DAE / IA - 2016

MATH-113 APPLIED MATHEMATICS - I

PAPER 'B' PART - A (OBJECTIVE)

Time: 30 Minutes Marks: 15

Q.1: *Encircle the correct answer.*

1. Area of an equilibrium triangle with side 'x' is;
[a] $\frac{\sqrt{3}}{4}x^2$ [b] $\frac{\sqrt{3}}{2}x^2$
[c] $\frac{2}{\sqrt{3}}x^2$ [d] None of these

2. Area of right triangle if base = 4cm, height = 6cm is:
[a] 24 sq.cm [b] 12 sq.cm
[c] 6 sq.cm [d] 10 sq.cm

3. Area of parallelogram having base 2cm and height 5cm is:
[a] 20 sq.cm [b] 30 sq.cm
[c] 15 sq.cm [d] 10 sq.cm

4. If diagonals of rhombus are 6cm and 5cm, then area is:
[a] 15 sq.cm [b] 30 sq.cm
[c] 10 sq.cm [d] 8 sq.cm

5. If the area of circle is 16π then radius is:
[a] 4 [b] 2 [c] 16 [d] 8

6. The length of diagonal of cube if edge of cube is 'a':
[a] $\sqrt{3}a$ [b] a^3
[c] $3a$ [d] $3a^2$

7. Lateral surface area of right circular cylinder is:
[a] πr^2 [b] πrh
[c] $2\pi rh$ [d] $2\pi r^2$

8. Volume of pyramid whose area of base $6a^2$ and height 'h' is:
[a] $\frac{1}{3}a^2h$ [b] $2a^2h$
[c] $3a^2h$ [d] a^2h

9. Volume of cone of radius of base 3cm and height 12cm is;
[a] 108π cu.cm [b] 36π cu.cm
[c] 12π cu.cm [d] 54π cu.cm

10. Unit vector $\mathbf{i}+\mathbf{j}+\mathbf{k}$ is;
[a] $i+j+k$ [b] $\frac{1}{3}(i+j+k)$
[c] $\frac{1}{\sqrt{3}}(i+j+k)$ [d] $\frac{1}{2}(i+j+k)$

11. If $\hat{i}$, $\hat{j}$ and $\hat{k}$ are orthogonal unit vector, then $\hat{j}\times\hat{i}=?$
[a] $\hat{k}$ [b] $-\hat{k}$
[c] 1 [d] -1

12. If $\vec{a}\times\vec{b}=0$ then $\vec{a}$ and $\vec{b}$ is;
[a] Non parallel [b] Parallel
[c] Perpendicular
[d] None of these

13. In an identity matrix all the diagonal elements are:
[a] zero [b] 2
[c] 1 [d] None of these

14. ~~If two rows of a determinant are identical then its value is;~~
[a] 1 [b] 0 [c] -1 [d] 2

15. Value of m for which matrix $\begin{bmatrix} 2 & 3 \\ 6 & m \end{bmatrix}$ is singular;
[a] 6 [b] 3
[c] 8 [d] 9

Answer Key

1	a	2	b	3	d	4	a	5	a
6	a	7	c	8	b	9	b	10	c
11	b	12	b	13	c	14	b	15	d

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DAE / IA - 2016

MATH - 113 APPLIED MATHEMATICS - I

PAPER 'B' PART - B (SUBJECTIVE)

Time: 2:30 Hrs Marks: 60

Section - I

Q.1. Write short answers to any Eighteen (18) questions.

1. Define equilateral triangle.

Sol. A triangle whose all sides are equal in length is called equilateral triangle.

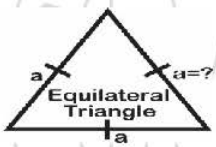
2. What is the side of the equilateral triangle whose area is $9\sqrt{3}$ sq.cm.

Sol. Let 'a' be length of each side of an equilateral triangle.

As, Area of equilateral triangle $= 9\sqrt{3}$ sq.cm

$$\Rightarrow \frac{\sqrt{3}}{4}a^2 = 9\sqrt{3}$$

$$\Rightarrow a^2 = 9\sqrt{3}\left(\frac{4}{\sqrt{3}}\right)$$



$$\Rightarrow a^2 = 36 \Rightarrow \sqrt{a^2} = \sqrt{36} \Rightarrow \boxed{a = 6\text{ cm}}$$

3. Write an area and perimeter of a square of sides 'a'.

Sol. Let each side of square has length equal to 'a'

Area = a^2 sq. unit &

Perimeter = 4a unit

4. The diagonals of a rhombus are 40cm and 30cm, find its area.

Sol. Here $d_1 = 40m$, & $d_2 = 30m$

$$\text{Area of Rhombus} = \frac{d_1 \times d_2}{2}$$

$$\text{Area} = \frac{40 \times 30}{2} = \boxed{600\text{ sq.m}}$$

5. Find the interior angle of hexagon.

Sol. Here n = 6

$$\text{Interior angle} = \theta = \frac{2n-4}{n} \times 90^\circ$$

$$\theta = \frac{2(6)-4}{6} \times 90^\circ$$

$$\theta = \frac{8}{6} \times 90^\circ = \boxed{120^\circ}$$

6. The perimeter of a regular hexagon is 12cm, find its area.

Sol. Perimeter of hexagon = 12 cm

$$6a = 12$$

$$\Rightarrow a = 12/6 = 2\text{cm}$$

$$\text{Area} = \frac{na^2}{4}\cot\left(\frac{180^\circ}{n}\right)$$

$$A = \frac{6(2)^2}{4}\cot\left(\frac{180^\circ}{6}\right) = 6\cot 30^\circ$$

$$A = \frac{6}{\tan 30^\circ} = \boxed{10.39\text{ sq.cm}}$$

7. What is the area and circumference of circle?

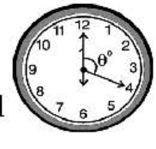
Sol. Area of Circle $= A = \boxed{\pi r^2 \text{ sq.unit}}$

Circumference of Circle $= C = \boxed{2\pi r \text{ unit}}$

8. The minute hand of a clock is 12cm long. Find the area which is described on the clock face between 6 A.M to 6.20 A.M.

Sol. Here: radius = r = 12cm

$$\theta = \frac{360^\circ}{12} \times 4 = 120^\circ$$



$$\theta = 120 \times \frac{\pi}{180} = \frac{2\pi}{3}\text{rad}$$

$$\text{Area of sector} = \frac{1}{2}r^2\theta = \frac{1}{2}(12)^2\left(\frac{2\pi}{3}\right)$$

$$\text{Area} = \frac{144\pi}{3} = \boxed{150.7\text{cm}^2}$$

9. If base of a field is 50m and number of ordinates are 11, then find breadth of strip.

Sol. Length of base = 50 m

As, No. of ordinates = 11

so, No. of strips = 10

S = Width of each strip

$$S = \frac{\text{Length of base}}{\text{No. of strips}} = \frac{50}{10} = \boxed{5\text{m}}$$

10. How many match box each 80mm by 75 mm by 18mm, can be packed into a box 72cm by 45cm, 60cm internally?

Sol. ℓ = Length of brick = 80mm = 8cm

b = breath of Match box = 75mm = 7.5cm

h = Height of Match box = 18mm = 1.8cm

L = Length of box = 72cm

B = breath of wall = 45cm

H = Height of wall = 60cm

Volume of brick = $V_1 = \ell bh$

$$V_1 = 8 \times 7.5 \times 1.8 = 108\text{cm}^3$$

Volume of wall = V_2 = LBH

$$V_2 = 72 \times 45 \times 60 = 1944000\text{cm}^3$$

$$\text{No. of Bricks} = \frac{V_2}{V_1} = \frac{1944000}{108} = \boxed{1800}$$

11. The volume of the cube is 95 cu.cm. Find the surface area and the edge of the cube.

Sol. Let 'a' be edge of cube

As, volume = 95

$$a^3 = 95$$

$$\left(a^3\right)^{1/3} = (95)^{1/3}$$

Edge of cube = a = $\boxed{4.56\,\text{cm}}$

Surface area of cube = $6a^2$

$$\text{S.A.} = 6(4.56)^2 = \boxed{124.92\,\text{cm}^2}$$

12. Write the formula of total surface of cylinder.

Sol. $\text{T.S.A.} = 2\pi rh + 2\pi r^2$ sq.unit

13. The diameter of the base of a right circular cylinder is 14cm and its height is 10cm. Find the volume of cylinder.

Sol. Here : d = 14cm & h = 10cm

$$\text{As, } d = 14\text{cm} \Rightarrow r = \frac{d}{2} = \frac{14}{2} = 7\text{cm}$$

$$\text{Volume} = \pi r^2 h$$

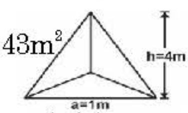
$$V = \pi(7)^2 10 = \boxed{1539.38\,\text{cm}^3}$$

14. Find the volume of pyramid whose base is an equilateral triangle of side 1m and height is 4m.

Sol. Here: V=?, a=1m, & h=4m

Area of base (Equilateral Triangle)

$$= \frac{\sqrt{3}}{4}a^2 = \frac{\sqrt{3}}{4}(1)^2 = 0.43\text{m}^2$$



$$\text{Volume} = \frac{1}{3} \times \text{area of base} \times \text{height}$$

$$V = \frac{1}{3} \times 0.43 \times 4 = \boxed{0.58\,\text{m}^3}$$

15. Find volume of a pentagonal based pyramid whose area of base is 15 sq.cm and height is 15cm.

Sol. Here: V=? Area of base = 15 cm² & height = 15cm

$$\text{Volume} = \frac{1}{3} \times \text{area of base} \times \text{height}$$

$$V = \frac{1}{3} \times 15 \times 15 = \boxed{75\,\text{cm}^3}$$

16. Find the cost of painting @ Rs.7.5 per sq.cm on a conical spire 64cm in circumference at the base and 108cm in slant height.

Sol. Here: C = 64cm, ℓ = 108cm & Cost=?

As, $C = 64$

$$\Rightarrow 2\pi r = 64 \Rightarrow r = \frac{64}{2\pi} \Rightarrow r = \frac{32}{\pi}$$

$$L.S.A. = \pi r\ell = \pi\left(\frac{32}{\pi}\right)108 = 3456\,cm^2$$

As, cost of painting per $cm^2 = Rs.750$

Total cost = Rate × L.S.A

$$= 7.50 \times 3456 = \boxed{Rs.25920}$$

17. How many lead balls, each of radius 1cm can be made from a sphere whose radius is 8cm.

Sol. Let r_1 = Radius of sphere = 8cm

& r_2 = Radius of ball = 1cm

$$V_1 = \text{Volume of sphere} = \frac{4}{3}\pi r_1^3$$

$$V_1 = \frac{4}{3}\pi(8)^3 = 2144.66\,cm^3$$

$$V_2 = \text{Volume of Ball} = \frac{4}{3}\pi r_2^3$$

$$V_2 = \frac{4}{3}\pi(1)^3 = 4.19\,cm^3$$

$$\text{No. of balls made} = \frac{V_1}{V_2}$$

$$= \frac{2144.66}{4.19} = \boxed{512\,balls}$$

18. Find 'α' so that $\left|\alpha i + (\alpha+1)j + 2k\right| = 3$

Sol. $\left|\alpha\hat{i} + (\alpha+1)\hat{j} + 2\hat{k}\right| = 3$

$$\sqrt{(\alpha)^2 + (\alpha+1)^2 + (2)^2} = 3$$

$$\sqrt{\alpha^2 + \alpha^2 + 2\alpha + 1 + 4} = 3$$

$$\sqrt{2\alpha^2 + 2\alpha + 5} = 3$$

Squaring both sides, we get :

$$2\alpha^2 + 2\alpha + 5 = 9$$

$$2\alpha^2 + 2\alpha + 5 - 9 = 0$$

$$2\alpha^2 + 2\alpha - 4 = 0$$

$$2\alpha^2 + 4\alpha - 2\alpha - 4 = 0 \text{ \{By Factorization\}}$$

$$2\alpha(\alpha+2) - 2(\alpha+2) = 0$$

$$(\alpha+2)(2\alpha-2) = 0$$

Either $\alpha + 2 = 0$, $\boxed{\alpha = -2}$

OR $2\alpha - 2 = 0$, $2\alpha = 2 \Rightarrow \boxed{\alpha = 1}$

19. Find unit vector along the vector $4i - 3j - 5k$

Sol. Let $\vec{a} = 4i - 3j - 5k$

$$|\vec{a}| = \sqrt{(4)^2 + (-3)^2 + (-5)^2}$$

$$|\vec{a}| = \sqrt{16 + 9 + 25} = \sqrt{50}$$

$$|\vec{a}| = \sqrt{25 \times 2} = 5\sqrt{2}$$

$$\text{Unit Vector} = \hat{a} = \frac{\vec{a}}{|\vec{a}|} = \boxed{\frac{4i - 3j - 5k}{5\sqrt{2}}}$$

20. Find $(\vec{a} + \vec{b}).(\vec{a} - \vec{b})$ if $\vec{a} = 2i + 2j + 3k$ & $\vec{b} = 2i - j + k$

Sol.

$$\vec{a} + \vec{b} = (2i + 2j + 3k) + (2i - j + k)$$

$$\vec{a} + \vec{b} = 2i + 2j + 3k + 2i - j + k$$

$$\vec{a} + \vec{b} = 4i + j + 4k$$

$$\vec{a} - \vec{b} = (2i + 2j + 3k) - (2i - j + k)$$

$$\vec{a} - \vec{b} = 2i + 2j + 3k - 2i + j - k$$

$$\vec{a} - \vec{b} = 3j + 2k$$

$$(\vec{a} + \vec{b}) \bullet (\vec{a} - \vec{b})$$

$$= (4i + j + 4k) \bullet (3j + 2k)$$

$$= (4)(0) + (1)(3) + (4)(2)$$

$$= 0 + 3 + 8 = \boxed{11}$$

21. Define Vector product.

Sol. The vector product of two vectors $\vec{a}$ & $\vec{b}$ is denoted by $\vec{a} \times \vec{b}$ and is defined as $\vec{a} \times \vec{b} = |\vec{a}||\vec{b}|\sin\theta\,\hat{n}$.

22. If $\vec{a} = 2i + 3j + 4k$ & $\vec{b} = i - j + k$

Find $|\vec{a} \times \vec{b}|$

Sol. $\vec{a} \times \vec{b} = \begin{vmatrix} i & j & k \\ 2 & 3 & 4 \\ 1 & -1 & 1 \end{vmatrix}$

$= i\begin{vmatrix} 3 & 4 \\ -1 & 1 \end{vmatrix} - j\begin{vmatrix} 2 & 4 \\ 1 & 1 \end{vmatrix} + k\begin{vmatrix} 2 & 3 \\ 1 & -1 \end{vmatrix}$

$= i(3+4) - j(2-4) + k(-2-3)$

$= i(7) - j(-2) + k(-5)$

$= 7i + 2j - 5k$

$|\vec{a} \times \vec{b}| = \sqrt{49+4+25} = \boxed{\sqrt{78}}$

23. **Define Diagonal matrix.**

Sol. A square matrix in which all elements except diagonal elements are zero is called diagonal matrix.

24. **Show that** $A = \begin{bmatrix} 2 & 3 & -1 \\ 1 & 1 & 0 \\ 2 & -3 & 5 \end{bmatrix}$ **is singular matrix.**

Sol. $A = \begin{bmatrix} 2 & 3 & -1 \\ 1 & 1 & 0 \\ 2 & -3 & 5 \end{bmatrix}$

$= 2\begin{bmatrix} 1 & 0 \\ -3 & 5 \end{bmatrix} - 3\begin{bmatrix} 1 & 0 \\ 2 & 5 \end{bmatrix} - 1\begin{bmatrix} 1 & 1 \\ 2 & -3 \end{bmatrix}$

$= 2(5-0) - 3(5-0) - 1(-3-2)$

$= 10 - 15 + 15 = 15 - 15 = \boxed{0}$

25. **Find x and y if**

$\begin{bmatrix} x+3 & 1 \\ -3 & 3y-4 \end{bmatrix} = \begin{bmatrix} y & 1 \\ -3 & 2x \end{bmatrix}$

Sol. $\begin{bmatrix} x+3 & 1 \\ -3 & 3y-4 \end{bmatrix} = \begin{bmatrix} y & 1 \\ -3 & 2x \end{bmatrix}$

Comparing corresponding elements of both matrices :

$x + 3 = y$ and $3y - 4 = 2x$

$x - y = -3 \rightarrow (i)$ | $2x - 3y = -4 \rightarrow (ii)$

CMultipling eq. (i) by 2 and subtracting eq. (ii)

$$\begin{aligned} 2x - 2y &= -6 \\ -2x \mp 3y &= \mp 4 \\ \hline y &= -2 \end{aligned}$$

$\boxed{y = -2}$

Put $y = -2$ in eq. (i)

$x - (-2) = -3$

$x = -3 - 2$

$\boxed{x = -5}$

26. **If** $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$

then find AB.

Sol. $AB = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$

$= \begin{bmatrix} 2+8 & 3+10 \\ 6+16 & 9+20 \end{bmatrix}$

$= \boxed{\begin{bmatrix} 10 & 13 \\ 22 & 29 \end{bmatrix}}$

27. **Find** A^{-1} **if** $A = \begin{bmatrix} 5 & 3 \\ 1 & 1 \end{bmatrix}$

Sol. $A = \begin{bmatrix} 5 & 3 \\ 1 & 1 \end{bmatrix}$

$|A| = \begin{vmatrix} 5 & 3 \\ 1 & 1 \end{vmatrix} = 5 - 3 = 2$

$\text{Adj } A = \begin{bmatrix} 1 & -3 \\ -1 & 5 \end{bmatrix}$

$A^{-1} = \dfrac{\text{Adj}(A)}{|A|}$

$A^{-1} = \dfrac{\begin{bmatrix} 1 & -3 \\ -1 & 5 \end{bmatrix}}{2} = \boxed{\begin{bmatrix} \frac{1}{2} & -\frac{3}{2} \\ -\frac{1}{2} & \frac{5}{2} \end{bmatrix}}$

Section - II

Note : Attemp any three (3) questions $3 \times 8 = 24$

Q.2.(a) Find area of a triangle whose

sides are in the ratio of 9:40:41 and whose perimeter is 180 meters.

Sol. See Q.7 of Ex # 10 (Page # 466)

(b) **The difference between two parallel sides of a trapezoid is 8m. The perpendicular distance between them is 24m and the area of the trapezoid is 312 square meter. Find the two parallel sides.**

Sol. See Q.6 of Ex # 11 (Page # 476)

Q.3.(a) **What is the length of the side and area of the largest hexagon that can be cut from 8cm round bar.**

Sol. See Q.9 of Ex # 12 (Page # 489)

(b) **Find the area of the field whose ordinates are 0, 20, 22.5, 33.5, 45, 42, 33, 5, 25.5 and 0 meter respectively. The width of each strip is 14m. Also find the approximately cost or purchasing the field at a cost of Rs. 5000 per sq.m.**

Sol. See example # 03 of Ch # 14

Q.4.(a) **A regular hexagonal pyramid has the perimeter of its base 12cm and its altitude is 15m. Find its volume.**

Sol. See Q.3 of Ex # 17[A] (Page # 545)

(b) **The radius of the base of a right circular cone in 6m and its height is 6.5m, find the volume and lateral surface area.**

Sol. See Q.1 of Ex # 18[A] (Page # 565)

Q.5.(a) **Given the vectors $\vec{\mathbf{a}} = 3\mathbf{i} - 2\mathbf{j} + 4\mathbf{k}$ and $\vec{\mathbf{b}} = 2\mathbf{i} + \mathbf{j} + 3\mathbf{k}$, find the magnitude and the direction cosines $\vec{\mathbf{a}} - \vec{\mathbf{b}}$.**

Sol. See Q.9(i) of Ex # 8.1 (Page # 374)

(b) **Find the sine of the angle and unit vector perpendicular to each.**

$$\vec{\mathbf{a}} = 2\mathbf{i} - \mathbf{j} + \mathbf{k} \ \& \ \vec{\mathbf{b}} = 3\mathbf{i} + 4\mathbf{j} - \mathbf{k}$$

Sol. See Q.19(ii) of Ex # 8.2 (Page # 389)

Q.6. **Solve the following system of equation by Cramer's Rule.**

$x + y + z = 0$, $2x - y - 4z = 15$, $x - 2y - z = 7$

Sol. See Q.8(v) of Ex # 9.2 (Page # 431)

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