DAE / IA - 2016

MATH-123 APPLIED MATHEMATICS - I

PAPER 'B' PART - A (OBJECTIVE)

Time : 30 Minutes Marks : 15

Q.1: *Encircle the correct answer.*

1. Conjugate of $(2+3i)+(1-i)$ is:

[a] $3-2i$ [b] $3+4i$
[c] $3-4i$ [d] $3+2i$

2. Ordered pair form of $-3-2i$ is:

[a] (3, 2) [b] (–3, –2)
[c] (–3, 2) [d] (3, –2)

3. If $z=a+bi$ then $z+\bar{z}$ is equal to:

[a] $2a$ [b] $2b$
[c] 0 [d] $2a+2bi$

4. The equivalent partial fractions of $\frac{x+11}{(x+1)(x-3)^2}$ is:

[a] $\frac{A}{x+1}+\frac{B}{(x-3)^2}$

[b] $\frac{A}{x-1}+\frac{B}{(x-3)}$

[c] $\frac{A}{x+1}+\frac{B}{(x-3)}+\frac{C}{(x-3)^2}$

[d] $\frac{A}{x+1}+\frac{Bx+C}{(x-3)^2}$

5. The fraction $\frac{(x-1)(x-2)(x-3)}{(x-4)(x-5)(x-6)}$ is:

[a] Proper [b] Improper
[c] Both proper and improper
[d] None of these

6. $(25)_{10}$ when converted to octal is called:

[a] $(31)_8$ [b] $(2.5)_8$
[c] $(13)_8$ [d] None of these

7. ~~In Boolean Algebra $X+\bar{X}$ is equal to:~~

[a] X [b] $\bar{X}$ [c] 0 [d] 1

8. If switch is off it is represented by:

[a] 0 [b] 1
[c] OR [d] NOT

9. Slope of the line $\frac{x}{a}+\frac{y}{b}=1$ is:

[a] $\frac{a}{b}$ [b] $\frac{b}{a}$ [c] $\frac{-b}{a}$ [d] $\frac{-a}{b}$

10. Distance between (4, 3) and (7, 5) is:

[a] 25 [b] $\sqrt{13}$
[c] 5 [d] None of these

11. Equation of the line in the slope intercept form is:

[a] $\frac{x}{a}+\frac{y}{b}=1$ [b] $y-y_1=m(x-x_1)$
[c] $y=mx+c$ [d] None of these

12. Given three points are collinear, if their slopes are:

[a] Equal [b] Unequal
[c] $m_1m_2=-1$ [d] None of these

13. Straight line from center to the circumference is:

[a] Circle [b] Radius
[c] Diameter [d] None of these

14. Radius of the circle $x^2+y^2+2gx+2fy+c=0$ is:

[a] c [b] c^2
[c] $\sqrt{g^2+f^2-c}$ [d] None of these

15. Center of circle $(x-1)^2+(y-2)^2=16$ is:

[a] $(1, 2)$ [b] $(2, 1)$
[c] $(4, 0)$ [d] None of these

Answer Key

1	a	2	b	3	a	4	c	5	b
6	a	7	d	8	a	9	c	10	b
11	c	12	a	13	b	14	c	15	d

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DAE / IA - 2016

MATH-123 APPLIED MATHEMATICS-I

PAPER 'B' PART-B (SUBJECTIVE)

Time: 2:30 Hrs Marks: 60

Section - I

Q.1. Write short answers to any Eighteen (18) questions.

1. Simplify the complex number $\frac{-9+4i}{8-3i}$

Sol. $\frac{-9+4i}{8-3i} = \frac{-9+4i}{8-3i} \times \frac{8+3i}{8+3i}$

$= \frac{-72-27i+32i+12i^2}{(8)^2-(3i)^2}$

$= \frac{-72+5i-12}{64+9}$

$= \frac{-84+5i}{73} = \boxed{-\frac{84}{73}+\frac{5}{73}i}$

2. Find the multiplicative of $-3+4i$.

Sol. Let $z = (-3, 4) = -3+4i$

Multiplicative Inverse of $Z = \frac{1}{Z}$

$= \frac{1}{-3+4i} = \frac{1}{-3+4i} \times \frac{-3-4i}{-3-4i}$

$= \frac{-3-4i}{(-3)^2-(4i)^2} = \frac{-3-4i}{9+16}$

$= \frac{-3-4i}{25} = \boxed{-\frac{3}{25}-\frac{4}{25}i}$

3. Factorize $36a^2+100b^2$

Sol. $36a^2+100b^2$

$= 36a^2-100b^2i^2$

$= (6a)^2-(10bi)^2$

$= \boxed{(6a-10bi)(6a+10bi)}$

4. Write the conjugate and modulus of $-\frac{2}{3}-\frac{4}{9}i$

Sol. Let $z = -\frac{2}{3}-\frac{4}{9}i$

Conjugate $= \bar{z} = \overline{-\frac{2}{3}-\frac{4}{9}i} = \boxed{-\frac{2}{3}+\frac{4}{9}i}$

As, $a = -\frac{2}{3}$ & $b = -\frac{4}{9}$

Modulus $= |z| = \sqrt{a^2+b^2}$

$|z| = \sqrt{\left(-\frac{2}{3}\right)^2+\left(-\frac{4}{9}\right)^2} = \sqrt{\frac{4}{9}+\frac{16}{81}}$

$|z| = \sqrt{\frac{36+16}{81}} = \sqrt{\frac{52}{81}} = \boxed{\frac{\sqrt{52}}{9}}$

5. ~~Express the complex number in the form $a+bi$, When $|z| = 2, \arg z = \frac{\pi}{3}$~~

Sol. $z = r\text{ cis }\theta = 2\text{cis}\left(\frac{\pi}{3}\right) = 2\text{ cis }60°$

$z = 2[\cos(60°)+i\sin(60°)]$

$z = 2\left[\frac{1}{2}+i\frac{\sqrt{3}}{2}\right] = 2\left[\frac{1+\sqrt{3}i}{2}\right]$

$\boxed{z = 1+\sqrt{3}i}$

6. Define improper fraction and give one example.

Sol. A fraction in which the degree of the numerator is greater then or equal to the degree of denominator is called improper fraction.

Example: $\frac{x^2+1}{(x+1)(x-1)}$

7. **Resolve $\frac{1}{x^2-x}$ into partial fractions.**

Sol. $\frac{1}{x^2-x}=\frac{1}{x(x-1)}$

$\frac{1}{x(x-1)}=\frac{A}{x}+\frac{B}{x-1}\to(i)$

$1=A(x-1)+Bx\to(ii)$

Put $x=0$ in eq.(ii)

$1=A(0-1)+B(0)$

$1=-A \Rightarrow \boxed{A=-1}$

Put $x=1$ in eq.(ii)

$1=A(1-1)+B(1)$

$1=A(0)+B \Rightarrow \boxed{B=1}$

Put values of A, & B in eq. (i), we get: $\boxed{-\frac{1}{x}+\frac{1}{x-1}}$

8. **Write an identity equation of $\frac{2x+5}{x^2+5x+6}$**

Sol. $\frac{2x+5}{x^2+5x+6}$

$=\frac{2x+5}{(x+2)(x+3)}$

$=\boxed{\frac{A}{x+2}+\frac{B}{x+3}}$

x^2+5x+6
$=x^2+3x+2x+6$
$=x(x+3)+2(x+3)$
$=(x+3)(x+2)$

9. **Form of partial fraction $\frac{1}{(x+1)^2(x-2)}$ is:**

Sol.

$$\frac{1}{(x+1)^2(x-2)}=\boxed{\frac{A}{(x+1)}+\frac{B}{(x+1)^2}+\frac{C}{(x-2)}}$$

10. **Define Decimal number.**

Sol. The Decimal number system is a number system of base equal to 10.

11. **Convert binary number $(10101)_2$ to decimal number.**

Sol. 10101_2

$=1\times2^4+0\times2^3+1\times2^2+0\times2^1+1\times2^0$

$=16+0+4+0+1=\boxed{21}$

12. **Prove by Boolean algebra rules:**

$\mathbf{X+\overline{X}Y=X+Y}$

Sol. $L.H.S.=X+\overline{X}Y$

$=(X+\overline{X})(X+Y)$ {By Dual of Distributive law}

$=1(X+Y) \quad \because X+\overline{X}=1$

$=X+Y=R.H.S.$ **Proved.**

13. **Prove by Boolean algebra rules:**

$\mathbf{XY+YZ+\overline{Y}Z=XY+Z}$

Sol. $L.H.S.=XY+YZ+\overline{Y}Z$

$=XY+Z(Y+\overline{Y})$

$=XY+Z(1) \quad \because Y+\overline{Y}=1$

$=XY+Z=R.H.S.$ **Proved.**

14. **Prepare a truth table for**

$\mathbf{X(X+Y)=X}$

Sol. L.H.S. R.H.S.

X	Y	X+Y	X(X+Y)
0	0	0	0
0	1	1	0
1	0	1	1
1	1	1	1

15. **Show that the points A(1, 2), B(7, 6) and C(4, 4) lie on a same straight line.**

Sol.
$$\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 2 & 1 \\ 7 & 6 & 1 \\ 4 & 4 & 1 \end{vmatrix}$$
$$= 1\begin{vmatrix} 6 & 1 \\ 4 & 1 \end{vmatrix} - 2\begin{vmatrix} 7 & 1 \\ 4 & 1 \end{vmatrix} + 1\begin{vmatrix} 7 & 6 \\ 4 & 4 \end{vmatrix}$$
$$= 1(6-4) - 2(7-4) + 1(28-24)$$
$$= 1(2) - 2(3) + 1(4) = 2 - 6 - 4 = 0$$
Hence given points are collinear. **Proved.**

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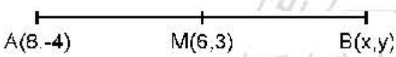
16. **If the mid-point of a segment is (6, 3) and one end point is (8, – 4), what are the coordinates of the other end point.**

Sol. Let B(x, y) be require end point.



$$\text{As, Mid-point} = (6,\ 3)$$
$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = (6,\ 3)$$
$$\left(\frac{8 + x}{2}, \frac{-4 + y}{2}\right) = (6,\ 3)$$
Comparing both order pairs, we have :
$$\frac{8+x}{2} = 6 \text{ and } \frac{-4+y}{2} = 3$$

$8 + x = 12$	$-4 + y = 6$
$x = 12 - 8$	$y = 6 + 4$
$x = 4$	$y = 10$

Hence other end point $= \boxed{(4,\ 10)}$

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17. ~~**Find the equation of a line through the points (–1 , 2) and (3 , 4).**~~

Sol. $$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-2}{3-(-1)} = \frac{2}{4} = \frac{1}{2}$$

Equation of line in point - slope form :
$$y - y_1 = m(x - x_1)$$
$$y - 2 = \frac{1}{2}(x - (-1))$$
$$2(y-2) = 1(x+1)$$
$$2y - 4 = x + 1 \Rightarrow 2y - 4 - x - 1 = 0$$
$$-x + 2y - 5 = 0 \Rightarrow \boxed{x - 2y + 5 = 0}$$

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18. ~~**Find the angle between the lines having slopes –3 and 2.**~~

Sol. Let, $m_1 = -3$ and $m_2 = 2$
$$\theta = \tan^{-1}\left(\frac{m_2 - m_1}{1 + m_2 m_1}\right)$$
$$\theta = \tan^{-1}\left(\frac{2-(-3)}{1+(2)(-3)}\right)$$
$$\theta = \tan^{-1}\left(\frac{2+3}{1-6}\right) = \tan^{-1}\left(\frac{5}{-5}\right)$$
$$\theta = \tan^{-1}(-1) = \boxed{135^\circ}$$

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19. **Show that the lines passing through the points (0, -7), (8, -5) and (5, 7), (8, -5) are perpendicular.**

Sol. $\ell_1 : (0,-7)\ \&\ (8,-5)$
$$\text{Slope of } \ell_1 = m_1 = \frac{y_2 - y_1}{x_2 - x_1}$$
$$m_1 = \frac{-5-(-7)}{8-0}$$
$$m_1 = \frac{-5+7}{8} = \frac{2}{8} = \frac{1}{4}$$
$\ell_2 : (5,7)\ \&\ (8,-5)$
$$\text{Slope of } \ell_2 = m_2 = \frac{y_2 - y_1}{x_2 - x_1}$$
$$m_2 = \frac{-5-7}{8-5} = -\frac{12}{3} = -4$$

As, $m_1m_2 = \left(\frac{1}{4}\right)(-4) = -1$

Hence both lines ℓ_1 & ℓ_2 are **perpendicular**. **Proved.**

20. **Find the distance between the points $(-3, 1)$ and $(3, -2)$.**

Sol. Distance between $(-3, 1)$ & $(3, -2)$.

$$= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$
$$= \sqrt{(-3-3)^2 + (1-(-2))^2}$$
$$= \sqrt{(-6)^2 + (3)^2}$$
$$= \sqrt{36+9} = \sqrt{45}$$
$$= \sqrt{9\times 5} = \boxed{3\sqrt{5}}$$

21. **Show that the points (1, 9), (-2, 3) and (-5, -3) are collinear.**

Sol.
$$\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 9 & 1 \\ -2 & 3 & 1 \\ -5 & -3 & 1 \end{vmatrix}$$
$$= 1\begin{vmatrix} 3 & 1 \\ -3 & 1 \end{vmatrix} - 9\begin{vmatrix} -2 & 1 \\ -5 & 1 \end{vmatrix} + 1\begin{vmatrix} -2 & 3 \\ -5 & -3 \end{vmatrix}$$
$$= 1(3-(-3)) - 9(-2-(-5)) + 1(6-(-15))$$
$$= 1(3+3) - 9(-2+5) + 1(6+15)$$
$$= 1(6) - 9(3) + 1(21) = 6 - 27 + 21 = 0$$

Hence given points are collinear. **Proved.**

22. ~~**Find the equation of line having x-intercept -2 and y-intercept 3.**~~

Sol. Let, $x - \text{intercept} = a = -2$

& $y - \text{intercept} = b = 3$

Equation of line in intercept form: $\frac{x}{a} + \frac{y}{b} = 1$

$$\frac{x}{-2} + \frac{y}{3} = 1$$
$$\frac{-3x + 2y}{6} = 1$$
$$-3x + 2y = 6$$
$$-3x + 2y - 6 = 0 \Rightarrow \boxed{3x - 2y + 6 = 0}$$

23. **Write the equation of circle with, center at (h, k) and radius 'r'.**

Sol. $\boxed{(x-h)^2 + (y-k)^2 = r^2}$

24. **Find the equation of circle with center $(0, 0)$ and radius 'r'.**

Sol. Standard form of equation of circle:

$$(x-h)^2 + (y-k)^2 = r^2$$

Put $h = 0,\ k = 0\ \&\ r = r$

$$(x-0)^2 + (y-0)^2 = r^2 \Rightarrow \boxed{x^2 + y^2 = r^2}$$

25. **Find the center and radius of the circle $6x^2 + 6y^2 - 18y = 0$**

Sol. $6x^2 + 6y^2 - 18y = 0$

Dividing each term by 6, we get:

$$x^2 + y^2 - 3y = 0$$

Comparing with general form:

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

$2g = 0$	$2f = -3$	$c = 0$
$g = 0$	$f = -\frac{3}{2}$	

Center $= (-g, -f)$

Center $= \left(0, -\left(-\frac{3}{2}\right)\right) = \boxed{\left(0, \frac{3}{2}\right)}$

Radius $= r = \sqrt{g^2 + f^2 - c}$

$$r = \sqrt{(0)^2 + \left(-\frac{3}{2}\right)^2 - 0} = \sqrt{\frac{9}{2}} = \boxed{\frac{3}{2}}$$

26. **Find the equation of a circle with center at $(-1, 3)$ and tangent to x-axis.**

Sol. Here : Centre $= (h,k) = (-1,3)$

& Radius $= r = 3$

Standard form of eq. of circle :

$(x-h)^2 + (y-k)^2 = r^2$

Put $h = -1$, $k = 3$ & $r = 3$

$(x+1)^2 + (y-3)^2 = (3)^2$

$(x)^2 + 2(x)(1) + (1)^2 + (y)^2 - 2(y)(3) + (3)^2 = 9$

$x^2 + 2x + 1 + y^2 - 6y + 9 - 9 = 0$

$\boxed{x^2 + y^2 + 2x - 6y + 1 = 0}$

27. **Reduce the equation of the circle $x^2 + y^2 - 4x + 6y - 12 = 0$ into standard form.**

Sol. As given equation :

$x^2 + y^2 - 4x + 6y - 12 = 0$

$x^2 - 4x + y^2 + 6y = 12$

Adding the square of one half of the coefficient of x & y on both sides :

$x^2 - 4x + (2)^2 + y^2 + 6y + (3)^2 = 12 + (2)^2 + (3)^2$

$(x-2)^2 + (y+3)^2 = 12 + 4 + 9$

$(x-2)^2 + (y+3)^2 = 25$

$\boxed{(x-2)^2 + (y+3)^2 = (5)^2}$

Section - II

Note : Attemp any three (3) questions $3 \times 8 = 24$

Q.2.(a) **Reduce the complex number $\dfrac{(2+3i)(3+2i)}{4-3i}$ to the form $a+bi$.**

Sol. See Q.6(ii) of Ex# 8.1 (Page # 308)

(b) **Prove that :**

$$\frac{1}{\cos\theta - i\sin\theta} = \cos\theta + i\sin\theta$$

Sol. See Q.10 of Ex# 8.1 (Page # 313)

Q.3. **Resolve $\dfrac{4+7x}{(2+3x)(1+x)^2}$ into partial fraction.**

Sol. See example 08 of Chapter 09

Q.4.(a) **Convert $(39.4475)_{10}$ to octal number.**

Sol. See Q.4[g] of Ex# 10 (Page # 408)

(b) **Prepare a truth table for the Boolean expression $XYZ + \overline{X}.\overline{Y}.\overline{Z}$**

Sol. See Q.1(i) of Ex# 11 (Page # 425)

Q.5.(a) **Is the point $(0, 4)$ inside or outside the circle of radius 4 with center at $(-3, 1)$?**

Sol. See Q.3 of Ex# 12.1 (Page # 449)

(b) **For the triangle $A(1, 3)$, $B(-2, 1)$, $C(0, -4)$. Find the slope of the line perpendicular to $\overline{AB}$.**

Sol. See Q.3[a] of Ex# 12.3 (Page # 467)

Q.6. **Find the equation of the circle passing through $(9, -7)$, $(-3, -1)$ and $(6, 2)$.**

Sol. See example 05 of Chapter 13

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