DAE / IA - 2017

MATH-113 APPLIED MATHEMATICS-I

PAPER 'B' PART - A (OBJECTIVE)

Time: 30 Minutes Marks: 15

Q.1: *Encircle the correct answer.*

1. If a = 2cm, b = 3cm, c = 5cm, are sides of a triangle then perimeter of triangle is:
[a] 8cm [b] 6cm
[c] 10cm [d] 30cm

2. Area of a square having a side 4cm is equal to:
[a] 30sq.cm [b] 16sq.cm
[c] 8sq.cm [d] 20sq.cm

3. A seven sided figure is called:
[a] square [b] octagon
[c] heptagon [d] pentagon

4. If 2a and 2b are the major and minor axis of the ellipse, then area of ellipse is:
[a] ab [b] πab
[c] $\frac{\pi}{2}ab$ [d] $\pi^2 ab$

5. In Simpson's rule, the number of ordinates are:
[a] Odd [b] Even
[c] In fraction [d] None of these

6. The cube is a right prism with:
[a] Square base
[b] Rectangular base
[c] Triangular base
[d] None of these

7. Lateral surface area of right circular cylinder is:
[a] πr^2 [b] πrh
[c] $2\pi rh$ [d] $\pi r^2 h$

8. ~~Each of the side of frustum of the pyramid is a:~~
[a] Triangle [b] Rectangle
[c] Trapezium [d] Square

9. The surface area of a sphere of radius 'r' is:
[a] $4\pi r^3$ [b] $4\pi r^2$
[c] πr^2 [d] $\frac{4}{3}\pi r^2$

10. $\vec{a}.\vec{b}$ is a;
[a] Vector quantity
[b] Scalar quantity
[c] Unity [d] None of these

11. If $\vec{a}\times\vec{b}$ then $\vec{a}$ and $\vec{b}$ are:
[a] Parallel [b] None parallel
[c] Perpendicular
[d] None of these

12. Magnitude of the vector $\underline{i} - 3\underline{j} + 5\underline{k}$ is:
[a] 3 **[b]** 25
[c] 35 **[d]** $\sqrt{35}$

13. The value of determinant $\begin{vmatrix} 2 & 0 \\ 1 & 3 \end{vmatrix}$ is:
[a] 6 [b] –6
[c] 1 [d] 0

14. The order of the matrix $\begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$ is:
[a] 3×1 [b] 1×3
[c] 3×3 [d] 2×3

15. ~~If all the elements of a row or a column are zero, then its value is:~~
[a] 1 [b] 2
[c] zero [d] 4

Answer Key

1	c	2	b	3	c	4	b	5	a
6	a	7	c	8	c	9	b	10	b
11	a	12	d	13	a	14	b	15	c

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DAE / IA - 2017

MATH - 113 APPLIED MATHEMATICS - I

PAPER 'B' PART - B (SUBJECTIVE)

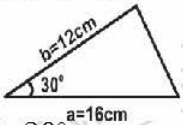
Time : 2 : 30 Hrs Marks : 60

Section - I

Q.1. Write short answers to any Eighteen (18) questions.

1. **Find the area of a triangle whose two adjacent sides are 16cm and 12cm and their included angle is 30º.**

Sol. Let, $a = 16\text{cm}, b = 12\text{cm} \ \& \ \theta = 30^\circ$



$$\text{Area} = \frac{1}{2}ab\sin\theta$$

$$\text{Area} = \frac{1}{2}(16)(12)\sin 30^\circ$$

$$\text{Area} = \boxed{48 \text{ sq.cm}}$$

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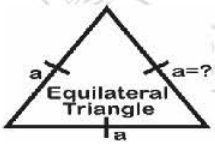
2. **What is the side of the equilateral triangle whose area is $9\sqrt{3}$ sq.cm.**

Sol. Let 'a' be length of each side of an equilateral triangle.

As, Area of equilateral triangle $= 9\sqrt{3}$ sq.cm



$$\Rightarrow \quad \frac{\sqrt{3}}{4}a^2 = 9\sqrt{3}$$

$$\Rightarrow \quad a^2 = 9\sqrt{3}\left(\frac{4}{\sqrt{3}}\right)$$

$$\Rightarrow \ a^2 = 36 \Rightarrow \sqrt{a^2} = \sqrt{36} \Rightarrow \boxed{a = 6\text{ cm}}$$

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3. **Find the area of trapezoid whose parallel sides are 20cm and 30cm and perpendicular distance between them is 4cm.**

Sol. Area of Trapezoid

$$= \frac{\text{Sum of parallel sides}}{2} \times \text{height}$$

$$= \frac{20+30}{2} \times 4 = \boxed{100 \text{ sq.cm}}$$

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4. **Define a cyclic quadrilateral and write its area.**

Sol. A quadrilateral inscribed in a circle is known as a cyclic quadrilateral.

$$\text{Area} = \sqrt{(s-a)(s-b)(s-c)(s-d)}$$

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5. **Define circumscribed polygon.**

Sol. If a polygon is drawn outside a circle, so that every side of the polygon touches the circle, then this polygon is called circumscribed polygon.

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6. **Find the interior angle of hexagon.**

Sol. Here n = 6

$$\text{Interior angle} = \theta = \frac{2n-4}{n} \times 90^\circ$$

$$\theta = \frac{2(6)-4}{6} \times 90^\circ = \frac{8}{6} \times 90^\circ = \boxed{120^\circ}$$

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7. **Find the radius of a circle the area of which is 9.3129 sq.cm.**

Sol. As, Area of circle = 9.3129 sq.cm

$$\pi r^2 = 9.3129 \ \Rightarrow \ r^2 = \frac{9.3129}{\pi}$$

$$r^2 = 2.96 \ \Rightarrow \ \boxed{r = 1.72\text{ cm}}$$

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8. **Write the area of the segment in terms of height and length of the chord of the segment.**

Sol. $\text{Area of segment} = \frac{h}{6c}\left(3h^2 + 4c^2\right)$

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9. **If base of a field is 50m and number of ordinates are 11, then find breadth of strip.**

Sol. Length of base = 50 m

As, No. of ordinates = 11

so, No. of strips = 10

S = Width of each strip

$$S = \frac{\text{Length of base}}{\text{No. of strips}} = \frac{50}{10} = \boxed{5\text{m}}$$

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10. **The volume of the cube is 95 cu.cm. Find the surface area and the edge of the cube.**

Sol. Let 'a' be edge of cube

As, volume = 95

$a^3 = 95$

$(a^3)^{1/3} = (95)^{1/3}$

Edge of cube $= a = \boxed{4.56\,cm}$

Surface area of cube $= 6a^2$

$S.A. = 6(4.56)^2 = \boxed{124.92\,cm^2}$

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11. **Find the height of the cylinder if volume is 528cm³ and diameter is 4cm.**

Sol. Here : $h = ?\ V = 528cm^3$ & $d = 4cm$

$As,\ r = \frac{d}{2} = \frac{4}{2} = 2cm$

As, Volume of Cylinder $= 528cm^3$

$\Rightarrow \quad \pi r^2 h = 528$

$h = \frac{528}{\pi(2)^2} \Rightarrow \boxed{h = 42\,cm}$

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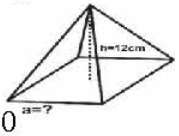
12. **The height of pyramid with square base is 12cm, and its volume is 100cu.cm. Find length of side of square base.**

Sol. $Here: h = 12cm, V = 100cm^3$ & $a = ?$

As, Volume of Pyramid $= 100cm^3$

$\frac{1}{3} \times$ Area of base $\times$ Height $= 100$

$\frac{1}{3} \times a^2 \times 12 = 100$



$4a^2 = 100 \Rightarrow a^2 = \frac{100}{4}$

$a^2 = 25 \Rightarrow \boxed{a = 5cm}$

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13. **The circumference of base of a 9m high comical tent is 44m, find the volume of the air contained in it.**

Sol. Here: C = 44 m, h = 9m, & V=?

As, circumference of base $= C = 44m$

$2\pi r = 44$

$r = \frac{44}{2\pi} \quad \Rightarrow \quad r = 7m$

Volume of Air $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi(7)^2(9) = \boxed{461.81\,m^3}$

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14. **Write formula for volume of a cone.**

Sol. Volume of cone $= V = \boxed{\frac{1}{3}\pi r^2 h \text{ cu.unit}}$

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15. **A solid cylinder of glass the radius of whose base is 9cm and height 12cm is melted and turned into a sphere. Find the radius of the sphere so formed.**

Sol. Let, r = Radius of the sphere = 9cm,

h = Height of cylinder = 12cm,

r_1 = Radius sphere = ?

Volume of Cylinder $= \pi r^2 h$

$V = \pi(9)^2 12 = 972\pi\,cm^3$

As, Volume of Sphere = Volume of Cylinder

$\frac{4}{3}\pi r_1^3 = 927\pi$

$r_1^3 = \frac{927\pi \times 3}{4\pi} = 729 \Rightarrow \boxed{r_1 = 9\,cm}$

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16. **Find the volume of a segment of a sphere whose height is $4\frac{1}{2}$ cm and diameter for whose base is 8cm.**

Sol. $Here: \ h = 4\frac{1}{2} = \frac{9}{2} = 4.5cm,$

$\& \quad d = 8cm \Rightarrow \ r = 4cm$

Volume of segment of shpere $= \frac{\pi h}{6}\left[h^2 + 3r^2\right]$

$V = \frac{\pi(4.5)}{6}\left[(4.5)^2 + 3(4)^2\right] = \boxed{160.81\,cm^3}$

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17. **The base of a right prism is an equilateral triangle with a side of 4cm and its height is 25cm, find its volume.**

Sol. Here: $a = 4\text{cm}, h = 25\text{cm}$ & $V = ?$

Area of base (equilateral triangle)

$$= \frac{\sqrt{3}}{4}a^2 = \frac{\sqrt{3}}{4}(4)^2 = 6.92\text{cm}^2$$

Volume = Area of base × height

$$V = 6.92 \times 25 = \boxed{173.2\text{cm}^3}$$

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18. **Find unit vector along the vector $4\mathbf{i} - 3\mathbf{j} - 5\mathbf{k}$**

Sol. Let $\vec{a} = 4i - 3j - 5k$

$$|\vec{a}| = \sqrt{(4)^2 + (-3)^2 + (-5)^2}$$

$$|\vec{a}| = \sqrt{16 + 9 + 25} = \sqrt{50}$$

$$|\vec{a}| = \sqrt{25 \times 2} = 5\sqrt{2}$$

$$\text{Unit Vector} = \hat{a} = \frac{\vec{a}}{|\vec{a}|} = \boxed{\frac{4i - 3j - 5k}{5\sqrt{2}}}$$

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19. **Find $(\vec{a} + \vec{b}).(\vec{a} - \vec{b})$ if $\vec{a} = 2\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$ & $\vec{b} = 2\mathbf{i} - \mathbf{j} + \mathbf{k}$**

Sol.

$$\vec{a} + \vec{b} = (2i + 2j + 3k) + (2i - j + k)$$

$$\vec{a} + \vec{b} = 2i + 2j + 3k + 2i - j + k$$

$$\vec{a} + \vec{b} = 4i + j + 4k$$

$$\vec{a} - \vec{b} = (2i + 2j + 3k) - (2i - j + k)$$

$$\vec{a} - \vec{b} = 2i + 2j + 3k - 2i + j - k$$

$$\vec{a} - \vec{b} = 3j + 2k$$

$$(\vec{a} + \vec{b}) \bullet (\vec{a} - \vec{b})$$

$$= (4i + j + 4k) \bullet (3j + 2k)$$

$$= (4)(0) + (1)(3) + (4)(2)$$

$$= 0 + 3 + 8 = \boxed{11}$$

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24. **If $\vec{a} = 2\mathbf{i} + 3\mathbf{j} + 4\mathbf{k}$ & $\vec{b} = \mathbf{i} - \mathbf{j} + \mathbf{k}$**

Find $|\vec{a} \times \vec{b}|$

Sol.

$$\vec{a} \times \vec{b} = \begin{vmatrix} i & j & k \\ 2 & 3 & 4 \\ 1 & -1 & 1 \end{vmatrix}$$

$$= i\begin{vmatrix} 3 & 4 \\ -1 & 1 \end{vmatrix} - j\begin{vmatrix} 2 & 4 \\ 1 & 1 \end{vmatrix} + k\begin{vmatrix} 2 & 3 \\ 1 & -1 \end{vmatrix}$$

$$= i(3 + 4) - j(2 - 4) + k(-2 - 3)$$

$$= i(7) - j(-2) + k(-5)$$

$$= 7i + 2j - 5k$$

$$|\vec{a} \times \vec{b}| = \sqrt{49 + 4 + 25} = \boxed{\sqrt{78}}$$

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21. **Prove that $\vec{a}$ and $\vec{b}$ are perpendicular to each other if $\vec{a} = \mathbf{i} + 3\mathbf{j} - 2\mathbf{k}$ & $\vec{b} = \mathbf{i} - \mathbf{j} - \mathbf{k}$**

Sol.

$$\vec{a}.\vec{b} = (i + 3j - 2k).(i - j - k)$$

$$= (1)(1) + (3)(-1) + (-2)(-1)$$

$$= 1 - 3 + 2 = \boxed{0}$$

Hence $\vec{a}$ and $\vec{b}$ are perpendicular.

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22. **Find 'α' so that $|\alpha i + (\alpha + 1)j + 2k| = 3$**

Sol.

$$|\alpha i + (\alpha + 1)j + 2k| = 3$$

$$\sqrt{(\alpha)^2 + (\alpha + 1)^2 + (2)^2} = 3$$

$$\sqrt{\alpha^2 + \alpha^2 + 2\alpha + 1 + 4} = 3$$

$$\sqrt{2\alpha^2 + 2\alpha + 5} = 3$$

Squaring both sides, we get:

$$2\alpha^2 + 2\alpha + 5 = 9$$

$$2\alpha^2 + 2\alpha + 5 - 9 = 0$$

$$2\alpha^2 + 2\alpha - 4 = 0$$

$$2\alpha^2 + 4\alpha - 2\alpha - 4 = 0 \left\{ \text{By Factorization} \right\}$$

$$2\alpha(\alpha + 2) - 2(\alpha + 2) = 0$$

$$(\alpha + 2)(2\alpha - 2) = 0$$

Either **OR**

$\alpha + 2 = 0$ | $2\alpha - 2 = 0$

$\boxed{\alpha = -2}$ | $2\alpha = 2 \Rightarrow \boxed{\alpha = 1}$

23. **Define row and column vectors.**

Sol. A matrix has only one row is called **row matrix**.

A matrix has only one column is called **column matrix.**

24. **Find x and y if** $\begin{bmatrix} 2 & 1 \\ -3 & 2 \end{bmatrix} = \begin{bmatrix} x+3 & 1 \\ -3 & 3y-4 \end{bmatrix}$

Sol. $\begin{bmatrix} 2 & 1 \\ -3 & 2 \end{bmatrix} = \begin{bmatrix} x+3 & 1 \\ -3 & 3y-4 \end{bmatrix}$

Comparing corresponding elements of both matrices :

$x + 3 = 2$ | $3y - 4 = 2$

$x = 2 - 3$ | $3y = 2 + 4$

$\boxed{x = -1}$ | $3y = 6$

$y = \frac{6}{3}$

$\boxed{y = 2}$

25. **Find** A^{-1} **if** $A = \begin{bmatrix} 5 & 3 \\ 1 & 1 \end{bmatrix}$

Sol. $A = \begin{bmatrix} 5 & 3 \\ 1 & 1 \end{bmatrix}$

$|A| = \begin{vmatrix} 5 & 3 \\ 1 & 1 \end{vmatrix} = 5 - 3 = 2$

$\text{Adj } A = \begin{bmatrix} 1 & -3 \\ -1 & 5 \end{bmatrix}$

$A^{-1} = \frac{\text{Adj}(A)}{|A|}$

$A^{-1} = \frac{\begin{bmatrix} 1 & -3 \\ -1 & 5 \end{bmatrix}}{2}$

$A^{-1} = \begin{bmatrix} \frac{1}{2} & -\frac{3}{2} \\ -\frac{1}{2} & \frac{5}{2} \end{bmatrix}$

26. **Find 'k' if** $\begin{vmatrix} k-2 & 1 \\ 5 & k+2 \end{vmatrix} = 0$

Sol. $\begin{vmatrix} k-2 & 1 \\ 5 & k+2 \end{vmatrix} = 0$

$\begin{vmatrix} k-2 & 1 \\ 5 & k+2 \end{vmatrix} = 0$

$(k-2)(k+2) - (1)(5) = 0$

$k^2 + \cancel{2k} - \cancel{2k} - 4 - 5 = 0$

$k^2 - 9 = 0$

$k^2 = 9$

$\sqrt{k^2} = \pm\sqrt{9}$

$\boxed{k = \pm 3}$

27. **What is the cofactor of 3 in matrix.** $\begin{bmatrix} 3 & 1 & -4 \\ 2 & 5 & 6 \\ 1 & 4 & 8 \end{bmatrix}$

Sol. $\begin{bmatrix} 3 & 1 & -4 \\ 2 & 5 & 6 \\ 1 & 4 & 8 \end{bmatrix}$

Here : $3 = a_{11}$, so

Minor of $3 = M_{11}$

$= \begin{vmatrix} 5 & 6 \\ 4 & 8 \end{vmatrix} = 40 - 24 = \boxed{16}$

Cofactor of $3 = C_{11}$

$= (-1)^{1+1} M_{11} = (-1)^2 (16) = \boxed{16}$

Section - II

Note : Attemp any three (3) questions $3 \times 8 = 24$

Q.2.(a) The hypotenuse of a right triangle is 10cm and its height is twice of its base. Find the area of triangle.

Sol. See Q.1 of Ex # 10 (Page # 462)

(b) The diagonals of a rhombus are 80cm and 60cm respectively. Find the area and length of each side.

Sol. See Q.5 of Ex # 11 (Page # 476)

Q.3.(a) The area of regular octagonal room is 51sq.cm. Find the length of its side.

Sol. See Q.1 of Ex # 12 (Page # 486)

(b) The axis of an ellipse are 40cm and 60cm. Find its perimeter and area.

Sol. See Q.9 of Ex # 13 (Page # 499)

Q.4. Find area of the field whose ordinates are 0, 20, 22.5, 33.5, 45, 42, 33.5, 25.5 and 0 meter respectively. The width of each strip is 14m. Find also the approximate cost of purchasing the field at a cost of Rs. 5,000/per sq.m.

Sol. See example # 03 of Ch # 14

Q.5.(a) For what value of m the vector

$4\underline{i} + 2\underline{j} - 3\underline{k}$ and $m\underline{i} - \underline{j} + \sqrt{3}\underline{k}$

have same magnitude.

Sol. See Q.3 of Ex # 8.1 (Page # 371)

(b) Find the sine of the angle and the unit vector perpendicular to each if

$\vec{a} = \underline{i} + \underline{j} + \underline{k}$ and $\vec{b} = 2\underline{i} + 3\underline{j} - \underline{k}$

Sol. See Q.19(i) of Ex # 8.2 (Page # 388)

Q.6.(a) Find the inverse of

$$\begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 4 \\ 0 & 2 & 2 \end{bmatrix}$$

Sol. See Q.4(iii) of Ex # 9.3 (Page # 440)

(b) Use Cramer's rule to solve the system of equations.

$3x - 4y = -2$

$x + y = 6$

Sol. See Q.8(ii) of Ex # 9.2 (Page # 429)

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