DAE / IA - 2017

MATH-123 APPLIED MATHEMATICS-I

PAPER 'B' PART - A (OBJECTIVE)

Time: 30 Minutes Marks: 15

Q.1: *Encircle the correct answer.*

1. The number of partial fractions of $\frac{6x+27}{4x^3-9x}$ are:

[a] 2 [b] 3
[c] 4 [d] None of these

2. Partial fractions of $\frac{3x+3}{(x-2)(x+5)}$ is:

[a] $\frac{2}{x-2}+\frac{1}{x+5}$ [b] $\frac{3}{x-2}+\frac{1}{x+5}$
[c] $\frac{2}{x-2}+\frac{3}{x+5}$ [d] $\frac{1}{x-2}+\frac{1}{x+5}$

3. Sum of $-3+5i$ and $4-7i$ is:

[a] $1-2i$ [b] $-1-2i$
[c] $1-12i$ [d] $-7+12i$

4. Ordered pair form of $-3-2i$ is:

[a] $(3, 2)$ [b] $(-3, -2)$
[c] $(-3, 2)$ [d] $(3, -2)$

5. $i(1-2i)$ is equal to:

[a] $1+2i$ [b] $2i+i^2$
[c] 3 [d] $3i$

6. According to Boolean Algebra $X+\overline{X}$ is equal to:

[a] X [b] $\overline{X}$
[c] 0 [d] 1

7. If the switch is on it is represented by:

[a] 0 [b] 1 [c] OR [d] Not

8. Symbol

[a] Not gate [b] NOR
[c] OR [d] None of these

9. Equation of line is slope intercept form is:

[a] $\frac{x}{y}+\frac{x}{b}=1$ [b] $y=mx+c$
[c] $y-y_1=m(x-x_1)$
[d] None of these

10. $y-y_1=m(x-x_1)$ is the:

[a] Slope intercept form
[b] Intercept form
[c] Point slope form
[d] None of these

11. ~~Slope of line through~~ (x_1, y_1) ~~and~~ (x_2, y_2) ~~is:~~

[a] $\frac{x_1}{y_1}+\frac{x_2}{y_2}$ [b] $\frac{y_2+y_1}{x_2+x_1}$
[c] $\frac{y_2-y_1}{x_2-x_1}$ [d] None of these

12. Distance between (4, 3) and (7, 5) is:

[a] 25 [b] $\sqrt{15}$
[c] 5 [d] None of these

13. ~~Ratio formula for y-coordinate is:~~

[a] $\frac{x_1r_2+x_2r_1}{r_1+r_2}$ [b] $\frac{y_1r_2+y_2r_1}{r_1+r_2}$
[c] $\frac{x-y}{2}$ [d] None of these

14. Center of circle $(x-1)^2+(y-2)^2=16$ is:

[a] $(1, 2)$ [b] $(2, 1)$
[c] $(4, 0)$ [d] None of these

15. Equation of unit circle is:

[a] $x^2+y^2+2x+2y+1=0$
[b] $x^2+y^2=1$ [c] $x^2+y^2+r^2$
[d] None of these

Answer Key

1	b	2	d	3	a	4	b	5	a
6	d	7	b	8	d	9	b	10	c
11	c	12	b	13	b	14	a	15	b

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DAE / IA - 2017

MATH-123 APPLIED MATHEMATICS-I

PAPER 'B' PART-B (SUBJECTIVE)

Time: 2:30 Hrs Marks: 60

Section - I

Q.1. Write short answers to any Eighteen (18) questions.

1. If $Z = 2 + 3i$, prove that $Z\bar{Z} = 13$

Sol. As, $Z = 2 - 3i$ then $\bar{Z} = 2 - 3i$

$$\text{L.H.S.} = Z\bar{Z} = (2+3i)(2-3i)$$

$$= (2)^2 - (3i)^2$$

$$= 4 + 9 = 13 = \text{R.H.S.}$$ **Proved.**

2. Find the multiplicative of $(-3, 4)$.

Sol. Let $z = (-3, 4) = -3 + 4i$

Multiplicative Inverse of $Z = \dfrac{1}{Z} = \dfrac{1}{-3+4i}$

$$= \frac{1}{-3+4i} \times \frac{-3-4i}{-3-4i} = \frac{-3-4i}{(-3)^2 - (4i)^2}$$

$$= \frac{-3-4i}{9+16} = \frac{-3-4i}{25} = \boxed{-\frac{3}{25} - \frac{4}{25}i}$$

3. Factorize $2x^2 + 5y^2$

Sol. $2x^2 + 5y^2 = 2x^2 - 5y^2 i^2$

$$= \left(\sqrt{2}x\right)^2 - \left(\sqrt{5}yi\right)^2$$

$$= \boxed{\left(\sqrt{2}x - \sqrt{5}yi\right)\left(\sqrt{2}x + \sqrt{5}yi\right)}$$

4. ~~**Express the complex number $3 - \sqrt{3}i$ in polar form.**~~

Sol. Let, $z = 3 - \sqrt{3}i$

Here: $a = 3$ & $b = -\sqrt{3}$

$$r = |z| = \sqrt{a^2 + b^2}$$

$$r = \sqrt{(3)^2 + \left(-\sqrt{3}\right)^2}$$

$$r = \sqrt{9+3} = \sqrt{12}$$

$$r = \sqrt{4 \times 3} = 2\sqrt{3}$$

$$\theta = \tan^{-1}\left(\frac{b}{a}\right)$$

$$\theta = \tan^{-1}\left(\frac{-\sqrt{3}}{3}\right)$$

$$\theta = -30^\circ$$

$$z = r\,\text{cis}\,\theta = 2\sqrt{3}\,\text{cis}(-30^\circ)$$

$$z = \boxed{2\sqrt{3}\left(\cos 30^\circ - i \sin 30^\circ\right)}$$

5. Find the values of x & y from the equation:

$$(2x - y - 1) - i(x - 3y) = (y - x) - i(2 - 2y)$$

Sol. $(2x - y - 1) - i(x - 3y) = (y - x) - i(2 - 2y)$

Comparing real & imaginary parts:

$2x - y - 1 = y - x$	$x - 3y = 2 - 2y$
$2x - y - y + x = 1$	$x - 3y + 2y = 2$
$3x - 2y = 1 \rightarrow (i)$	$x - y = 2 \rightarrow (ii)$

Multiply eq. (ii) by 2 & subtracting eq. (i)

$$2x - 2y = 4$$

$$\underline{-3x \mp 2y = -1}$$

$$-x \quad = 3 \Rightarrow \boxed{x = -3}$$

Put $x = -3$ in eq. (i), we have:

$$-3 - y = 2$$

$$-y = 2 + 3$$

$$-y = 5 \Rightarrow \boxed{y = -5}$$

6. Define proper fraction and give example.

Sol. A fraction in which the degree of the numerator is less than the degree of the denominator is called proper fraction.

Example: $\dfrac{2x}{(x-2)(x+5)}$

7. **Resolve into partial fractions:** $\dfrac{1}{x^2-x}$

Sol. $\dfrac{1}{x^2-x}=\dfrac{1}{x(x-1)}$

$$\frac{1}{x(x-1)}=\frac{A}{x}+\frac{B}{x-1}\to(i)$$

$$1=A(x-1)+Bx\to(ii)$$

Put $x=0$ in eq.(ii)

$$1=A(0-1)+B(0)$$

$$1=-A \Rightarrow \boxed{A=-1}$$

Put $x=1$ in eq.(ii)

$$1=A(1-1)+B(1)$$

$$1=A(0)+B \Rightarrow \boxed{B=1}$$

Put values of A, & B

in eq. (i), we get: $\boxed{-\dfrac{1}{x}+\dfrac{1}{x-1}}$

8. **Write an identity equation of** $\dfrac{6x^3+5x^2-7}{3x^2-2x-1}$

Sol. $\dfrac{6x^3+5x^2-7}{3x^2-2x-1}$ {Improper Fraction}

$$\begin{array}{r|l} 3x^2-2x-1 & 6x^3+5x^2-7 \quad (2x+3 \\ & \underline{-6x^3 \mp 4x^2 \mp 2x} \\ & 9x^2+2x-7 \\ & \underline{-9x^2 \mp 6x \mp 3} \\ & 8x-4 \end{array}$$

$$3x^2-2x-1$$
$$=3x^2-3x+x-1$$
$$=3x(x-1)+1(x-1)$$
$$=(x-1)(3x+1)$$

$$=2x+3+\frac{8x-4}{3x^2-2x-1}$$

$$=2x+3+\frac{8x-4}{(x-1)(3x+1)}$$

$$=\boxed{2x+3+\frac{A}{x-1}+\frac{B}{3x+1}}$$

9. ~~**Form of partial fractions of**~~ ~~$\dfrac{2x^4-3x^2-4x}{(x+1)(x^2+2)^2}$~~

Sol. $\dfrac{2x^4-3x^2-4x}{(x+1)(x^2+2)^2}$

$$=\boxed{\frac{A}{(x+1)}+\frac{Bx+C}{(x^2+2)}+\frac{Dx+E}{(x^2+2)^2}}$$

10. **Prove by Boolean Algebra rules:** $X(X+Y)=X$

Sol. $L.H.S.=X(X+Y)$

$=XX+XY$

$=X+XY \qquad \because XX=X$

$=X(1+Y)$

$=X(1) \qquad \because 1+Y=1$

$=X=R.H.S.$ **Proved.**

11. **Prove by Boolean algebra rules:** $XY+YZ+\overline{Y}Z=XY+Z$

Sol. $L.H.S.=XY+YZ+\overline{Y}Z$

$=XY+Z(Y+\overline{Y})$

$=XY+Z(1) \quad \because Y+\overline{Y}=1$

$=XY+Z=R.H.S.$ **Proved.**

12. **Convert the binary number** $(110110.011)_2$ **to octal number.**

Sol. $\overbrace{110}\,\overbrace{110}\,.\,\overbrace{011}$

$$110=1\times2^2+1\times2^1+0\times2^0=4+2+0=6$$

$$110=1\times2^2+1\times2^1+0\times2^0=4+2+0=6$$

$$011=0\times2^2+1\times2^1+1\times2^0=0+2+1=3$$

$$\boxed{(66.3)_8}$$

13. **Add the binary numbers**

$$(1101)_2 + (1011_2)$$

Sol.

$$\begin{array}{r} 1101 \\ +1011 \\ \hline 11000 \end{array}$$

$$\boxed{(11000)_2}$$

14. **Prepare a truth table for**

$$\mathbf{X(X+Y)=X}$$

Sol. $X(X+Y)=X$ L.H.S. R.H.S.

X	Y	X+Y	X(X+Y)	X
0	0	0	0	0
0	1	1	0	0
1	0	1	1	1
1	1	1	1	1

15. **Find the distance between the points $(-3\ ,1)$ and $(3\ ,-2)$.**

Sol. Distance between $(-3\ ,1)$ & $(3\ ,-2)$.

$$= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

$$= \sqrt{(-3-3)^2 + (1-(-2))^2}$$

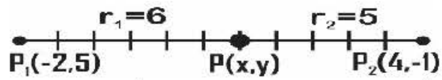
$$= \sqrt{(-6)^2 + (3)^2} = \sqrt{36+9}$$

$$= \sqrt{45} = \sqrt{9\times 5} = \boxed{3\sqrt{5}}$$

16. ~~**Find the coordinates of the point $P(x, y)$ which divide internally the segment through $P_1(-2, 5)$ and $P_2(4, -1)$ of the ratio of $\frac{r_1}{r_2} = \frac{6}{5}$.**~~

Sol. Here : $r_1 = 6,\ r_2 = 5,\ (x_1, y_1) = (-2,5)$

& $(x_2, y_2) = (4,-1)$



$$P(x\ ,\ y) = \left(\frac{r_1 x_2 + r_2 x_1}{r_1 + r_2}, \frac{r_1 y_2 + r_2 y_1}{r_1 + r_2}\right)$$

$$= \left(\frac{6(4)+5(-2)}{6+5}, \frac{6(-1)+5(5)}{6+5}\right)$$

$$= \left(\frac{24-10}{11}, \frac{-6+25}{11}\right) = \boxed{\left(\frac{14}{11}, \frac{19}{11}\right)}$$

17. ~~**Find the equation of the line passing through the point $(1, -2)$ making an angle of 135º with the X-axis.**~~

Sol. Let, $\theta = 135°$

Slope $= m = \tan\theta$

$m = \tan 135° = -1$

Equation of line in point - slope form :

$$y - y_1 = m(x - x_1)$$

$$y - (-2) = -1(x - 1)$$

$$y + 2 = -x + 1$$

$$y + 2 + x - 1 = 0$$

$$\boxed{x + y + 1 = 0}$$

18. **If a line ℓ_1 contains points $(2\ ,\ 6)$ and $(0\ ,\ y)$. Find 'y' if ℓ_1 is parallel to ℓ_2 and the slope of $\ell_2 = \frac{3}{4}$.**

Sol. Slope of line

$$\ell_1 = m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{y-6}{0-2} = \frac{y-6}{-2}$$

& slope of line $\ell_2 = m_2 = \frac{3}{4}$

As, both lines are parallel,

So, $m_1 = m_2$

$$\frac{y-6}{-2} = \frac{3}{4}$$

$$4(y-6) = 3(-2)$$

$$4y - 24 = -6$$

$$4y = -6 + 24$$

$$4y = 18$$

$$y = \frac{18}{4} \Rightarrow \boxed{y = \frac{9}{2}}$$

19. ~~**Find the distance from the point $(1, 3)$ to the line $3x - 2y + 12 = 0$**~~

Sol. Distance between point & line

$$D = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

$$D = \frac{|3(1) - 2(3) + 12|}{\sqrt{(3)^2 + (-2)^2}}$$

$$D = \frac{|3 - 6 + 12|}{\sqrt{9+4}} = \frac{|9|}{\sqrt{13}} = \boxed{\frac{9}{\sqrt{13}}}$$

20. **Show that the point $(3, \sqrt{7})$ is on a circle with center of the origin and radius 4.**

Sol. Here: $C = (0, 0), P = (3, \sqrt{7})$ & $r = 4$

$$D = |\overline{PC}| = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

$$D = \sqrt{(0-3)^2 + (0-\sqrt{7})^2}$$

$$D = \sqrt{(-3)^2 + (-\sqrt{7})^2}$$

$$D = \sqrt{9+7} = \sqrt{16} = 4$$

Hence the given point $(3, \sqrt{7})$ lie on the circle. **Proved.**

21. **Find an equation of the line with scope $-\frac{2}{3}$ and having y-intercept 3.**

Sol. Here: $m = -\frac{2}{3}$, $c = 3$

By using slope-intercept form:

$$y = mx + c$$

$$y = \left(-\frac{2}{3}\right)x + 3$$

Multipling each term by '3', we get:

$$3y = -2x + 9$$

$$\boxed{2x + 3y - 9 = 0}$$

22. **Find the slope and y-intercept of $ax + by = b,\ b \neq 0$**

Sol. As, $ax + by = b$

$$by = -ax + b$$

Dividing each term by 'b', we get:

$$y = -\frac{a}{b}x + 1$$

Comparing it with slope intercept form:

$y = mx + c$, we get

$$\boxed{\text{Slope} = m = -\frac{a}{b}\ \&\ \text{y-intercept} = c = 1}$$

23. **Find the equation for the line through $(-1, -2)$ and parallel to y-axis.**

Sol. As require line is parallel to y-axis:

So, slope $= m = \frac{1}{0}$

Equation of line in point-slope form:

$$y - y_1 = m(x - x_1)$$

$$y - (-2) = \frac{1}{0}(x - (-1))$$

$$0(y+2) = 1(x+1)$$

$$0 = x + 1 \Rightarrow \boxed{x + 1 = 0}$$

24. Find the equation of circle with center (1, -3) and r = 3.

Sol. Here: $\text{Center} = (h, k) = (1, -3)$

& $\text{Radius} = r = 3$

Standard form of equation of circle:

$$(x-h)^2 + (y-k)^2 = r^2$$

Put $h = 1, \quad k = -3 \quad \& \quad r = 3$

$$(x-1)^2 + (y+3)^2 = (3)^2$$

$$(x)^2 - 2(x)(1) + (1)^2 + (y)^2 + 2(y)(3) + (3)^2 = 9$$

$$x^2 - 2x + 1 + y^2 + 6y + 9 - 9 = 0$$

$$\boxed{x^2 + y^2 - 2x + 6y + 1 = 0}$$

25. Find the equation of the circle concenter with the circle $\mathbf{x^2 + y^2 - 6x + 4y - 12 = 0}$ with radius 6 units.

Sol. Given circle: $x^2 + y^2 - 6x + 4y - 12 = 0$

Comparing with general form:

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

$2g = -6$	$2f = 4$	$c = -12$
$g = -\frac{6}{2}$	$f = \frac{4}{2}$	
$g = -3$	$f = 2$	

$$\text{Center} = (-g, -f) = \boxed{(3, -2)}$$

So, center of require circle is also (3, –2) and radius 6.

Standard form of equation of circle:

$$(x-h)^2 + (y-k)^2 = r^2$$

Put $h = 3, \quad k = -2 \quad \& \quad r = 6$

$$(x-3)^2 + (y+2)^2 = (6)^2$$

$$(x)^2 - 2(x)(3) + (3)^2 + (y)^2 + 2(y)(2) + (2)^2 = 36$$

$$x^2 - 6x + 9 + y^2 + 4y + 4 - 36 = 0$$

$$\boxed{x^2 + y^2 - 6x + 4y - 23 = 0}$$

26. Find which of the two circles: $x^2 + y^2 - 3x + 4y = 0$ and $x^2 + y^2 - 6x - 8y = 0$ is greater?

Sol. Equation of 1st circle:

$x^2 + y^2 - 3x + 4y = 0$

Comparing with general form:

$$x^2 + y^2 + 2g_1x + 2f_1y + c_1 = 0$$

$2g_1 = -3$	$2f_1 = 4$	$c_1 = 0$
$g_1 = -3/2$	$f_1 = 2$	

Radius of 1st circle =

$$r_1 = \sqrt{g_1^2 + f_1^2 - c_1}$$

$$r_1 = \sqrt{\left(-\frac{3}{2}\right)^2 + (2)^2 - 0}$$

$$r_1 = \sqrt{\frac{9}{4} + 4} = \sqrt{\frac{9+16}{4}}$$

$$r_1 = \sqrt{\frac{25}{4}} \Rightarrow \boxed{r_1 = 5/2}$$

Equation of 2nd circle:

$x^2 + y^2 - 6x - 8y = 0$

Comparing with general form:

$$x^2 + y^2 + 2g_2x + 2f_2y + c_2 = 0$$

$2g_2 = -6$	$2f_2 = -8$	$c_2 = 0$
$g_2 = -3$	$f_2 = -4$	

Radius of 2nd circle =

$$r_2 = \sqrt{g_2^2 + f_2^2 - c}$$

$$r_2 = \sqrt{(-3)^2 + (-4)^2 + (0)}$$

$$r_2 = \sqrt{9 + 16 - 0}$$

$$r_2 = \sqrt{25} \Rightarrow \boxed{r_2 = 5}$$

As, $r_2 > r_1$

Hence 2nd circle is greater than 1st circle.

27. Find the equation of the circle having $(-2, 5)$ and $(3, 4)$ as the end point of its diameter.

Sol. Let A(-2, 5) & B(3, 4)

Center = Midpoint of

$$A(-2, 5) \ \& \ B(3, 4)$$

$$\text{Center} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$\text{Center} = \left(\frac{-2+3}{2}, \frac{5+4}{2}\right) = \boxed{\left(\frac{1}{2}, \frac{9}{2}\right)}$$

Diameter = Distance between

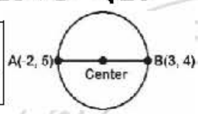
$$A(-2, 5) \ \& \ B(3, 4)$$

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

$$d = \sqrt{(-2-3)^2 + (5-4)^2}$$

$$d = \sqrt{(-5)^2 + (1)^2} = \sqrt{25+1} = \sqrt{26}$$

So, Radius $= r = \boxed{\dfrac{\sqrt{26}}{2}}$



Standard form of eq. of circle:

$$(x-h)^2 + (y-k)^2 = r^2$$

$$\text{Put } h = \frac{1}{2}, k = \frac{9}{2} \ \& \ r = \frac{\sqrt{26}}{2}$$

$$\left(x - \frac{1}{2}\right)^2 + \left(y - \frac{9}{2}\right)^2 = \left(\frac{\sqrt{26}}{2}\right)^2$$

$$(x)^2 - 2(x)\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)^2 + (y)^2 - 2(y)\left(\frac{9}{2}\right) + \left(\frac{9}{2}\right)^2 = \frac{26}{4}$$

$$x^2 - x + \frac{1}{4} + y^2 - 9y + \frac{81}{4} = \frac{26}{4}$$

$$x^2 + y^2 - x - 9y + \frac{1}{4} + \frac{81}{4} - \frac{26}{4} = 0$$

$$x^2 + y^2 - x - 9y + \frac{1+81-26}{4} = 0$$

$$\boxed{x^2 + y^2 - x - 9y + 14 = 0}$$

Section - II

Note : Attemp any three (3) questions $3 \times 8 = 24$

Q.2. **Extract the square root of the complex number $21 - 20i$.**

Sol. See example 09 of Chapter 08

Q.3. **Resolve into partial fractions $\dfrac{9x-7}{(x+3)(x^2+1)}$**

Sol. See example 09 of Chapter 09

Q.4.(a) **Prepare the truth table for the Boolean expression $AB + \overline{A}\,\overline{B}$**

Sol. See Q.1(v) of Ex# 11 (Page # 426)

(b) **Minimize the expression by use of Boolean rule**

$$X = ABC + \overline{A}B + AB\overline{C}$$

Sol. See example 05[a] of chapter 11

Q.5. **(a)** ~~**Find the point which is $\frac{7}{10}$ of the way from the point $(4, 5)$ to the point $(-6, 10)$.**~~

Sol. See Q.4 of Ex# 12.2 (Page # 458)

(b) ~~**Find the equation of the perpendicular bisector of the line segment joining the points $(2, 4)$ & $(6, 8)$.**~~

Sol. See Q.16[a] of Ex # 12.4 (Page # 484)

Q.6. **Find equation of circle passing through the points $(-2, 1)$, $(-4, -3)$ and $(3, 0)$.**

Sol. See Q.3[c] of Ex # 13 (Page # 526)

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