DAE / IA - 2017

MATH-113 APPLIED MATHEMATICS-I

PAPER 'A' PART - A (OBJECTIVE)

Time: 30 Minutes Marks: 15

Q.1: *Encircle the correct answer.*

1. To make $x^2 - 5x$ a complete square, we should add:

[a] 25 [b] $\frac{25}{4}$ [c] $\frac{25}{9}$ [d] $\frac{25}{16}$

2. Sum of the roots of $ax^2 - bx + c = 0$ is:

[a] $-\frac{c}{a}$ [b] $\frac{c}{a}$

[c] $-\frac{b}{a}$ [d] $\frac{b}{a}$

3. The 10th term $7, 17, 27, \ldots$ is:

[a] 97 [b] 98 [c] 99 [d] 100

4. The nth term of a G.P. $a, ar, ar^2, \ldots\ldots$ is;

[a] nr^2 [b] nr^{n+1}

[c] $\frac{1}{a}r^{n-1}$ [d] nr^{n-1}

5. The sum of infinite terms of G.P $a, ar, ar^2, \ldots\ldots$ If $|r|<1$

[a] $\frac{a}{1-r}$ [b] $\frac{a(1-r^n)}{1-r}$

[c] ar^{n-1} [d] None of these

6. The number of terms in the expansion $(a + b)^{13}$ are:

[a] 12 [b] 13 [c] 14 [d] 15

7. The value of $\binom{2n}{n}$ is:

[a] $\left(\begin{matrix}2n\\ n!n!\end{matrix}\right)$ [b] $\frac{(2n)!}{n!n!}$

[c] $\frac{(2n)!}{n!}$ [d] $\frac{(2n)!}{n(n-1)!}$

8. The number of partial fractions of $\frac{x^3 - 3x^2 + 1}{(x-1)(x+1)(x^2-1)}$

[a] 2 [b] 3 [c] 4 [d] 5

9. One degree is equal to:

[a] πrad. [b] $\frac{\pi}{180}$rad.

[c] $\frac{180}{\pi}$rad. [d] $\frac{\pi}{360}$rad.

10. If $\sin\theta$ is positive and $\cos\theta$ is negative, then the terminal side of the angle lies in quadrant.

[a] 1st [b] 2nd [c] 3rd [d] 4th

11. If $\sin\theta = \frac{2}{\sqrt{7}}$ and $\cos\theta = \frac{-1}{\sqrt{7}}$ then $\cot\theta$ is equal to:

[a] 2 [b] −1 [c] $-\frac{1}{2}$ [d] 2

12. $\sin(\pi - x)$ is equal to:

[a] $-\sin x$ [b] $\sin x$

[c] $\cos x$ [d] $-\cos x$

13. $2\cos^2\frac{\theta}{2}$ is equal to:

[a] $1+\cos\theta$ [b] $1-\cos\theta$

[c] $1+\sin\theta$ [d] $1-\sin\theta$

14. Law of sine is:

[a] $\frac{a}{\sin B} = \frac{b}{\sin A} = \frac{c}{\sin C}$

[b] $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

[c] $\frac{a}{\sin B} = \frac{b}{\sin A} = \frac{c}{\sin C}$

[d] $\frac{a}{\sin B} = \frac{b}{\sin C} = \frac{c}{\sin A}$

15. If a = 2, b = 2, A = 30º, then B is equal to:

[a] $45°$ [b] $30°$

[c] $60°$ [d] $90°$

Answer Key

1	b	2	c	3	a	4	d	5	a
6	b	7	b	8	c	9	b	10	b
11	c	12	b	13	a	14	b	15	b

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DAE / IA - 2017

MATH-113 APPLIED MATHEMATICS-I

PAPER 'A' PART-B (SUBJECTIVE)

Time: 2:30 Hrs Marks: 60

Section-I

Q.1. Write short answers to any Eighteen (18) questions.

1. Solve the quadratic equation $x^2 - 3x - 18 = 0$ by quadratic formula.

Sol. $x^2 - 3x - 18 = 0$

Here: $a = 1, b = -3, c = -18$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(-18)}}{2(1)}$$

$$x = \frac{3 \pm \sqrt{9 + 72}}{2} = \frac{3 \pm \sqrt{81}}{2} = \frac{3 \pm 9}{2}$$

Either

$$x = \frac{3+9}{2}$$

$$x = \frac{12}{2} = 6$$

OR

$$x = \frac{3-9}{2}$$

$$x = \frac{-6}{2} = -3$$

$$S.S. = \{-3, 6\}$$

2. Discuss the nature of the roots of the equation $2x^2 - 7x + 3 = 0$

Sol. Here: $a = 2, b = -7, c = 3$

$$Disc. = b^2 - 4ac$$

$$= (-7)^2 - 4(2)(3) = 49 - 24 = 25$$

∴ The roots are **Rational unequal and Real.**

3. ~~For what value of k, the sum of roots of~~ $3x^2 + kx + 5 = 0$ ~~may be equal to the product of roots.~~

Sol. Here: $a = 3, b = k, c = 5$

As, Sum of roots = Product of roots

$$-\frac{b}{a} = \frac{c}{a} \Rightarrow -\frac{k}{3} = \frac{5}{3}$$

$$-k = 5 \Rightarrow \boxed{k = -5}$$

4. Find the 7^{th} term of an A.P. 1 , 4 , 7 , . . .

Sol. Here: $a_1 = 1$ & $d = 4 - 1 = 3$

$$a_7 = a_1 + 6d$$

$$a_7 = 1 + 6(3)$$

$$a_7 = 1 + 18 \Rightarrow \boxed{a_7 = 19}$$

5. ~~Find the sum of the series 3 + 11 + 19 + ... to 16 terms.~~

Sol. Here: $a_1 = 3, d = 11 - 3 = 8$ & $n = 16$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{16} = \frac{16}{2}[2(3) + (16-1)(8)]$$

$$S_{16} = 8[6 + 120] = 8(126) = \boxed{1008}$$

6. Find the 6^{th} term in G.P. $1, 3^3, 3^6, \ldots$

Sol. Here: $a_1 = 1, r = \frac{3^3}{1} = 3^3$ & $a_6 = ?$

$$a_6 = a_1 r^5 = 1(3^3)^5 = \boxed{(27)^5}$$

7. Find the geometric mean between $4/3$ and 243.

Sol. Here: $a = \frac{4}{3}$, $b = 243$

$$G = \pm\sqrt{ab} = \pm\sqrt{\frac{4}{3} \times 243}$$

$$G = \pm\sqrt{324} \Rightarrow \boxed{G = \pm 18}$$

8. ~~Write the formula of sum of the first 'n' terms of a geometric sequence.~~

Sol. Formulas:

$$(i)\ S_n = \frac{a(1-r^n)}{1-r} \text{ if } |r|<1$$

$$(ii)\ S_n = \frac{a(r^n-1)}{r-1} \text{ if } |r|>1$$

9. ~~Find the sum of infinite geometric series in which~~ $a = 128$ and $r = -\frac{1}{2}$.

Sol. $S_\infty = \frac{a}{1-r}$

$$S_\infty = \frac{128}{1-\left(-\frac{1}{2}\right)} = \frac{128}{1+\frac{1}{2}} = \frac{128}{\frac{2+1}{2}}$$

$$S_\infty = \frac{128}{3/2} = 128\times\frac{2}{3} = \boxed{\frac{256}{3}}$$

10. Expand $\left(\frac{x}{2}-\frac{2}{y}\right)^4$ by using binomial theorem.

Sol. $\left(\frac{x}{2}-\frac{2}{y}\right)^4$

$$= \binom{4}{0}\left(\frac{x}{2}\right)^4\left(\frac{2}{y}\right)^0 - \binom{4}{1}\left(\frac{x}{2}\right)^3\left(\frac{2}{y}\right)^1 + \binom{4}{2}\left(\frac{x}{2}\right)^2\left(\frac{2}{y}\right)^2 - \binom{4}{3}\left(\frac{x}{2}\right)^1\left(\frac{2}{y}\right)^3 + \binom{4}{4}\left(\frac{x}{2}\right)^0\left(\frac{2}{y}\right)^4$$

$$= (1)\left(\frac{x^4}{16}\right)(1) - 4\left(\frac{x^3}{8}\right)\left(\frac{2}{y}\right) + 6\left(\frac{x^2}{4}\right)\left(\frac{4}{y^2}\right) - 4\left(\frac{x}{2}\right)\left(\frac{8}{y^3}\right) + (1)(1)\left(\frac{16}{y^4}\right)$$

$$= \boxed{\frac{x^4}{16} - \frac{x^3}{y} + 6\frac{x^2}{y^2} - 16\frac{x}{y^3} + \frac{16}{y^4}}$$

11. Calculate $(1.04)^5$ by binomial theorem upto two decimal places.

Sol. $(1.04)^5 = (1+0.04)^5$

$$= \binom{5}{0}(1)^5(0.04)^0 + \binom{5}{1}(1)^4(0.04)^1 + \binom{5}{2}(1)^3(0.04)^2 + \ldots$$

$$= (1)(1)(1) + 5(1)(0.04) + 10(1)(0.0016) + \ldots$$

$$= 1 + 0.2 + 0.016 + \ldots = 1.2160 = \boxed{1.22}$$

12. ~~Expand~~ $\frac{1}{(1+x)^2}$ ~~to three terms.~~

Sol. $\frac{1}{(1+x)^2} = (1+x)^{-2}$

Put $b = x$ & $n = -2$ in Binomial series Formula, we have

$$= 1 + (-2)(x) + \frac{(-2)(-2-1)}{2!}(x)^2 + \ldots$$

$$= 1 - 2x + \frac{(-2)(-3)}{2}x^2 + \ldots$$

$$= \boxed{1 - 2x + 3x^2 + \ldots}$$

13. Find the 6th term in the expansion of $(x+3y)^{10}$

Sol. Here: $a = x$, $b = 3y$, $n = 10$ & $r = 5$

Using general term formula,

$$T_{r+1} = \binom{n}{r}a^{n-r}b^r$$

$$T_{5+1} = \binom{10}{5}(x)^{10-5}(3y)^5$$

$$T_6 = (252)(x^5)(243y^5)$$

$$\boxed{T_6 = 61236x^5y^5}$$

14. Define proper fraction and give one example.

Sol. A fraction in which the degree of the numerator is less than the degree of the denominator is called proper fraction.

Example: $\frac{2x}{(x-2)(x+5)}$

15. Resolve $\frac{1}{x^2-x}$ into partial fractions.

Sol. $\frac{1}{x^2-x} = \frac{1}{x(x-1)}$

$$\frac{1}{x(x-1)}=\frac{A}{x}+\frac{B}{x-1}\rightarrow(i)$$

$$1=A(x-1)+Bx\rightarrow(ii)$$

Put $x=0$ in eq.(ii)

$$1=A(0-1)+B(0)$$

$$1=-A \Rightarrow \boxed{A=-1}$$

Put $x=1$ in eq.(ii)

$$1=A(1-1)+B(1)$$

$$1=A(0)+B \Rightarrow \boxed{B=1}$$

Put values of A, & B

in eq. (i), we get: $\boxed{-\frac{1}{x}+\frac{1}{x-1}}$

16. ~~Convert~~ $12°40'$ ~~into radian measure.~~

Sol. $12°40'=\left(12+\frac{40}{60}\right)^{\circ}$

$$=(12+0.666)^{\circ}=12.6666^{\circ}$$

$$=12.6666^{\circ}\times\frac{\pi}{180}=\boxed{0.22\text{ rad}}$$

17. Find 'x' if

$$\tan^2 45°-\cos^2 60°=x\sin 45°\cos 45°\tan 60°$$

Sol. $\tan^2 45°-\cos^2 60°=x\sin 45°\cos 45°\tan 60°$

$$(1)^2-\left(\frac{1}{2}\right)^2=x\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)\left(\sqrt{3}\right)$$

$$1-\frac{1}{4}=x\left(\frac{2\sqrt{3}}{4}\right)\Rightarrow\frac{4-1}{4}\times\frac{4}{2\sqrt{3}}=x$$

$$\frac{3}{2\sqrt{3}}=x \Rightarrow \boxed{x=\frac{\sqrt{3}}{2}}$$

18. ~~Find the length of arc cut off on a circle of radius 3cm by a central angle of 2 radians.~~

Sol. Here ℓ = ?, r = 3cm, θ = 2 rad

By using formula: $\ell=r\theta$

$$\ell=r\theta=(3)(2)=\boxed{6\text{cm}}$$

19. Show that:

$$\cot^4\theta+\cot^2\theta=\operatorname{cosec}^4\theta-\operatorname{cosec}^2\theta$$

Sol. $\text{L.H.S.}=\cot^4\theta+\cot^2\theta$

$$=\cot^2\theta\left(\cot^2\theta+1\right)$$

$$=\cot^2\theta\left(\operatorname{cosec}^2\theta\right)\ \because \cot^2\theta+1=\operatorname{cosec}^2\theta$$

$$=\left(\operatorname{cosec}^2\theta-1\right)\left(\operatorname{cosec}^2\theta\right)$$

$$=\operatorname{cosec}^4\theta-\operatorname{cosec}^2\theta=\text{R.H.S.}$$ **Proved.**

20. Prove that: $\sin(-\theta)=-\sin\theta$

Sol. We know that

$$\sin(\alpha-\beta)=\sin\alpha\cos\beta-\cos\alpha\sin\beta$$

Put $\alpha=0$ & $\beta=\theta$ we have:

$$\sin(0-\theta)=\sin(0)\cos\theta-\cos(0)\sin\theta$$

$$\sin(-\theta)=0.\cos\beta-1.\sin\theta$$

$$\sin(-\theta)=0-\sin\theta$$

$$\boxed{\sin(-\theta)=-\sin\theta}$$ **Proved.**

21. Show that:

$$\cos(\alpha+\beta)-\cos(\alpha-\beta)=-2\sin\alpha\sin\beta$$

Sol. $\text{L.H.S.}=\cos(\alpha+\beta)-\cos(\alpha-\beta)$

$$=\left[\cos\alpha\cos\beta-\sin\alpha\sin\beta\right]-\left[\cos\alpha\cos\beta+\sin\alpha\sin\beta\right]$$

$$=\cancel{\cos\alpha\cos\beta}-\sin\alpha\sin\beta-\cancel{\cos\alpha\cos\beta}-\sin\alpha\sin\beta$$

$$=-2\sin\alpha\sin\beta=\text{R.H.S.}$$ **Proved.**

22. Express $\cos(a+b)\cos(a-b)-\sin(a+b)\sin(a-b)$ as single term.

Sol.

$$\cos(a+b)\cos(a-b)-\sin(a+b)\sin(a-b)$$

$$=\cos(a+b+a-b)$$

$$=\boxed{\cos 2a}$$

23. ~~Express the sum~~ $\cos 12\theta-\cos 4\theta$ ~~as product.~~

Sol. $\cos 12\theta-\cos 4\theta$

$$= -2\sin\left(\frac{12\theta+4\theta}{2}\right)\sin\left(\frac{12\theta-4\theta}{2}\right)$$
$$= -2\sin\left(\frac{16\theta}{2}\right)\sin\left(\frac{8\theta}{2}\right)$$
$$= \boxed{-2\sin 8\theta \sin 4\theta}$$

24. ~~Given that, $\gamma = 90^\circ$, $\alpha = 35^\circ$, $a = 5$, find angle β.~~

Sol. We know that in any triangle:

$$\alpha+\beta+\gamma=180^\circ$$
$$\beta=180^\circ-\alpha-\gamma$$
$$\beta=180^\circ-35^\circ-90^\circ \Rightarrow \boxed{\beta=55^\circ}$$

25. **Define angle of elevation.**

Sol. **Angle of Elevation:**
If the line of sight is upward from the horizontal, the angle is called angle of Elevation.

26. **In any triangle ABC in which a = 16, b = 17, γ = 25º, find 'c'.**

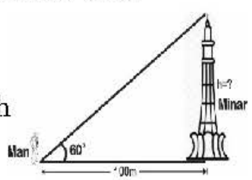
Sol. By using law of cosines:

$$c^2 = a^2+b^2-2ab\cos\gamma$$
$$c^2 = (16)^2+(17)^2-2(16)(17)\cos 25^\circ$$
$$c^2 = 256+289-493.03$$
$$c^2 = 51.97 \Rightarrow \sqrt{c^2} = \sqrt{51.97} \Rightarrow \boxed{c=7.2}$$

27. ~~A minaret stands on a horizontal ground. A man on the ground 100m from the minaret, the angle of elevation of the top of the minaret to be 60º. Find its height.~~

Sol. From figure, we know that:

$$\tan 60^\circ = \frac{h}{100}$$
$$100\tan 60^\circ = h$$
$$\boxed{h = 173.20\,m}$$



Section - II

Note: Attemp any three (3) questions $3\times 8 = 24$

Q.2. **Show that the roots of the equation $(mx+c)^2 = 4ax$ will be equal if $c = \frac{a}{m}$.**

Sol. See Q.3(ii) of Ex # 1.2 (Page # 33)

(b) ~~If α, β are the roots of $ax^2+bx+c=0$, find the value of $\alpha^3+\beta^3$.~~

Sol. See Q.4(i) of Ex # 1.3 (Page # 44)

Q.3.(a) **The A.M of two positive integral numbers exceeds their positive G.M by 2 and their sum is 20. Find the numbers.**

Sol. See Q.8 of Ex # 2.5 (Page # 115)

(b) ~~Sum the series 51 + 50 + 49 + ... + 21.~~

Sol. See Q.1(ii) of Ex # 2.3 (Page # 89)

Q.4.(a) **Find the term involving x^5 in the expansion of $\left(2x^2-\frac{3}{x}\right)^{10}$**

Sol. See Q.7(i) of Ex # 3.1 (Page # 152)

(b) **Resolve $\frac{1}{(x+1)(x^2-1)}$ into partial fractions.**

Sol. See Q.2 of Ex # 4.2 (Page # 194)

Q.5.(a) **Prove that:**

$$\sqrt{\frac{1-\sin\theta}{1+\sin\theta}} = \sec\theta-\tan\theta$$

Sol. See Q.9 of Ex # 5.3 (Page # 256)

(b) **Prove that: $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$**

Sol. See Triple angles proof (Page # 294)

Q.6.(a) ~~Show that:~~

$$\cos 20^\circ+\cos 100^\circ+\cos 140^\circ = 0$$

Sol. See Q.13(iv) of Ex # 6.3 (Page # 314)

(b) ~~A man 18dm tall observes that the angle of elevation of the top of a tree at a distance of 12m from the man is 32º. What is the height of the tree.~~

Sol. See Q.3 of Ex # 7.2 (Page # 336)

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