DAE (1st Year)
MATH-123
CH # 01---13

IMPORTANT—FORMULAE

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Chapter # 01

Exercise # 1.1

Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Exercise # 1.2

Discriminant

$$\text{Disc} = b^2 - 4ac$$

Exercise # 1.3

Sum & Product of the Roots

$$\text{Sum of the Roots} = \alpha + \beta = -\frac{\text{coefficient of } x}{\text{coefficient of } x^2} = -\frac{b}{a}$$

$$\text{Product of the Roots} = \alpha\beta = \frac{\text{constant term}}{\text{coefficient of } x^2} = \frac{c}{a}$$

Exercise # 1.4

Formation of Quadratic Equation

$$x^2 - Sx + P = 0$$

Where (S = Sum of the roots & P = Product of the roots.)

Chapter # 02

Exercise # 2.1

Binomial Theorem

1. $n! = n(n-1)(n-2)\ldots 3.2.1$
2. ${}^{n}C_{r} = \binom{n}{r} = \frac{n!}{r!(n-r)!}$
3. Binomial Theorem

$$(a+b)^n = \binom{n}{0}a^n b^o + \binom{n}{1}a^{n-1}b^1 + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{n}a^o b^n$$

Exercise # 2.2

Binomial Series

$$(1+b)^n = 1 + \frac{n}{1!}b + \frac{n(n-1)}{2!}b^2 + \frac{n(n-1)(n-2)}{3!}b^3 + \ldots$$

Chapter # 03

Exercise # 3.1

Relation between ℓ and θ

$$\ell = r\theta$$

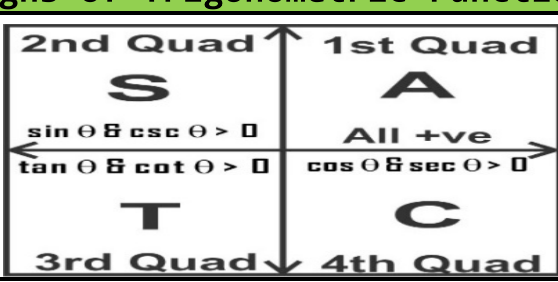
where θ is in Radian, ℓ and r measured in terms of same unit.

Conversion of Degree ⇔ Radian

$$1^\circ = \frac{\pi}{180}\text{rad} \quad \& \quad 1\text{rad} = \frac{180^\circ}{\pi}$$

Exercise # 3.2

Signs of Trigonometric Functions



Values of Trigonometric Functions

θ	0°	30°	45°	60°	90°
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞

Exercise # 3.3

Fundamental Identities

1. $\sin^2\theta + \cos^2\theta = 1$ or $\begin{cases}\sin^2\theta = 1-\cos^2\theta \\ \cos^2\theta = 1-\sin^2\theta\end{cases}$
2. $1+\tan^2\theta = \sec^2\theta$ or $\begin{cases}\tan^2\theta = \sec^2\theta - 1 \\ \sec^2\theta - \tan^2\theta = 1\end{cases}$
3. $1+\cot^2\theta = \operatorname{cosec}^2\theta$ or $\begin{cases}\cot^2\theta = \operatorname{cosec}^2\theta - 1 \\ \operatorname{cosec}^2\theta - \cot^2\theta = 1\end{cases}$
4. $\tan\theta = \frac{\sin\theta}{\cos\theta}$ & 5. $\cot\theta = \frac{\cos\theta}{\sin\theta}$
6. $\operatorname{cosec}\theta = \frac{1}{\sin\theta}$ & 7. $\sec\theta = \frac{1}{\cos\theta}$

Chapter # 04

Exercise # 4.1

Fundamental Laws of Trigonometry

1. $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$
2. $\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$
3. $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$
4. $\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$
5. $\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1-\tan\alpha\tan\beta}$
6. $\tan(\alpha-\beta) = \frac{\tan\alpha - \tan\beta}{1+\tan\alpha\tan\beta}$

Exercise # 4.2

Double Angles Identities

1. $\sin 2\theta = 2\sin\theta\cos\theta$
2. $\cos 2\theta = \begin{cases}\cos^2\theta - \sin^2\theta \\ 2\cos^2\theta - 1 \\ 1 - 2\sin^2\theta\end{cases}$
3. $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$

Triple Angles Identities

1. $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$
2. $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$
3. $\tan 3\theta = \frac{3\tan\theta - \tan^3\theta}{1-3\tan^2\theta}$

Exercise # 4.3

Product ⇒ Sum & Difference

1. $2\sin\alpha\cos\beta = \sin(\alpha+\beta) + \sin(\alpha-\beta)$
2. $2\cos\alpha\sin\beta = \sin(\alpha+\beta) - \sin(\alpha-\beta)$
3. $2\cos\alpha\cos\beta = \cos(\alpha+\beta) + \cos(\alpha-\beta)$
4. $-2\sin\alpha\sin\beta = \cos(\alpha+\beta) - \cos(\alpha-\beta)$

Sum & Difference ⇒ Product

1. $\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$
2. $\sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$
3. $\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$
4. $\cos A - \cos B = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$

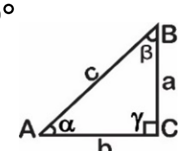
Chapter # 05

Exercise # 5.1 , 5.2

Right Angle Triangle

1. $\sin\alpha = \dfrac{\text{Perpendicular}}{\text{Hypotenuse}} = \dfrac{a}{c} = \dfrac{\text{عمود}}{\text{وتر}}$ (عو)
2. $\cos\alpha = \dfrac{\text{Base}}{\text{Hypotenuse}} = \dfrac{b}{c} = \dfrac{\text{قاعدہ}}{\text{وتر}}$ (قو)
3. $\tan\alpha = \dfrac{\text{Perpendicular}}{\text{Base}} = \dfrac{a}{b} = \dfrac{\text{عمود}}{\text{قاعدہ}}$ (عق)
4. Sum of angles of the triangle is $180°$

   $\boxed{\alpha+\beta+\gamma = 180°}$



5. The Pythagoras Theorem :

$\boxed{(\text{Base})^2 + (\text{Perpendicular})^2 = (\text{Hypotenus})^2}$

Exercise # 5.3, 5.4, 5.5

Oblique Triangles

1. Sum of angles of a triangle : $\alpha+\beta+\gamma = 180°$
2. The Law of Sines : $\left\{\dfrac{a}{\sin\alpha} = \dfrac{b}{\sin\beta} = \dfrac{c}{\sin\gamma}\right\}$
3. The Law of cosines : $\left\{\begin{array}{ll} \text{i.} & a^2 = b^2 + c^2 - 2bc\cos\alpha \\ \text{ii.} & b^2 = c^2 + a^2 - 2ca\cos\beta \\ \text{iii.} & c^2 = a^2 + b^2 - 2ab\cos\gamma \end{array}\right\}$

Chapter # 06

Exercise # 6.1

1. Magnitude of a vector $\vec{r} = xi + yj + zk$ is

   $$|\vec{r}| = \sqrt{x^2+y^2+z^2}$$

2. Units vector of a vector $\vec{a} = \hat{a} = \dfrac{\vec{a}}{|\vec{a}|}$

3. Direction Cosines of a vector $\vec{r} = xi + yj + zk$ are

   $$\frac{x}{\sqrt{x^2+y^2+z^2}}, \frac{y}{\sqrt{x^2+y^2+z^2}}, \frac{z}{\sqrt{x^2+y^2+z^2}}$$

Exercise # 6.2

1. Cosine of the angle between $\vec{a}$ & $\vec{b} = \cos\theta = \dfrac{\vec{a}.\vec{b}}{|\vec{a}|.|\vec{b}|}$
2. Sine of the angle between $\vec{a}$ & $\vec{b} = \sin\theta = \dfrac{|\vec{a}\times\vec{b}|}{|\vec{a}|\ |\vec{b}|}$
3. Units vector perpendicular to both $\vec{a}$ & $\vec{b} = \hat{u} = \dfrac{\vec{a}\times\vec{b}}{|\vec{a}\times\vec{b}|}$
4. Area of the Parallelogram $= |\vec{a}\times\vec{b}|$
5. Two vectors $\vec{a}$ & $\vec{b}$ are perpendicular iff $\vec{a}.\vec{b} = 0$
6. Two vectors $\vec{a}$ & $\vec{b}$ are parallel iff $\vec{a}\times\vec{b} = 0$
7. Projection of $\vec{a}$ along $\vec{b} = \dfrac{\vec{a}.\vec{b}}{|\vec{b}|}$

Chapter # 07

Rectangular or Cartesian form of Phasor

$$Z = x + jy\ , \text{ where } j = \sqrt{-1}$$

Trigonometric or Polar form of Phasor

$$Z = r(\cos\theta + j\sin\theta) = r\angle\theta$$

Where $r = |Z| = \sqrt{a^2+b^2}$ & $\theta = \tan^{-1}\left(\dfrac{y}{x}\right)$

Exponential form of Phasor

$$Z = x + jy = r(\cos\theta + j\sin\theta) = r\angle\theta = re^{j\theta}$$

Chapter # 08

Exercise # 8.1

Powers of i

$$i = \sqrt{-1}\ ,\ i^2 = -1\ ,\ i^3 = -i\ ,\ i^4 = 1$$

Rectangular form of Complex Number

$$Z = x + iy\ , \text{ where } i = \sqrt{-1}$$

Conjugate of Complex Number

If $z = a + bi$ then $\overline{z} = \overline{a+bi} = a - bi$

Multiplication of two Complex Numbers

If $z_1 = a + ib$ & $z_2 = c + id$ then

$$z_1 . z_2 = (a+ib).(c+id) = (ac-bd) + i(ad+bc)$$

Exercise # 8.2

Polar Form of Complex Number

If $z = a + bi$ then $r = |z| = \sqrt{a^2+b^2}$, $\theta = \tan^{-1}\left(\dfrac{b}{a}\right)$

Principle Argument of Complex Number

b

II – Quadrant	I – Quadrant
{When a – ve & b + ve}	{When a + ve & b + ve}
$\theta = 180° - \tan^{-1}\left(\left\lvert\frac{b}{a}\right\rvert\right)$	$\theta = \tan^{-1}\left(\left\lvert\frac{b}{a}\right\rvert\right)$
III – Quadrant	**IV – Quadrant**
{When a – ve & b – ve}	{When a + ve & b – ve}
$\theta = 180° + \tan^{-1}\left(\left\lvert\frac{b}{a}\right\rvert\right)$	$\theta = 360° - \tan^{-1}\left(\left\lvert\frac{b}{a}\right\rvert\right)$

–a a

–b

Exercise # 8.3

Multiplication of two Complex Numbers in Polar form

$$z_1 . z_2 = [r_1(\cos\theta_1 + i\sin\theta_1)][r_2(\cos\theta_2 + i\sin\theta_2)]$$

$$z_1 . z_2 = r_1 r_2[\cos(\theta_1 + \theta_2) + i\sin(\theta_1 + \theta_2)]$$

Division of two Complex Numbers in Polar form

$$\frac{z_1}{z_2} = \frac{r_1(\cos\theta_1 + i\sin\theta_1)}{r_2(\cos\theta_2 + i\sin\theta_2)} = \frac{r_1}{r_2}[\cos(\theta_1 - \theta_2) + i\sin(\theta_1 - \theta_2)]$$

Chapter # 09

How to resolve a Rational Fraction into Partial Fraction

1. The degree of N(x) must be less than that of degree of D(x). if not, divide and work with the remainder theorem.
2. Multiply both sides by D(x).
3. Equate the coefficients of like powers of x.
4. solve the resulting equations for the coefficients.

Chapter # 10

Octal and Binary Number Correspondence

Octal Number	Binary Number
0	000
1	001
2	010
3	011
4	100
5	101
6	110
7	111

Chapter # 11

Boolean Algebra

Logical - Addition	Logical - Multiplication
$0 + 0 = 0$	$0 \times 0 = 0$
$0 + 1 = 1$	$0 \times 1 = 0$
$1 + 0 = 1$	$1 \times 0 = 0$
$1 + 1 = 1$	$1 \times 1 = 1$

OR - Operations

1. $X + 0 = X$	**2.** $X + 1 = 1$
3. $X + X = X$	**4.** $X + \overline{X} = 1$

AND - Operations

5. $X.0 = 0$	**6.** $X.1 = X$
7. $X.X = X$	**8.** $X.\overline{X} = 0$

Double Complement

9. $\overline{\overline{X}} = X$

Dual of Distributive Law

10. $X + YZ = (X + Y).(X + Z)$

De Morgan's Theorems

11. $\overline{X + Y} = \overline{X}.\overline{Y}$

12. $\overline{X.Y} = \overline{X} + \overline{Y}$

Chapter # 12

Exercise # 12.1

Distance Formula

Distance Formula: $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

Here '(x_1, y_1) & (x_2, y_2)' be two different points in a plene.

Exercise # 12.2

Mid-Point Formula

$$\text{Mid-Point} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Here '(x_1, y_1) & (x_2, y_2)' be two different points in a plene.

The Ratio Formulas

When P divide the given two points Internally

$$P(x, y) = P\left(\frac{r_1 x_2 + r_2 x_1}{r_1 + r_2}, \frac{r_1 y_2 + r_2 y_1}{r_1 + r_2}\right)$$

When P divide the given two points Externally

$$P(x, y) = P\left(\frac{r_1 x_2 - r_2 x_1}{r_1 - r_2}, \frac{r_1 y_2 - r_2 y_1}{r_1 - r_2}\right)$$

Exercise # 12.3

Slope of a line Formula

$$\text{Slope} = m = \frac{y_2 - y_1}{x_2 - x_1} \quad \text{or} \quad m = \tan\theta$$

Parallel & Perpendicular Theorems

1. Two lines are Parallel iff $m_1 = m_2$

2. Two lines are Perpendicular iff $m_1 m_2 = -1$

Angle Between two Lines

$$\theta = \tan^{-1}\left(\frac{m_2 - m_1}{1 + m_2 m_1}\right)$$

Here 'm_1' is the slope of 1st line & 'm_2' is the slope of 2nd line.

Exercise # 12.4

Equation of Line in Point Slope Form

$$y - y_1 = m(x - x_1)$$

Here 'm' is the slope & '(x_1, y_1)' is the point on the line.

Equation of Line in Two Points Form

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

Here '(x_1, y_1)' & '(x_2, y_2)' are the two points on the line.

Exercise # 12.5

Slope Intercept Form

$$y = mx + c$$

Here 'm' is the slope & 'c' is the y-intercept of the line.

Intercepts Form

$$\frac{x}{a} + \frac{y}{b} = 1$$

Here 'a' is the x-intercept & 'b' is the y-intercept of the line.

Normal Form

$$x\cos\alpha + y\sin\alpha = p$$

Here 'α' is the angle of inclination of the line & 'p' is the length of the perpendicular from origin to the line.

Condition that Three Points be Collinear

The three points $'(x_1, y_1), (x_2, y_2)$ & $(x_3, y_3)'$ are collinear if and only if : $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$

Condition of Concurrency to the Three Straight Lines

The three non-parallel lines $'\begin{cases} a_1x + b_1y + c_1 = 0 \\ a_2x + b_2y + c_2 = 0 \\ a_3x + b_3y + c_3 = 0 \end{cases}'$ are concurrent if and only if : $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_2 \end{vmatrix} = 0$

Exercise # 12.6

Distance of a point from a line

The distance of a point $'P(x_1, y_1)'$ from the line $'\ell : ax + by + c = 0'$ is :

$$\text{Distance} = d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

Position of a point with respect to a line

Let $'\ell : ax + by + c = 0'$ be a given line then a point $'P(x_1, y_1)'$ lies :

(i) Above the line if: $ax_1 + by_1 + c > 0$

(ii) Below the line if : $ax_1 + by_1 + c < 0$

Chapter # 13

Forms of the circle:

$$\begin{cases} \text{If } r > 0, & \text{the circle is a Real Circle.} \\ \text{If } r = 0, & \text{the circle is a Point Circle.} \\ \text{If } r < 0, & \text{the circle is an Imaginary Circle.} \end{cases}$$

Standard form of the equation of a circle

$$(x - h)^2 + (y - k)^2 = r^2$$

Centre $= (h, k)$ & Radius $= r$

General form of the equation of a circle

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

Centre $= (-g, -f)$ & Radius $= \sqrt{g^2 + f^2 - c}$

Twe Circles touches Externally & Internally

Let $'r_1'$ be radius and $'C_1'$ be centre of the 1st Circle and let $'r_2'$ be radius and $'C_2'$ be centre of the 2nd Circle respectively :

$$\begin{cases} \text{Then two circle touches 'Externally'} & \text{iff} & |\overline{C_1C_2}| = r_1 + r_2 \\ \text{Then two circle touches 'Internally'} & \text{iff} & |\overline{C_1C_2}| = r_1 - r_2 \end{cases}$$