$$

$$= \cos^2\alpha - \sin^2\alpha$$

$$= \cos^2\alpha - \left(1-\cos^2\alpha\right)$$

$$= \cos^2\alpha - 1 + \cos^2\alpha = 2\cos^2\alpha - 1$$

$$\Rightarrow\ 2\cos^2\alpha = 1+\cos 2\alpha$$

$$\Rightarrow\ \cos^2\alpha = \frac{1+\cos 2\alpha}{2}$$ **Proved.**

18. ~~**Express**~~ $\sin(x+30^\circ)+\sin(x-30^\circ)$ ~~**as product.**~~

Sol. $$\sin(x+30^\circ)+\sin(x-30^\circ)$$

$$= 2\sin\left(\frac{x+30^\circ+x-30^\circ}{2}\right)\cos\left(\frac{x+30^\circ-x+30^\circ}{2}\right)$$

$$= 2\sin\left(\frac{2x}{2}\right)\cos\left(\frac{60^\circ}{2}\right) = \boxed{2\sin x\cos 30^\circ}$$

19. ~~**Given that, $\gamma = 90^\circ$, $\alpha = 35^\circ$, a = 5, find angle β.**~~

Sol. We know that in any triangle:

$$\alpha+\beta+\gamma = 180^\circ$$

$$\beta = 180^\circ - \alpha - \gamma$$

$$\beta = 180^\circ - 35^\circ - 90^\circ \Rightarrow \boxed{\beta = 55^\circ}$$

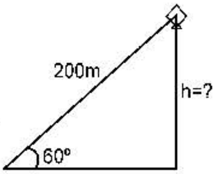
20. ~~**A string of a flying kite is 200 meters long, and its angle of elevation is 60º. Find the height of the kite above the ground taking the string to be fully stretched.**~~

Sol. In this figure:

$$\sin 60^\circ = \frac{h}{200}$$

$$200\sin 60^\circ = h$$

$$\boxed{h = 173.20\,m}$$



22. **In any triangle ABC in which b = 45, c = 34, ∝ = 52º, find a.**

Sol. By using Law of cosines:

$$a^2 = b^2 + c^2 - 2bc\cos 52^\circ$$

$$a^2 = (45)^2 + (34)^2 - 2(45)(34)\cos 52^\circ$$

$$a^2 = 2025 + 1156 - 1883.92$$

$$a^2 = 1297.07$$

$$\sqrt{a^2} = \sqrt{1297.07} \Rightarrow \boxed{a = 36.01}$$

22. **In any triangle ABC if a = 3, b = 7, β = 85º find α.**

Sol. By using law of sines:

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$$

$$\text{We take: } \frac{a}{\sin\alpha} = \frac{b}{\sin\beta}$$

$$\sin\alpha = \frac{a\sin\beta}{b} \Rightarrow \sin\alpha = \frac{3\sin 85^\circ}{7}$$

$$\sin\alpha = 0.4269$$

$$\alpha = \sin^{-1}(0.4269) \Rightarrow \boxed{\alpha = 25^\circ 16'}$$

23. **If $\cos\alpha,\ \cos\beta,\ \cos\gamma$ are direction cosines of a vector $\vec{r} = xi + yj + zk$, then show that $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$**

Sol. As, $\vec{r} = xi + yj + zk \Rightarrow |\vec{r}| = \sqrt{x^2 + y^2 + z^2}$

Direction cosines of $\vec{r}$ are:

$$\cos\alpha = \frac{x}{\sqrt{x^2+y^2+z^2}} \Rightarrow \cos^2\alpha = \frac{x^2}{x^2+y^2+z^2} \to (i)$$

$$\cos\beta = \frac{y}{\sqrt{x^2+y^2+z^2}} \Rightarrow \cos^2\beta = \frac{y^2}{x^2+y^2+z^2} \to (ii)$$

$$\cos\gamma = \frac{z}{\sqrt{x^2+y^2+z^2}} \Rightarrow \cos^2\gamma = \frac{y^2}{x^2+y^2+z^2} \to (iii)$$

Adding eq. (i), eq. (ii) & eq. (iii), we have:

$$L.H.S. = \cos^2\alpha + \cos^2\beta + \cos^2\gamma$$

$$= \frac{x^2}{x^2+y^2+z^2} + \frac{y^2}{x^2+y^2+z^2} + \frac{z^2}{x^2+y^2+z^2}$$

$$= \frac{x^2+y^2+z^2}{x^2+y^2+z^2} = 1 = R.H.S.$$ **Proved.**

24. **Find a vector whose magnitude is 2 and is parallel to 5i + 3j + 2k**

Sol. Let $\vec{a}$ be a require vector, so $|\vec{a}| = 2$

$$\& \text{ Let } \vec{b} = 5i + 3j + 2k$$

$$|\vec{b}| = \sqrt{(5)^2 + (3)^2 + (2)^2}$$

$$|\vec{b}| = \sqrt{25 + 9 + 4} = \sqrt{38}$$

As $\vec{a}$ and $\vec{b}$ are parallel vectors:

$$\text{So, } \hat{a} = \hat{b} \Rightarrow \frac{\vec{a}}{|\vec{a}|} = \frac{\vec{b}}{|\vec{b}|} \Rightarrow \frac{\vec{a}}{2} = \frac{5i+3j+2k}{\sqrt{38}}$$

$$\vec{a} = \frac{2(5i+3j+2k)}{\sqrt{38}} \Rightarrow \vec{a} = \boxed{\frac{10i+6j+4k}{\sqrt{38}}}$$

25. **Prove that $\vec{a}$ and $\vec{b}$ are perpendicular to each other if $\vec{a} = i + 3j - 2k \ \& \ \vec{b} = i - j - k$**

Sol.

$$\vec{a}.\vec{b} = (i + 3j - 2k).(i - j - k)$$

$$= (1)(1) + (3)(-1) + (-2)(-1)$$

$$= 1 - 3 + 2 = \boxed{0}$$

Hence $\vec{a}$ and $\vec{b}$ are perpendicular.

26. ~~**Express**~~ $\sqrt{3} + j$ ~~**in Polar form.**~~

Sol. Let $Z = \sqrt{3} + j$

Here: $a = \sqrt{3}$ & $b = 1$

$r = \sqrt{a^2 + b^2}$

$r = \sqrt{(\sqrt{3})^2 + (1)^2}$

$r = \sqrt{3+1} = \sqrt{4} = 2$

$\theta = \tan^{-1}\left(\frac{b}{a}\right)$

$\theta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = 30^\circ$

$z = r\angle\theta = \boxed{2\ \angle 30^\circ}$

27. ~~Simplify the phasor $\frac{1}{4-j5} - \frac{1}{5-j4}$ and write the result in Rectangular form.~~

Sol. $\frac{1}{4-j5} - \frac{1}{5-j4}$

$$= \frac{1(5-j4) - 1(4-j5)}{(4-j5)(5-j4)}$$

$$= \frac{5-j4-4+j5}{20-j16-j25+j^2 20}$$

$$= \frac{1+j}{20-j41-20} = \frac{1+j}{-j41} = \frac{1+j}{-j41} \times \frac{j41}{j41}$$

$$= \frac{j41+j^2 41}{-j^2 41^2} = \frac{j41-41}{1681} = \frac{-41+j41}{1681}$$

$$= \frac{41(-1+j)}{1681} = \frac{-1+j}{41} = \boxed{-\frac{1}{41} + j\frac{1}{41}}$$

Section - II

Note : Attemp any three (3) questions $3 \times 8 = 24$

Q.2.(a) Solve the equation

$$\frac{1}{x+1} + \frac{1}{x+2} + \frac{1}{x+3} = \frac{3}{x}$$

by using quadratic formula.

Sol. See Q.3(iii) of Ex# 1.1 (Page # 20)

(b) ~~If the difference of the roots of $x^2 - 7x + k - 4 = 0$ is 5, find the value of k and the roots.~~

Sol. See Q.3(i) of Ex# 1.3 (Page # 42)

Q.3.(a) Find the constant term in the expansion or $\left(x^2 - \frac{1}{x}\right)^9$

Sol. See Q.9(i) of Ex# 2.1 (Page # 95)

(b) If x is nearly equal to unity, prove that $\frac{mx^n - nx^m}{x^n - x^m} = \frac{1}{1-x}$

Sol. See Q.4 of Ex# 2.2 (Page # 104)

Q.4.(a) ~~A flywheel rotates at 300 rev/min. If the radius is 6cm, through what total distance does a point on the rim travel in 30 seconds?~~

Sol. See Q.11 of Ex# 3.1 (Page # 118)

(b) If $\cot\theta = 2/3$, and the terminal side of the angle does not lie in the first quadrant, find the remaining trigonometric ratios of θ.

Sol. See Q.7 of Ex# 3.2 (Page # 125)

Q.5. **(a)** ~~Prove that: $\cos 20^\circ \cos 40^\circ \cos 60^\circ \cos 80^\circ = \frac{1}{16}$~~

Sol. See Q.17 of Ex# 4.3 (Page # 196)

(b) A town B is 15 km due North of a town A. The road from A to B runs North 27º, East to G, then North 34º, West to B. Find the distance by road from town A to Be.

Sol. See example # 11 of Ch# 03

Q.6. **(a)** Show that the vectors $4\hat{i} - 6\hat{j} + 9\hat{k}$ and $-6\hat{i} + 9\hat{j} - \frac{27}{2}\hat{k}$ are parallel.

Sol. See Q.7 of Ex# 6.1 (Page # 249)

(b) Find $\left|(\vec{a} x \vec{b}) x \vec{c}\right|$ if $\vec{a} = i - 2j - 3k$, $\vec{b} = 2i + j - k$, $\vec{c} = i + 3j - 2k$.

Sol. See Q.18 of Ex# 6.2 (Page # 264)

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