DAE / IA - 2017

MATH - 123 APPLIED MATHEMATICS - I

PAPER 'A' PART - A (OBJECTIVE)

Time : 30 Minutes Marks : 15

Q.1: *Encircle the correct answer.*

1. $\ell x^2 + mx + n = 0$ will be a pure quadratic if:
[a] $\ell = 0$ [b] $n = 0$
[c] $m = 0$ [d] Both ℓ, $m = 0$

2. If the discriminant of $ax^2 + bx + c = 0$ is positive and perfect square then its roots will be:
[a] Imaginary [b] Rational
[c] Equal [d] Irrational

3. ~~Product of roots of~~ $2x^2 - 3x - 5 = 0$ ~~is:~~
[a] $-\frac{5}{2}$ [b] $\frac{5}{2}$
[c] $\frac{2}{5}$ [d] $-\frac{2}{5}$

4. Number of terms in the expansion of $(a+b)^{13}$ are:
[a] 12 [b] 13
[c] 14 [d] 15

5. $\binom{6}{4}$ will have the value:
[a] 10 [b] 15 [c] 20 [d] 25

6. $\frac{2\pi}{3}$ ~~radians are equal to:~~
[a] 60° [b] 90°
[c] 120° [d] 150°

7. ~~If an arc of a circle has length ℓ, and subtends an angle θ then its radius will be:~~
[a] $\frac{\theta}{\ell}$ [b] $=\frac{\ell}{\theta}$
[c] $\ell\theta$ [d] $\ell + \theta$

8. If $\sin\theta = \frac{\sqrt{3}}{2}$ and terminal ray of angle lies in 1st quadrant then cos θ is equal to:
[a] $\frac{1}{\sqrt{2}}$ [b] $-\frac{1}{2}$
[c] $\frac{1}{2}$ [d] $-\frac{1}{\sqrt{2}}$

9. $\sin(90^\circ - \theta)$ is equal to:
[a] $\cos\theta$ [b] $-\cos\theta$
[c] $-\sin\theta$ [d] $\sin\theta$

11. $\sin 2\alpha$ is equal to:
[a] $\cos^2\alpha - \sin^2\alpha$
[b] $\cos 2\alpha$
[c] $1 - \cos^2\alpha$ [d] $2\sin\alpha\cos\alpha$

11. $\cos(\alpha+\beta) - \cos(\alpha-\beta)$ is equal to:
[a] $2\sin\alpha\cos\beta$ [b] $2\cos\alpha\sin\beta$
[c] $2\cos\alpha\cos\beta$ [d] $-25\sin\alpha\sin\beta$

12. In a right triangle if one angle is 45º then the other will be:
[a] 45º [b] 50º
[c] 60º [d] 75º

13. Magnitude of the vector $i - 3j + 5k$ is:
[a] 3 [b] 25 [c] 35 [d] $\sqrt{35}$

14. If i, j and are orthogonal unit vectors then i × j is equal is:
[a] k [b] –k [c] 1 [d] -1

15. If $\underline{a} \times \underline{b} = 0$ implies that $\underline{a}$ and $\underline{b}$ are:
[a] Unparallel [b] Parallel
[c] Perpendicular [d] Oblique

Answer Key

1	b	2	b	3	a	4	c	5	b
6	c	7	b	8	c	9	a	10	c
11	c	12	a	13	d	14	a	15	b

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DAE / IA - 2017

MATH - 123 APPLIED MATHEMATICS - I

PAPER 'A' PART - B (SUBJECTIVE)

Time : 2 : 30 Hrs Marks : 60

Section - I

Q.1. Write short answers to any Eighteen (18) questions.

1. Solve the equation by factorization: $3x^2 + 5x = 2$

Sol. $3x^2 + 5x = 2$

$3x^2 + 5x - 2 = 0$

$3x^2 + 6x - x - 2 = 0$

$3x(x+2) - 1(x+2) = 0$

$(x+2)(3x-1) = 0$

Either OR

$x + 2 = 0 \quad | \quad 3x - 1 = 0$

$x = -2 \quad | \quad 3x = 1 \Rightarrow x = \frac{1}{3}$

$$S.S. = \boxed{\left\{-2, \frac{1}{3}\right\}}$$

2. Discuss the nature of roots of $x^2 + x + 1 = 0$

Sol. Here : $a = 1,\ b = 1,\ c = 1$

$\text{Disc.} = b^2 - 4ac$

$= (1)^2 - 4(1)(1) = 1 - 4 = -3$

$\therefore$ The roots are **Imaginary.**

3. ~~For what value of k, the sum of roots of~~ $3x^2 + kx + 5 = 0$ ~~may be equal to the product of roots.~~

Sol. Here : $a = 3,\ b = k,\ c = 5$

As, Sum of roots = Product of roots

$$\frac{-b}{a} = \frac{c}{a}$$

$$\frac{-k}{3} = \frac{5}{3}$$

$$-k = 5 \Rightarrow \boxed{k = -5}$$

4. ~~From the quadratic equation whose roots are~~ $3\sqrt{5}$ and $-3\sqrt{5}$

Sol.

$S = 3\sqrt{5} + (-3\sqrt{5})$

$S = 3\sqrt{5} - 3\sqrt{5}$

$S = 0$

$P = (3\sqrt{5})(-3\sqrt{5})$

$P = -9(\sqrt{5})^2 = -45$

$$x^2 - Sx + P = 0$$

$$x^2 - (0)x + (-45) = 0 \Rightarrow \boxed{x^2 - 45 = 0}$$

5. ~~Find the sum and product of roots of~~ $x^2 - 9 = 0$.

Sol. Here : $a = 1,\ b = 0,\ c = -9$

Sum of the Roots $= S = -\frac{b}{a} = -\frac{0}{1} = \boxed{0}$

Product of the Roots $= P = \frac{c}{a} = \frac{-9}{1} = \boxed{-9}$

6. Expand the expression $\left(\frac{x}{y} + \frac{y}{x}\right)^4$

Sol. $\left(\frac{x}{y} + \frac{y}{x}\right)^4$

$$= \binom{4}{0}\left(\frac{x}{y}\right)^4\left(\frac{y}{x}\right)^0 + \binom{4}{1}\left(\frac{x}{y}\right)^3\left(\frac{y}{x}\right)^1 + \binom{4}{2}\left(\frac{x}{y}\right)^2\left(\frac{y}{x}\right)^2$$

$$+ \binom{4}{3}\left(\frac{x}{y}\right)^1\left(\frac{y}{x}\right)^3 + \binom{4}{4}\left(\frac{x}{y}\right)^0\left(\frac{y}{x}\right)^4$$

$$= (1)\left(\frac{x^4}{y^4}\right)(1) + 4\left(\frac{x^3}{y^3}\right)\left(\frac{y}{x}\right) + 6\left(\frac{x^2}{y^2}\right)\left(\frac{y^2}{x^2}\right)$$

$$+ 4\left(\frac{x}{y}\right)\left(\frac{y^3}{x^3}\right) + 1(1)\left(\frac{y^4}{x^4}\right)$$

$$= \boxed{\frac{x^4}{y^4} + 4\frac{x^2}{y^2} + 6 + 4\frac{y^2}{x^2} + \frac{y^4}{x^4}}$$

7. **Find the 7th term in the expansion of** $\left(\mathbf{x}-\dfrac{\mathbf{1}}{\mathbf{x}}\right)^{9}$

Sol. Here: $a = x,\ b = -\dfrac{1}{x},\ n = 9\ \&\ r = 6$

Using general term formula:

$$T_{r+1} = \binom{n}{r} a^{n-r} b^{r}$$

$$T_{6+1} = \binom{9}{6}(x)^{9-6}\left(-\frac{1}{x}\right)^{6}$$

$$T_7 = 84x^3\left(\frac{1}{x^6}\right) = \frac{84}{x^3}$$

8. ~~**Expand** $(1+2x)^{-2}$ **to three terms.**~~

Sol. $(1+2x)^{-2}$

$$= 1 + (-2)(2x) + \frac{(-2)(-2-1)}{2!}(2x)^2 + \ldots$$

$$= 1 - 4x + \frac{(-2)(-3)}{2}(4x^2) + \ldots$$

$$= \boxed{1 - 4x + 12x^2 + \ldots}$$

9. **Which will be the middle term/terms in the expansion of** $\left(\mathbf{x}+\dfrac{\mathbf{3}}{\mathbf{x}}\right)^{15}$

Sol. As $n = 15$ (Odd), so

Middle terms $= \left(\dfrac{n+1}{2}\right)^{th} + \left(\dfrac{n+3}{2}\right)^{th}$ terms.

Middle terms $= \left(\dfrac{15+1}{2}\right)^{th} + \left(\dfrac{15+3}{2}\right)^{th}$ terms.

Middle terms $= 8^{th} + 9^{th}$ terms.

$\boxed{\text{Hence } T_8\ \&\ T_9 \text{ are two middle terms.}}$

10. ~~**Convert**~~ $42^\circ\, 36'\, 12''$ ~~**into radian measure.**~~

Sol. $42^\circ\, 36'\, 12''$

$$= \left(42 + \frac{36}{60} + \frac{12}{3600}\right)^\circ$$

$$= (42 + 0.6 + 0.0033)^\circ$$

$$= 42.6033^\circ$$

$$= 42.6033 \times \frac{\pi}{180} = \boxed{0.74\text{ rad}}$$

11. ~~**Find the length of arc cut off on a circle of radius 3cm by a central angle of 2 radians.**~~

Sol. Here $\ell = ?$, $r = 3\text{cm}$, $\theta = 2$ rad

By using formula: $\ell = r\theta$

$\ell = r\theta = (3)(2) = \boxed{6\text{cm}}$

12. **Prove that:**

$$\mathbf{\cos 30^\circ \cos 60^\circ - \sin 30^\circ \sin 60^\circ = 0}$$

Sol. $\text{L.H.S.} = \cos 30^\circ \cos 60^\circ - \sin 30^\circ \sin 60^\circ$

$$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$$

$$= \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4}$$

$= 0 = \text{R.H.S.}$ **Proved.**

13. **If** $\sin\theta = 3/8$ **and terminal side of the angle lies in second quadrant find the remaining trigonometric ratios of θ.**

Sol. As, $\sin\theta = \dfrac{3}{8}$ then $\boxed{\operatorname{cosec}\theta = \dfrac{8}{3}}$

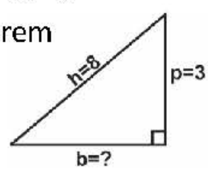
As, $\sin\theta = \dfrac{\text{perp}}{\text{hyp}} = \dfrac{3}{8}$

so, $p = 3\ \&\ h = 8$

By Pythagoras theorem

$b^2 + p^2 = h^2$

$b^2 + (3)^2 = (8)^2$



$$b^2 = 64 - 9$$

$$b^2 = 55 \Rightarrow \quad b = \sqrt{55}$$

As θ lies in the II-Quad.

$\text{As}, \cos\theta = -\dfrac{b}{h}$	$\text{As}, \tan\theta = -\dfrac{p}{b}$
$\Rightarrow \boxed{\cos\theta = -\dfrac{8}{\sqrt{55}}}$	$\Rightarrow \boxed{\tan\theta = -\dfrac{3}{\sqrt{55}}}$
$\Rightarrow \boxed{\sec\theta = -\dfrac{8}{\sqrt{55}}}$	$\Rightarrow \boxed{\cot\theta = -\dfrac{\sqrt{55}}{3}}$

14. Prove that:

$$\tan(45° + \theta)\tan(45° - \theta) = 1$$

Sol. $\text{L.H.S.} = \tan(45° + \theta)\tan(45° - \theta)$

$$= \frac{\tan 45° + \tan\theta}{1 - \tan 45° \tan\theta} \times \frac{\tan 45° - \tan\theta}{1 + \tan 45° \tan\theta}$$

$$= \frac{1 + \tan\theta}{1 - (1)\tan\theta} \times \frac{1 - \tan\theta}{1 + (1)\tan\theta} \quad \because \left\{\begin{matrix}\text{Using calculator} \\ \tan 45° = 1\end{matrix}\right\}$$

$$= \frac{1 + \tan\theta}{1 - \tan\theta} \times \frac{1 - \tan\theta}{1 + \tan\theta} = 1 = \text{R.H.S.}$$ **Proved.**

16. Express $\sin x \cos 2x - \sin 2x \cos x$ as single term.

Sol. $\sin x \cos 2x - \sin 2x \cos x$

$$= \sin x \cos 2x - \cos x \sin 2x$$

$$= \sin(x - 2x) \quad \because \left\{\begin{matrix}\sin(\alpha - \beta) \\ = \sin\alpha\cos\beta - \cos\alpha\sin\beta\end{matrix}\right\}$$

$$= \sin(-x) = \boxed{-\sin x}$$

16. ~~If~~ **cos θ =** $\dfrac{4}{5}$ ~~and terminal ray of~~ **θ** ~~is in the first quadrant find cos~~ $\dfrac{\theta}{2}$

Sol. As, $\cos\dfrac{\theta}{2} = \sqrt{\dfrac{1 + \cos\theta}{2}}$

$$= \sqrt{\frac{1 + \frac{4}{5}}{2}} = \sqrt{\frac{\frac{5+4}{5}}{2}} = \sqrt{\frac{9}{10}} = \boxed{\frac{3}{\sqrt{10}}}$$

17. ~~Express~~ **sin 5θ – sin θ** ~~as product.~~

Sol. $\sin 5\theta - \sin\theta$

$$= 2\cos\left(\frac{5\theta + \theta}{2}\right)\sin\left(\frac{5\theta - \theta}{2}\right)$$

$$= 2\cos\left(\frac{6\theta}{2}\right)\sin\left(\frac{4\theta}{2}\right)$$

$$= \boxed{2\cos 3\theta \sin 2\theta}$$

18. Define law of cosines.

Sol.

i. $a^2 = b^2 + c^2 - 2bc\cos\alpha$

ii. $b^2 = c^2 + a^2 - 2ca\cos\beta$

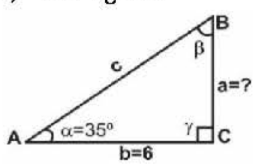
iii. $c^2 = a^2 + b^2 - 2ab\cos\gamma$

19. ~~In right triangle ABC, b = 6, α = 35°, γ = 90°, Find side 'a'.~~

Sol. We know that, from figure:

$$\tan\alpha = \frac{a}{b}$$

$$\tan 35° = \frac{a}{6}$$



$$6\tan 35° = a \Rightarrow \boxed{a = 4.2}$$

20. The sides of a triangle are 16, 20 and 33 meters respectively. Find its greatest angle.

Sol. Let a = 16, b = 20, c = 33

As side 'c' is greatest so, we will find angle 'γ'.

Using law of cosines:

$$\cos\gamma = \frac{a^2 + b^2 - c^2}{2ab}$$

$$\cos\gamma = \frac{(16)^2 + (20)^2 - (33)^2}{2(16)(20)}$$

$$\cos\gamma = -0.6765$$

$$\gamma = \cos^{-1}(-0.6765) \Rightarrow \boxed{\gamma = 132° 34'}$$

22. **In any triangle ABC in which b = 45, c = 34, ∝ = 52º, find a.**

Sol. By using Law of cosines:

$$a^2 = b^2 + c^2 - 2bc\cos 52^\circ$$

$$a^2 = (45)^2 + (34)^2 - 2(45)(34)\cos 52^\circ$$

$$a^2 = 2025 + 1156 - 1883.92$$

$$a^2 = 1297.07$$

$$\sqrt{a^2} = \sqrt{1297.07}$$

$$\boxed{a = 36.01}$$

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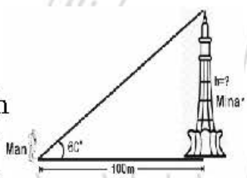
22. ~~**A minaret stands on a horizontal ground. A man on the ground 100m from the minaret, the angle of elevation of the top of the minaret to be 60º. Find its height.**~~

Sol. From figure, we know that:

$$\tan 60^\circ = \frac{h}{100}$$

$$100\tan 60^\circ = h$$

$$\boxed{h = 173.20\text{m}}$$



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24. **Find unit vector along the vector $\mathbf{4i - 3j - 5k}$**

Sol. Let $\vec{a} = 4i - 3j - 5k$

$$|\vec{a}| = \sqrt{(4)^2 + (-3)^2 + (-5)^2}$$

$$|\vec{a}| = \sqrt{16 + 9 + 25} = \sqrt{50}$$

$$|\vec{a}| = \sqrt{25 \times 2} = 5\sqrt{2}$$

$$\text{Unit Vector} = \hat{a} = \frac{\vec{a}}{|\vec{a}|} = \boxed{\frac{4i - 3j - 5k}{5\sqrt{2}}}$$

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24. **If $\underline{a}$ = 2i + 3j + 4k and $\underline{b}$ = i – j + k Find the magnitude of 3a – b.**

Sol. $3a - b = 3(2i + 3j + 4k) - (i - j + k)$

$$= 6i + 9j + 12k - i + j - k$$

$$= 5i + 10j + 11k$$

$$\therefore |3a - b| = \sqrt{(5)^2 + (10)^2 + (11)^2}$$

$$= \sqrt{25 + 100 + 121} = \sqrt{246}$$

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24. **Given the vectors $\vec{a} = 3i + j - k$ and $\vec{b} = 2i + j - k$ = 2i + j – k, find magnitude of $3\vec{a} - \vec{b}$.**

Sol. $3\vec{a} - \vec{b} = 3(3i + j - k) - (2i + j - k)$

$$3\vec{a} - \vec{b} = 9i + 3j - 3k - 2i - j + k$$

$$3\vec{a} - \vec{b} = 7i + 2j - 2k$$

$$|3\vec{a} - \vec{b}| = \sqrt{(7)^2 + (2)^2 + (-2)^2}$$

$$= \sqrt{49 + 4 + 4} = \boxed{\sqrt{57}}$$

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25. **Find $(\vec{a} + \vec{b}).(\vec{a} - \vec{b})$ if $\vec{a} = 2i + 2j + 3k$ & $\vec{b} = 2i - j + k$**

Sol.

$$\vec{a} + \vec{b} = (2i + 2j + 3k) + (2i - j + k)$$

$$\vec{a} + \vec{b} = 2i + 2j + 3k + 2i - j + k$$

$$\vec{a} + \vec{b} = 4i + j + 4k$$

$$\vec{a} - \vec{b} = (2i + 2j + 3k) - (2i - j + k)$$

$$\vec{a} - \vec{b} = 2i + 2j + 3k - 2i + j - k$$

$$\vec{a} - \vec{b} = 3j + 2k$$

$$(\vec{a} + \vec{b}) \bullet (\vec{a} - \vec{b})$$

$$= (4i + j + 4k) \bullet (3j + 2k)$$

$$= (4)(0) + (1)(3) + (4)(2)$$

$$= 0 + 3 + 8 = \boxed{11}$$

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26. **Find a vector whose magnitude is 2 and is parallel to 5i + 3j + 2k**

Sol. Let $\vec{a}$ be a require vector, so $|\vec{a}| = 2$

& Let $\vec{b} = 5i + 3j + 2k$

$$|\vec{b}| = \sqrt{(5)^2 + (3)^2 + (2)^2}$$

$$|\vec{b}| = \sqrt{25 + 9 + 4} = \sqrt{38}$$

As $\vec{a}$ and $\vec{b}$ are parallel vectors:

So, $\hat{a} = \hat{b} \Rightarrow \frac{\vec{a}}{|\vec{a}|} = \frac{\vec{b}}{|\vec{b}|}$

$$\frac{\vec{a}}{2} = \frac{5i + 3j + 2k}{\sqrt{38}}$$

$$\vec{a} = \frac{2(5i + 3j + 2k)}{\sqrt{38}}$$

$$\vec{a} = \boxed{\frac{10i + 6j + 4k}{\sqrt{38}}}$$

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27. ~~Simplify the phasor~~ $\frac{1}{4 - j5} - \frac{1}{5 - j4}$ ~~and write the result in Rectangular form.~~

Sol. $\frac{1}{4 - j5} - \frac{1}{5 - j4}$

$$= \frac{1(5 - j4) - 1(4 - j5)}{(4 - j5)(5 - j4)}$$

$$= \frac{5 - j4 - 4 + j5}{20 - j16 - j25 + j^2 20}$$

$$= \frac{1 + j}{20 - j41 - 20}$$

$$= \frac{1 + j}{-j41}$$

$$= \frac{1 + j}{-j41} \times \frac{j41}{j41}$$

$$= \frac{j41 + j^2 41}{-j^2 41^2}$$

$$= \frac{j41 - 41}{1681}$$

$$= \frac{-41 + j41}{1681}$$

$$= \frac{41(-1 + j)}{1681}$$

$$= \frac{-1 + j}{41} = \boxed{-\frac{1}{41} + j\frac{1}{41}}$$

Section - II

Note : Attemp any three (3) questions $3 \times 8 = 24$

Q.2. Solve by using quadratic formula. $mx^2 + (1 + m)x + 1 = 0$

Sol. See Q.3(vi) of Ex# 1.1 (Page # 22)

Q.3. Find the term independent of x in the expansion of $\left(2x^2 + \frac{1}{x}\right)^9$

Sol. See Q.10(ii) of Ex# 2.1 (Page # 97)

Q.4. Prove that

$$(\operatorname{cosec}\theta - \cot\theta)^2 = \frac{1 - \cos\theta}{1 + \cos\theta}$$

Sol. See Q.6 of Ex# 3.3 (Page # 133)

Q.5. If $\cos A = \frac{1}{5}$ and $\cos B = \frac{1}{2}$, A and B be acute angles, find $\cos(A - B)$

Sol. See Q.6 of Ex# 3.3 (Page # 133)

Q.6. If $\underline{a}$ = 2i – j + k and $\underline{b}$ = 3i + 4j – k Find sine of the angle between $\underline{a}$ and $\underline{b}$, and unit vector perpendicular to each.

Sol. See Q.19(ii) of Ex# 6.2 (Page # 265)

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