DAE / IA - 2016

MATH - 113 APPLIED MATHEMATICS - I

PAPER 'A' PART - A (OBJECTIVE)

Time: 30 Minutes Marks: 15

Q.1: *Encircle the correct answer.*

1. **A second degree equation is known as;**
[a] Linear [b] Quadratic
[c] Cubic [d] None of these

2. $\ell x^2 + mx + n = 0$ **will be a pure quadratic if:**
[a] $\ell = 0$ [b] $n = 0$
[c] $m = 0$ [d] Both $\ell, m = 0$

3. **The nth term of an A.P whose 1st term is 'a' and common difference is 'd' is:**
[a] $2a + (n+1)d$ [b] $a + (n+1)d$
[c] $a + (n-1)d$ [d] $2a + (n-1)d$

4. **Geometric mean between 3 and 27 is:**
[a] -9 [b] 12
[c] 15 [d] ± 9

5. **The sum of infinite geometric series** $1 + \frac{1}{3} + \frac{1}{9} + \ldots$ **is:**
[a] $\frac{2}{3}$ [b] $-\frac{2}{3}$
[c] $\frac{3}{2}$ [d] $-\frac{3}{2}$

6. **In the expansion** $(a+b)^n$ **the general term is:**
[a] $\binom{n}{r} a^r b^r$ [b] $\binom{n}{r} a^{n-r} b^r$
[c] $\binom{n}{r-1} a^{n-r+1} b^{r-1}$
[d] $\binom{n}{r} a^{n-r-1} b^{r-1}$

7. **The value of** $\binom{n}{n}$ **is equal to:**
[a] Zero [b] 1
[c] n [d] -n

8. **The fractions is** $\frac{2x+5}{x^2+5x+6}$ **known as:**
[a] Proper [b] Improper
[c] Both a & b [d] None of these

9. **If l = 12cm and r = 3cm, then θ is equal to:**
[a] 36rad [b] 4rad
[c] $\frac{1}{4}\text{rad}$ [d] 18rad

10. **If sin θ is positive tan θ is negative, then the terminal side of the angle lies in quadrant:**
[a] 1st [b] 2nd
[c] 3rd [d] 4th

11. $\sec^2\theta + \operatorname{cosec}^2\theta$ **is equal to:**
[a] $\sec^2\theta \operatorname{cosec}^2\theta$ [b] $\sin\theta\cos\theta$
[c] $2\sec^2$ [d] $2\operatorname{cosec}^2\theta$

12. $\sin(90° - \theta)$ **is equal to:**
[a] $-\sin\theta$ [b] $\sin\theta$
[c] $-\cos\theta$ [d] $\cos\theta$

13. $\frac{\sin(\alpha+\beta)}{\cos\alpha\cos\beta}$ **is equal to:**
[a] $\tan\alpha - \tan\beta$ [b] $\tan\alpha + \tan\beta$
[c] $\sin\alpha - \sin\beta$ [d] $\sin\alpha - \sin\beta$

14. **In a triangle ABC, ∠A = 70º, ∠B = 60º then ∠C is:**
[a] $30°$ [b] $40°$
[c] $50°$ [d] $60°$

15. **If B = 90º, b = 2, A = 30º, then side 'a' is;**
[a] 4 [b] 3
[c] 2 [d] 1

Answer Key

1	b	2	b	3	c	4	d	5	c
6	b	7	b	8	a	9	b	10	b
11	a	12	d	13	b	14	c	15	d

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DAE / IA - 2016

MATH - 113 APPLIED MATHEMATICS - I

PAPER 'A' PART - B (SUBJECTIVE)

Time: 2:30 Hrs Marks: 60

Section - I

Q.1. Write short answers to any Eighteen (18) questions.

1. Solve the quadratic equation $x(x+7)=(2x-1)(x+4)$ by factorization.

Sol. $x(x+7)=(2x-1)(x+4)$

$x^2+7x=2x^2+8x-x-4$

$x^2+7x-2x^2-8x+x+4=0$

$-x^2+4=0 \Rightarrow x^2-4=0$

$(x)^2-(2)^2=0 \Rightarrow (x-2)(x+2)=0$

Either $x-2=0$, $x=2$ OR $x+2=0$, $x=-2$

S.S. $=\{-2, 2\}$

2. Prove that the roots of the equation $(a+b)x^2-ax-b=0$ are rational.

Sol. Here: $A=(a+b)$, $B=-a$, $C=-b$

Disc. $=B^2-4AC$

$=(-a)^2-4(a+b)(-b)$

$=a^2+4ab+4b^2$

$=(a+2b)^2$ Which is a perfect square.

Hence roots are Rational. Proved.

3. ~~Form the quadratic equation whose roots are~~ $i\sqrt{3}$ and $-i\sqrt{3}$

Sol.

$S=i\sqrt{3}+(-i\sqrt{3})$

$S=i\sqrt{3}-i\sqrt{3}$

$S=0$

$P=(i\sqrt{3})(-i\sqrt{3})$

$P=-(i)^2(\sqrt{3})^2$

$P=-(-1)(3)=3$

$x^2-Sx+P=0$

$x^2-0x+3=0 \Rightarrow \boxed{x^2+3=0}$

4. Define common difference.

Sol. The difference between any two consecutive terms of an A.P. is called common difference.

5. ~~Write the formula to find the sum of 'n' terms of an arithmetic sequence.~~

Sol. $\boxed{S_n=\frac{n}{2}[2a_1+(n-1)d]}$

6. Find the 7th term of A.P., in which the first term is 7 and the common difference is -3.

Sol. Here: $a_7=?$, $a_1=7$ & $d=-3$

$a_7=a+6d$

$a_7=7+6(-3)$

$a_7=7-18 \Rightarrow \boxed{a_7=-11}$

7. ~~Find the sum of the series~~ $1+\frac{1}{3}+\frac{1}{9}+\ldots$ ~~to 6 terms.~~

Sol. Here: $a_1=1$, $r=\frac{1}{3}\div 1=\frac{1}{3}$, $n=6$ & $S_6=?$

$S_n=\frac{a(1-r^n)}{1-r} \Rightarrow S_6=\frac{1\left(1-\left(\frac{1}{3}\right)^6\right)}{1-\frac{1}{3}}$

$S_6=\frac{1-\frac{1}{729}}{\frac{3-1}{3}}=\frac{\frac{729-1}{729}}{\frac{2}{3}}=\frac{728}{729}\times\frac{3}{2}=\boxed{\frac{364}{243}}$

8. Find the geometric mean between 8 and 72.

Sol. Let: $a=8$ & $b=72$

As, $G=\pm\sqrt{ab}$

$G=\pm\sqrt{8\times 72}=\pm\sqrt{576}=\boxed{\pm 24}$

9. ~~Find the sum of infinite geometric series in which~~ $a=128$ and $r=-\frac{1}{2}$.

Sol. $S_\infty = \dfrac{a}{1-r}$

$$S_\infty = \frac{128}{1-\left(-\frac{1}{2}\right)} = \frac{128}{1+\frac{1}{2}} = \frac{128}{\frac{2+1}{2}}$$

$$S_\infty = \frac{128}{3/2} = 128 \times \frac{2}{3} = \boxed{\frac{256}{3}}$$

10. **Expand $(2x-3y)^4$ by Binomial theorem.**

Sol. $(2x-3y)^4$

$$= \binom{4}{0}(2x)^4(3y)^0 - \binom{4}{1}(2x)^3(3y)^1 + \binom{4}{2}(2x)^2(3y)^2 - \binom{4}{3}(2x)^1(3y)^3 + \binom{4}{4}(2x)^0(3y)^4$$

$$= (1)(16x^4)(1) - 4(8x^3)(3y) + 6(4x^2)(9y^2) - 4(2x)(27y^3) + (1)(1)(81y^4)$$

$$= \boxed{16x^4 - 96x^3y + 216x^2y^2 - 216xy^3 + 81y^4}$$

11. ~~**Compute $(1.02)^4$ to two decimal places by use of Binomial formula.**~~

Sol. $(1.02)^4 = (1+0.02)^4$

$$= \binom{4}{0}(1)^4(0.02)^0 + \binom{4}{1}(1)^3(0.02)^1 + \binom{4}{2}(1)^2(0.02)^2 + \dots$$

$$= (1)(1)(1) + 4(1)(0.02) + 6(1)(0.0004) + \dots$$

$$= 1 + 0.08 + 0.0024 + \dots = 1.0824 = \boxed{1.08}$$

12. ~~**Expand $\dfrac{1}{\sqrt{1+x}}$ to three terms.**~~

Sol. $\dfrac{1}{\sqrt{1+x}} = (1+x)^{-1/2}$

$$= 1 + \left(-\frac{1}{2}\right)(x) + \frac{\left(-\frac{1}{2}\right)\left(-\frac{1}{2}-1\right)}{2!}(x)^2 + \dots$$

$$= 1 - \frac{x}{2} + \frac{1}{2}\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)x^2 + \dots$$

$$= \boxed{1 - \frac{x}{2} + \frac{3}{8}x^2 + \dots}$$

13. **Find the 5th term in the expansion of $\left(2x - \dfrac{x^2}{4}\right)^7$**

Sol. Here: $a = 2x,\ b = -\dfrac{x^2}{4},\ n = 7\ \&\ r = 4$

$$T_{r+1} = \binom{n}{r}a^{n-r}b^r \Rightarrow T_{4+1} = \binom{7}{4}(2x)^{7-4}\left(-\frac{x^2}{4}\right)^4$$

$$T_5 = (35)(8x^3)\left(\frac{x^8}{256}\right) \Rightarrow \boxed{T_5 = \frac{35}{32}x^{11}}$$

14. **Resolve into partial fractions $\dfrac{2x}{(x-2)(x+5)}$**

Sol. $\dfrac{2x}{(x-2)(x+5)} = \dfrac{A}{x-2} + \dfrac{B}{x+5} \rightarrow (i)$

$2x = A(x+5) + B(x-2) \rightarrow (ii)$

Put $x = 2$ in eq. (ii)

$2(2) = A(2+5) + B(2-2)$

$4 = A(7) + B(0)$

$4 = 7A + 0 \Rightarrow \boxed{A = 4/7}$

Put $x = -5$ in eq. (ii)

$2(-5) = A(-5+5) + B(-5-2)$

$-10 = A(0) + B(-7)$

$-10 = 0 - 7B \Rightarrow \boxed{B = 10/7}$

Put values of A, & B in eq. (i),

we get: $\boxed{\dfrac{4}{7(x-2)} + \dfrac{10}{7(x+5)}}$

15. **Write an identity equation of $\dfrac{2x+5}{x^2+5x+6}$**

Sol. $\dfrac{2x+5}{x^2+5x+6}$

$= \dfrac{2x+5}{(x+2)(x+3)}$

$= \boxed{\dfrac{A}{x+2} + \dfrac{B}{x+3}}$

$x^2 + 5x + 6$
$= x^2 + 3x + 2x + 6$
$= x(x+3) + 2(x+3)$
$= (x+3)(x+2)$

16. ~~**Convert $\frac{\pi}{2}$ rad into degree measure.**~~

Sol. $\frac{\pi}{2}\text{rad} = \frac{\pi}{2}\times\frac{180^\circ}{\pi} = \frac{180^\circ}{2} = 90$

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17. **Prove that:**

$$\tan^2 30^\circ + \tan^2 45^\circ + \tan^2 60^\circ = \frac{13}{3}$$

Sol. $\text{L.H.S.} = \tan^2 30^\circ + \tan^2 45^\circ + \tan^2 60^\circ$

$$= \left(\frac{1}{\sqrt{3}}\right)^2 + (1)^2 + \left(\sqrt{3}\right)^2 = \frac{1}{3} + 1 + 3$$

$$= \frac{1+3+9}{3} = \frac{13}{3} = \text{R.H.S.}$$ **Proved.**

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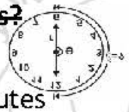
18. ~~**If a minute hand of a clock is 10cm long, how for does the tip of the hand moves in 30 minutes?**~~



Sol. Here: $r = 10\text{cm}$ & $\ell = ?$

θ = hand moves in 30 minutes

$$= 180^\circ = 180\times\frac{\pi}{180} = \pi \text{ rad}$$

By using formula: $\ell = r\theta$

$$\ell = r\theta = (10)\pi = \boxed{31.4\,\text{cm}}$$

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19. **Prove that:**

$$(1+\sin\theta)(1-\sin\theta) = \frac{1}{\sec^2\theta}$$

Sol. $\text{L.H.S.} = (1+\sin\theta)(1-\sin\theta)$

$$= (1)^2 - (\sin\theta)^2 = 1 - \sin^2\theta$$

$$= \cos^2\theta = \frac{1}{\sec^2\theta} = \text{R.H.S.}$$ **Proved.**

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20. **Prove that:** $\tan(-\beta) = -\tan\beta$

Sol. As, $\tan(\alpha-\beta) = \dfrac{\tan\alpha - \tan\beta}{1+\tan\alpha\tan\beta}$

Put $\alpha = 0$ & $\beta = \theta$ we have:

$$\tan(0-\theta) = \frac{\tan 0 - \tan\theta}{1+\tan 0\tan\theta}$$

$$\tan(-\theta) = \frac{0-\tan\theta}{1+(0)\tan\theta} \because \left\{\begin{matrix}\text{Using calculator}\\ \tan 0 = 0\end{matrix}\right\}$$

$$\tan(-\theta) = \frac{-\tan\theta}{1+0}$$

$$\boxed{\tan(-\theta) = -\tan\theta}$$ **Proved.**

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21. **Show that:**

$$\sin\left(\theta+\frac{\pi}{6}\right) + \cos\left(\theta+\frac{\pi}{3}\right) = \cos\theta$$

Sol. $\text{L.H.S.} = \sin\left(\theta+\frac{\pi}{6}\right) + \cos\left(\theta+\frac{\pi}{3}\right)$

$$= \sin(\theta+30^\circ) + \cos(\theta+60^\circ)$$

$$= \sin\theta\cos 30^\circ + \cos\theta\sin 30^\circ + \cos\theta\cos 60^\circ - \sin\theta\sin 60^\circ$$

$$= \sin\theta\left(\frac{\sqrt{3}}{2}\right) + \cos\theta\left(\frac{1}{2}\right) + \cos\theta\left(\frac{1}{2}\right) - \sin\theta\left(\frac{\sqrt{3}}{2}\right)$$

$$= \frac{\cos\theta}{2} + \frac{\cos\theta}{2}$$

$$= 2.\frac{\cos\theta}{2} = \cos\theta = \text{R.H.S.}$$ **Proved.**

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22. **Express** $\sin x\cos 2x - \sin 2x\cos x$ **as single term.**

Sol. $\sin x\cos 2x - \sin 2x\cos x$

$$= \sin x\cos 2x - \cos x\sin 2x$$

$$= \sin(x-2x) \because \left\{\begin{matrix}\sin(\alpha-\beta)\\ = \sin\alpha\cos\beta - \cos\alpha\sin\beta\end{matrix}\right\}$$

$$= \sin(-x) = \boxed{-\sin x}$$

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23. ~~**Express**~~ $2\cos 5\theta\sin 3\theta$ ~~**as sum of difference.**~~

Sol. $2\cos 5\theta\sin 3\theta$

$$= \sin(5\theta+3\theta) - \sin(5\theta-3\theta)$$

$$= \boxed{\sin 8\theta - \sin 2\theta}$$

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24. **Define the law of Sine.**

Sol. In any triangle ABC, with usual notations.

$$\boxed{\frac{a}{\sin\alpha}=\frac{b}{\sin\beta}=\frac{c}{\sin\gamma}}$$

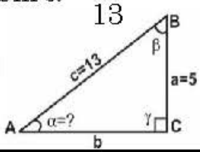
25. ~~**In right triangle ABC, γ = 90º, a = 5, c = 13 then find value of angle ∝.**~~

Sol. We know that, from figure:

$$\sin\alpha=\frac{a}{c}\ \Rightarrow\ \sin\alpha=\frac{5}{13}$$

$$\alpha=\sin^{-1}\left(\tfrac{5}{13}\right)$$

$$\boxed{\alpha=22^\circ 37'}$$



26. **In any triangle ABC in which b = 45, c = 34, ∝ = 52º, find a.**

Sol. By using Law of cosines:

$$a^2=b^2+c^2-2bc\cos 52^\circ$$

$$a^2=(45)^2+(34)^2-2(45)(34)\cos 52^\circ$$

$$a^2=2025+1156-1883.92$$

$$a^2=1297.07\ \Rightarrow\ \boxed{a=36.01}$$

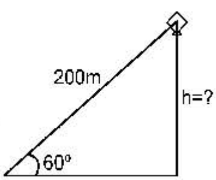
27. ~~**A string of a flying kite is 200 meters long, and its angle of elevation is 60º. Find the height of the kite above the ground taking the string to be fully stretched.**~~

Sol. In this figure:

$$\sin 60^\circ=\frac{h}{200}$$

$$200\sin 60^\circ=h$$

$$\boxed{h=173.20\,\text{m}}$$



Section - II

Note : Attemp any three (3) questions 3 × 8 = 24

Q.2.(a) Solve the equation

$$x^2+x(m-n)x-2(m-n)^2=0$$

by using Quadratic formula.

Sol. See Q.3(v) of Ex# 1.1 (Page # 22)

(b) **If the roots of the equation** $px^2+qx+q=0$ **are** α **and** β **prove that** $\sqrt{\frac{\alpha}{\beta}}+\sqrt{\frac{\beta}{\alpha}}+\sqrt{\frac{q}{p}}=0$

Sol. See Q.6 of Ex# 1.3 (Page # 46)

Q.3.(a) If 5,8 are two A.M's between a and b, find a and b.

Sol. See Q.6 of Ex# 2.2 (Page # 85)

(b) ~~**How many terms of the series 5 + 7 + 9 + ... Amount to 192?**~~

Sol. See Q.5(ii) of Ex# 2.3 (Page # 92)

Q.4.(a) Find the constant term in the expansion of $\left(x^2-\frac{1}{x}\right)^9$

Sol. See Q.9(i) of Ex# 3.1 (Page # 160)

(b) **Resolve** $\frac{1}{x^3+1}$ **into partial fractions.**

Sol. See Q.4 of Ex# 4.3 (Page # 207)

Q.5.(a) Prove that

$$(\sec\theta-\tan\theta)^2=\frac{I-\sin\theta}{I+\sin\theta}$$

Sol. See Q.5 of Ex# 5.3 (Page # 255)

(b) **Prove that**

$$\frac{\sin 3\theta}{\cos\theta}+\frac{\cos 3\theta}{\sin\theta}=2\cot 2\theta$$

Sol. See Q.14 of Ex# 6.2 (Page # 301)

Q.6.(a) ~~**Express sin 3θ + sin5θ + sin 7θ + sin 9θ as product.**~~

Sol. See Q.3 of Ex# 6.3 (Page # 307)

(b) **Find the angle of largest measure in the triangle ABC where a = 224, b = 380 and c = 340.**

Sol. See Q.6(i) of Ex# 7.5 (Page # 356)

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