DAE / IA - 2016

MATH - 123 APPLIED MATHEMATICS - I

PAPER 'A' PART - A (OBJECTIVE)

Time : 30 Minutes Marks : 15

Q.1: *Encircle the correct answer.*

1. The factors of $x^2 - 7x + 12 = 0$ are:

[a] $(x-4)(x+3)$
[b] $(x-4)(x-3)$
[c] $(x+4)(x+3)$
[d] $(x+4)(x-3)$

2. $\ell x^2 + mx + n = 0$ will be a pure quadratic if:

[a] $\ell = 0$ [b] $n = 0$
[c] $m = 0$ [d] Both $\ell, m = 0$

3. ~~The sum of the roots of~~ $ax^2 - bx + c = 0$ ~~is:~~

[a] $-\frac{c}{a}$ [b] $\frac{c}{a}$
[c] $-\frac{b}{a}$ [d] $\frac{b}{a}$

4. The number of terms in the expansion of $(a+b)^{13}$ are:

[a] 12 [b] 13 [c] 14 [d] 15

5. The value of $\binom{n}{n}$ is equal to:

[a] 0 [b] 1
[c] n [d] $-n$

6. The middle term of $\left(\frac{x}{y} - \frac{y}{x}\right)^4$ is:

[a] 6 [b] 8
[c] $\frac{4x^2}{y^2}$ [d] $\frac{4x}{y}$

7. ~~If an arc of a circle has length 'ℓ' and subtend an angle 'θ' then 'r' will be:~~

[a] $\frac{\theta}{\ell}$ [b] $\frac{\ell}{\theta}$
[c] $\ell\theta$ [d] $\ell + \theta$

8. $\sec^2\theta + \operatorname{cosec}^2\theta$ is equal to:

[a] $\sec^2\theta \operatorname{cosec}^2\theta$
[b] $\sin\theta\cos\theta$
[c] $-\cos\theta$ [d] $2\operatorname{cosec}^2\theta$

9. $\cos(\pi+\theta)$ is equal to:

[a] $\cos\theta$ [b] $-\sin\theta$
[c] $-\cos\theta$ [d] $\sin\theta$

10. $\sin 2\alpha$ is equal to:

[a] $\cos^2\alpha - \sin^2\alpha$
[b] $\cos 2\alpha$
[c] $1 - \cos^2\alpha$ [d] $2\sin\alpha\cos\alpha$

11. When angle of elevation is viewed by an observer the object is:

[a] Above [b] Below
[c] At the same level
[d] None of these

12. In right triangle if one angle is $45°$, then the other will be:

[a] $45°$ [b] $50°$
[c] $60°$ [d] $75°$

13. Unit vector of $\underline{i} + \underline{j} + \underline{k}$ is:

[a] $\underline{i} + \underline{j} + \underline{k}$ [b] $\frac{1}{3}(\underline{i} + \underline{j} + \underline{k})$
[c] $\frac{1}{\sqrt{3}}(\underline{i} + \underline{j} + \underline{k})$ [d] $\frac{1}{2}(\underline{i} + \underline{j} + \underline{k})$

14. $|\vec{a} \times \vec{b}|$ is area of the figure called:

[a] Triangle [b] Rectangle
[c] Sector [d] Parallelogram

15. ~~The value of $j^2 E$ is:~~

[a] jE [b] $-$jE [c] E [d] $-$E

Answer Key

1	b	2	c	3	d	4	c	5	b
6	a	7	b	8	a	9	c	10	d
11	a	12	a	13	c	14	d	15	b

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DAE / IA - 2016

MATH - 123 APPLIED MATHEMATICS - I

PAPER 'A' PART - B (SUBJECTIVE)

Time: 2:30 Hrs Marks: 60

Section - I

Q.1. Write short answers to any Eighteen (18) questions.

1. Solve the equation $x^2-3x=2x-6$ by using quadratic formula.

Sol. $x^2-3x=2x-6$

$x^2-3x-2x+6=0$

$x^2-5x+6=0$

Here: $a=1,\ b=-5,\ c=6$

$x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x=\dfrac{-(-5)\pm\sqrt{(-5)^2-4(1)(6)}}{2(1)}$

$x=\dfrac{5\pm\sqrt{25-24}}{2}=\dfrac{5\pm\sqrt{1}}{2}=\dfrac{5\pm1}{2}$

Either OR

$x=\dfrac{5+1}{2}$ | $x=\dfrac{5-1}{2}$

$x=\dfrac{6}{2}=3$ | $x=\dfrac{4}{2}=2$

$S.S.=\{2,3\}$

2. Discuss the nature of the roots of the equation $x^2-2\sqrt{2}x+2=0$

Sol. Here: $a=1,\ b=-2\sqrt{2},\ c=2$

Disc. $=b^2-4ac$

$=\left(-2\sqrt{2}\right)^2-4(2)(1)=8-8=0$

Hence roots are **Equal and Real.**

3. Prove that the roots of the equation $(a+b)x^2-ax-b=0$ are rational.

Sol. Here: $A=(a+b),\ B=-a,\ C=-b$

Disc. $=B^2-4AC$

$=(-a)^2-4(a+b)(-b)$

$=a^2+4ab+4b^2$

$=(a+2b)^2$ Which is a perfect square.

Hence roots are Rational. Proved.

4. ~~Find the sum and product of the roots of the equation~~ $9x^2+6x+1=0$

Sol. Here: $a=9,\ b=6,\ c=1$

Sum of Roots: $S=-\dfrac{b}{a}=-\dfrac{6}{9}=\boxed{-\dfrac{2}{3}}$

Product of Roots: $P=\dfrac{c}{a}=\boxed{\dfrac{1}{9}}$

5. ~~Form the quadratic equation whose roots are~~ $i\sqrt{3}$ and $-i\sqrt{3}$

Sol. $S=i\sqrt{3}+\left(-i\sqrt{3}\right)$ | $P=\left(i\sqrt{3}\right)\left(-i\sqrt{3}\right)$

$S=i\sqrt{3}-i\sqrt{3}$ | $P=-(i)^2\left(\sqrt{3}\right)^2$

$S=0$ | $P=-(-1)(3)=3$

$x^2-Sx+P=0$

$x^2-0x+3=0\Rightarrow\boxed{x^2+3=0}$

6. Expand $(2x-3y)^4$ by Binomial theorem.

Sol. $(2x-3y)^4$

$=\binom{4}{0}(2x)^4(3y)^0-\binom{4}{1}(2x)^3(3y)^1+\binom{4}{2}(2x)^2(3y)^2$

$-\binom{4}{3}(2x)^1(3y)^3+\binom{4}{4}(2x)^0(3y)^4$

$=(1)(16x^4)(1)-4(8x^3)(3y)+6(4x^2)(9y^2)$

$-4(2x)(27y^3)+(1)(1)(81y^4)$

$=\boxed{16x^4-96x^3y+216x^2y^2-216xy^3+81y^4}$

7. Calculate $(1.04)^5$ by binomial theorem up to two decimal places.

Sol. $(1.04)^5 = (1+0.04)^5$

$= \binom{5}{0}(1)^5(0.04)^0 + \binom{5}{1}(1)^4(0.04)^1 + \binom{5}{2}(1)^3(0.04)^2 + \dots$

$= (1)(1)(1) + 5(1)(0.04) + 10(1)(0.0016) + \dots$

$= 1 + 0.2 + 0.016 + \dots = 1.2160 = \boxed{1.22}$

8. ~~Expand $(1+2x)^{-2}$ to three terms.~~

Sol. $(1+2x)^{-2}$

$= 1 + (-2)(2x) + \frac{(-2)(-2-1)}{2!}(2x)^2 + \dots$

$= 1 - 4x + \frac{(-2)(-3)}{2}(4x^2) + \dots$

$= \boxed{1 - 4x + 12x^2 + \dots}$

9. Which will be the middle term in the expansion of $\left(x + \frac{3}{x}\right)^{15}$

Sol. As $n = 15$ (Odd), so

Middle terms $= \left(\frac{n+1}{2}\right)^{th} + \left(\frac{n+3}{2}\right)^{th}$ terms.

Middle terms $= \left(\frac{15+1}{2}\right)^{th} + \left(\frac{15+3}{2}\right)^{th}$ terms.

Middle terms $= 8^{th} + 9^{th}$ terms.

$\boxed{\text{Hence } T_8 \ \& \ T_9 \text{ are two middle terms.}}$

10. Find the 5^{th} term in the expansion of $\left(2x - \frac{x^2}{4}\right)^7$

Sol. Here : $a = 2x$, $b = -\frac{x^2}{4}$, $n = 7$ & $r = 4$

$T_{r+1} = \binom{n}{r} a^{n-r} b^r \Rightarrow T_{4+1} = \binom{7}{4}(2x)^{7-4}\left(-\frac{x^2}{4}\right)^4$

$T_5 = (35)(8x^3)\left(\frac{x^8}{256}\right) \Rightarrow \boxed{T_5 = \frac{35}{32}x^{11}}$

11. ~~Convert $22\frac{1}{2}°$ into radian measure.~~

Sol. $22\frac{1}{2}° = 22.5° = 22.5° \times \frac{\pi}{180} = \boxed{0.39\,\text{rad}}$

12. ~~Find the radius of the circle, when $\ell = 8.4$cm, $\theta = 2.8$ rad, r=?~~

Sol. We know that : $\ell = r\theta$

$\Rightarrow r = \frac{\ell}{\theta} = \frac{8.4}{2.8} = \boxed{3\,\text{cm}}$

13. Find 'x' if

$\tan^2 45° - \cos^2 60° = x \sin 45° \cos 45° \tan 60°$

Sol. $\tan^2 45° - \cos^2 60° = x \sin 45° \cos 45° \tan 60°$

$(1)^2 - \left(\frac{1}{2}\right)^2 = x\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)(\sqrt{3})$

$1 - \frac{1}{4} = x\left(\frac{2\sqrt{3}}{4}\right) \Rightarrow \frac{4-1}{4} \times \frac{4}{2\sqrt{3}} = x$

$\frac{3}{2\sqrt{3}} = x \Rightarrow \boxed{x = \frac{\sqrt{3}}{2}}$

14. Prove that:

$\cos^4\theta - \sin^4\theta = 1 - 2\sin^2\theta$

Sol. $\text{L.H.S.} = \cos^4\theta - \sin^4\theta$

$= (\cos^2\theta)^2 - (\sin^2\theta)^2$

$= (\cos^2\theta - \sin^2\theta)(\cos^2\theta + \sin^2\theta)$

$= (1 - \sin^2\theta - \sin^2\theta)(1)$

$= 1 - 2\sin^2\theta = \text{R.H.S.}$ **Proved.**

15. Prove that: $\cos(-\beta) = \cos\beta$

Sol. As, $\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

Put $\alpha = 0$, we have :

$\cos(0 - \beta) = \cos(0)\cos\beta + \sin(0)\sin\beta$

$\cos(-\beta) = 1.\cos\beta + 0.(\sin\beta)$

$\cos(-\beta) = \cos\beta + 0$

$\boxed{\cos(-\beta) = \cos\beta}$ **Proved.**

16. **Express $\sin x\cos 2x-\sin 2x\cos x$ as single term.**

Sol. $\sin x\cos 2x-\sin 2x\cos x$

$=\sin x\cos 2x-\cos x\sin 2x$

$=\sin(x-2x) \quad \because \left\{\begin{matrix}\sin(\alpha-\beta)\\ =\sin\alpha\cos\beta-\cos\alpha\sin\beta\end{matrix}\right\}$

$=\sin(-x)=\boxed{-\sin x}$

17. **Prove that: $\sin^2\alpha=\dfrac{1-\cos 2\alpha}{2}$**

Sol. Take $\cos 2\alpha=\cos(\alpha+\alpha)$

$\cos 2\alpha=\cos\alpha\cos\alpha-\sin\alpha\sin\alpha$

$\cos 2\alpha=\cos^2\alpha-\sin^2\alpha$

$\cos 2\alpha=1-\sin^2\alpha-\sin^2\alpha$

$\cos 2\alpha=1-2\sin^2\alpha$

$\Rightarrow 2\sin^2\alpha=1-\cos 2\alpha$

$\Rightarrow \sin^2\alpha=\dfrac{1-\cos 2\alpha}{2}$ Proved.

18. ~~**Express**~~ **$\cos 12\theta+\cos 4\theta$** ~~**as product.**~~

Sol. $\cos 12\theta+\cos 4\theta$

$=2\cos\left(\dfrac{12\theta+4\theta}{2}\right)\cos\left(\dfrac{12\theta-4\theta}{2}\right)$

$=\boxed{2\cos 8\theta\cos 4\theta}$

19. ~~**Given that, γ = 90º, α = 35º, a = 5, find angle β.**~~

Sol. We know that in any triangle:

$\alpha+\beta+\gamma=180^\circ$

$\beta=180^\circ-\alpha-\gamma$

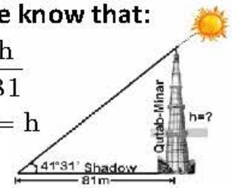
$\beta=180^\circ-35^\circ-90^\circ \Rightarrow \boxed{\beta=55^\circ}$

20. **Define angle of depression.**

Sol. If the line of sight is downward from the horizontal, the angle is called angle of Depression.

21. ~~**The shadow of Qutab Minar is 81m long when the measure of the angle of elevation of the sun is 41º31′. Find the height of the Qutab-Minar.**~~

Sol. From figure, we know that:



$\tan 41^\circ 31'=\dfrac{h}{81}$

$81\tan 41^\circ 31'=h$

$\boxed{h=71.70\,m}$

22. **In any triangle ABC, if a = 5, c = 6, a = 45º, then find γ.**

Sol. By using law of sines:

$\dfrac{a}{\sin\alpha}=\dfrac{b}{\sin\beta}=\dfrac{c}{\sin\gamma}$

Here we take: $\dfrac{a}{\sin\alpha}=\dfrac{c}{\sin\gamma}$

$5\sin\gamma=6\sin 45^\circ$

$\sin\gamma=\dfrac{6\sin 45^\circ}{5}=0.8485$

$\gamma=\sin^{-1}(0.8485) \Rightarrow \boxed{\gamma=58^\circ 3'}$

23. **Find the unit vector parallel to the sum of the vector.**

$\vec{a}=[2,4,-5]$ and $\vec{b}=[1,2,3]$

Sol. Let $\vec{v}$ = sum of the vectors

$\vec{a}\ \&\ \vec{b}=\vec{a}+\vec{b}=[2,4,-5]+[1,2,3]$

$\vec{v}=[3,6,-2]=3i+6j-2k$

$|\vec{v}|=\sqrt{(3)^2+(6)^2+(-2)^2}$

$|\vec{v}|=\sqrt{9+36+4}=\sqrt{49}=7$

Unit vector $=|\hat{v}|=\dfrac{\vec{v}}{|\vec{v}|}=\boxed{\dfrac{3i+6j-2k}{7}}$

24. **Find $(\vec{a}+\vec{b}).(\vec{a}-\vec{b})$ if**

$\vec{a}=2i+2j+3k$ & $\vec{b}=2i-j+k$

Sol. $\vec{a}+\vec{b}=(2i+2j+3k)+(2i-j+k)$

$\vec{a}+\vec{b}=2i+2j+3k+2i-j+k$

$\vec{a}+\vec{b}=4i+j+4k$

$\vec{a}-\vec{b}=(2i+2j+3k)-(2i-j+k)$

$\vec{a}+\vec{b}=2i+2j+3k+2i-j+k$

$\vec{a}+\vec{b}=4i+j+4k$

$\vec{a}-\vec{b}=(2i+2j+3k)-(2i-j+k)$

$\vec{a}-\vec{b}=2i+2j+3k-2i+j-k=3j+2k$

$(\vec{a}+\vec{b})\bullet(\vec{a}-\vec{b})=(4i+j+4k)\bullet(3j+2k)$

$=(4)(0)+(1)(3)+(4)(2)=0+3+8=\boxed{11}$

25. ~~Under what condition does the relation~~ $\vec{a}.\vec{b}=|\vec{a}||\vec{b}|$ ~~hold?~~

Sol. As, $\vec{a}.\vec{b}=|\vec{a}||\vec{b}|$

$\Rightarrow\quad |\vec{a}||\vec{b}|\cos\theta=|\vec{a}||\vec{b}| \because \left\{\begin{matrix}\text{By Definition}\\ \vec{a}.\vec{b}=|\vec{a}||\vec{b}|\cos\theta\end{matrix}\right\}$

$\Rightarrow\quad \cos\theta=\dfrac{|\vec{a}||\vec{b}|}{|\vec{a}||\vec{b}|} \quad\Rightarrow\quad \cos\theta=1$

$\Rightarrow\quad \theta=\cos^{-1}(1) \quad\Rightarrow\quad \boxed{\theta=0^\circ}$

26. ~~Express~~ $\sqrt{3}+j$ ~~in polar form.~~

Sol. Let $Z=\sqrt{3}+j$

Here : $a=\sqrt{3}$ & $b=1$

$r=\sqrt{a^2+b^2}$

$r=\sqrt{(\sqrt{3})^2+(1)^2}$

$r=\sqrt{3+1}=\sqrt{4}=2$

$\theta=\tan^{-1}\left(\dfrac{b}{a}\right)$

$\theta=\tan^{-1}\left(\dfrac{1}{\sqrt{3}}\right)=30^\circ$

$z=r\angle\theta=\boxed{2\ \angle 30^\circ}$

27. ~~Simplify the phasor~~ $\dfrac{-9+j4}{8-j3}$ ~~and write the result in rectangular form.~~

Sol. $\dfrac{-9+j4}{8-j3}$

$=\dfrac{-9+j4}{8-j3}\times\dfrac{8+j3}{8+j3}=\dfrac{-72-j27+j32+j^2 12}{(8)^2-(j3)^2}$

$=\dfrac{-72+j5-12}{64+9}=\dfrac{-84+j5}{73}=\boxed{\dfrac{-84}{73}+j\dfrac{5}{73}}$

Section - II

Note : Attemp any three (3) questions $3\times 8=24$

Q.2.(a) Solve the equation.

$\dfrac{a}{ax-1}+\dfrac{b}{bx-1}=a+b$ by factorization.

Sol. See Q.1(ix) of Ex # 1.1 (Page # 10)

(b) ~~Find the value of K given that if one root of $9x^2-15x+k=0$, exceeds the other by 3. Also find the roots.~~

Sol. See Q.8 of Ex # 1.3 (Page # 48)

Q.3. Find the coefficient of x^5 in the expansion of $\left(2x^2-\dfrac{3}{x}\right)^{10}$

Sol. See Q.5(i) of Ex # 2.1 (Page # 83)

Q.4.(a) If m = tanθ + sinθ and n = tan θ − sinθ, then prove that $m^2-n^2=4\sqrt{mn}$

Sol. See Q.22 of Ex # 3.3 (Page # 140)

(b) If $\cos=\dfrac{1}{5}$ and $\cos B=\dfrac{1}{2}$, A and B be acute angles, find cos (A − B)

Sol. See Q.8 of Ex # 4.1 (Page # 162)

Q.5.(a) ~~Prove that: $\cos 20^\circ+\cos 100^\circ+\cos 140^\circ=0$~~

Sol. See Q.13(iv) of Ex # 4.3 (Page # 192)

(b) Solve the triangle ABC when $c=4$, $a=70^\circ$ and $\gamma=45^\circ$

Sol. See Q.1 of Ex # 5.5 (Page # 232)

Q.6.(a) Find the magnitude and direction cosines of $3\bar{a}-2\bar{b}$ if

Sol. $\vec{a}=3\underline{i}-2\underline{j}+4\underline{k}$ and $\vec{b}=2\underline{i}+\underline{j}+3\underline{k}$

See Q.9(ii) of Ex # 6.1 (Page # 250)

(b) ~~Express~~ $\dfrac{(3+2j)(5-3j)}{3-4j}$ ~~in the form a+jb~~

Sol. See Q.2(d) of Ex # 07 (Page # 282)

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