Remarks:

i. The scalar product of two vectors is also called the dot product because the "." used to indicate this kind of multiplication. Sometimes it is also called the inner product.

ii. The scalar product of two non-zero vectors is zero if and only if they are at right angles to each other. For $\vec{a} \cdot \vec{b} = 0$ implies that Cos $\theta = 0$, which is the condition of perpendicularity of two vectors.

Deductions:

From the definition (1) we deduct the following:

i. If $\vec{a}$ and $\vec{b}$ have the same direction, then
$\theta = 0^o \Rightarrow \text{Cos } 0^o = 1$
$\vec{a} \cdot \vec{b} = |\vec{a}|\ |\vec{b}|$

ii. If $\vec{a}$ and $\vec{b}$ have opposite directions, then
$\theta = \pi \Rightarrow \cos \pi = -1$

$\therefore \quad \vec{a} \cdot \vec{b} = -|\vec{a}|\ |\vec{b}|$

Iii $\vec{a} \cdot \vec{b}$ will be positive if $0 \le \theta < \frac{\pi}{2}$

and negative if, $\frac{\pi}{2} < \theta \le \pi$

iv The dot product of $\vec{a}$ and $\vec{b}$ is equal to the product of magnitude of $\vec{a}$ and the projection of $\vec{b}$ on $\vec{a}$. This illustrate the geometrical meaning of $\vec{a} \cdot \vec{b}$. In the fig. 15 $|\vec{b}|$ Cos θ is the projection of $\vec{b}$ on $\vec{a}$.

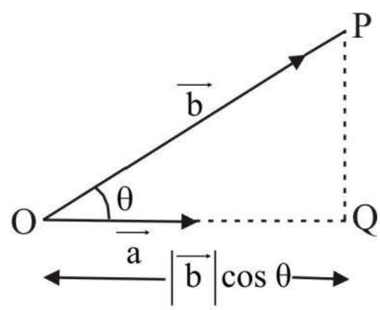

Fig. 15

v From the equation (1)

$$\vec{b} \cdot \vec{a} = |\vec{b}|\,|\vec{a}| \text{ Cos}\theta$$

$$= |\vec{a}|\ |\vec{b}| \text{ Cos}\theta$$

$$\vec{b} \cdot \vec{a} = \vec{a} \cdot \vec{b}$$

Hence the dot product is commutative.

Corollary 1:

If $\overline{a}$ be a vector, then the scalar product $\overline{a} \cdot \overline{a}$ can be expressed with the help of equation (1) as follows:

$$\vec{a} \cdot \vec{a} = |\vec{a}||\vec{a}| \text{Cos } 0^o = |\vec{a}|^2$$

Or $|\vec{a}| = = \sqrt{\vec{a} \cdot \vec{a}}$ ……………………………….(2)

This relation gives us the magnitude of a vector in terms of dot product.

Corollary 2:

If i, j and k are the unit vectors in the directions of X –, Y – and Z – axes, then from eq. (2)

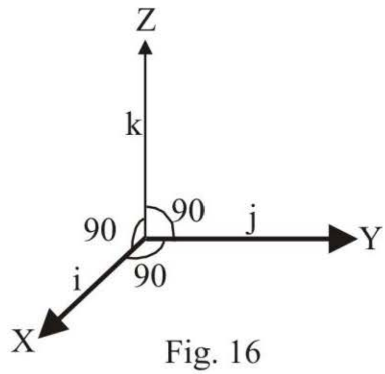

Fig. 16

$i^2 = i.i = |i| \; |i| \text{ Cos}0^o$

$i^2 = 1$

so $i^2 = j^2 = k^2 = 1$

and $i \cdot j = j \cdot i = 0$ Because $\text{Cos}90^o = 0$

$i \cdot k = k \cdot i = 0$

$k \cdot i = i \cdot k = 0$

Corollary 3:

(Analytical expression of $\vec{a} \cdot \vec{b}$)

Scalar product of two vectors in terms of their rectangular components.

For the two vectors

$$\overline{a} = a_1 i + a_2 j + a_3 k$$

and $$\overline{b} = b_1 i + b_2 j + b_3 k$$

the dot product is given as,

$$\vec{a} \cdot \vec{b} = (a_1 i + a_2 j + a_3 k) \cdot (b_1 i + b_2 j + b_3 k)$$
$$= a_1 b_1 + a_2 b_2 + a_3 b_3 \text{ as } i^2 = j^2 = k^2 = 1$$
$$\text{and } i \cdot j = j \cdot k = k \cdot i = 0$$

Also $\overline{a}$ and $\overline{b}$ are perpendicular if and only if $a_1 b_1 + a_2 b_2 + a_3 b_3 = 0$

Example 6:

If $\vec{a} = 3i + 4j - k$, $\vec{b} = -2i + 3j + k$ find $\vec{a} \cdot \vec{b}$

Solution:

$$\vec{a} \cdot \vec{b} = (3i + 4j - k) \cdot (-2i + 3j + k)$$
$$= -6 + 12 - 1$$
$$= 5$$

Example 7:

For what values of λ, the vectors 2i – j + 2k and 3i + 2λj are perpendicular?

Solution:

Let $\overrightarrow{a} = 2i - j + 2k$ and $\overrightarrow{b} = 3i + 2\lambda j$

Since $\overrightarrow{a}$ and $\overrightarrow{b}$ are perpendicular,

So $\overrightarrow{a} \cdot \overrightarrow{b} = 0$

$(2i - j + 2k) \cdot (3i + 2\lambda j) = 0$

$6 - 2\lambda = 0$

Or $\lambda = 3$

Example 8:

Find the angle between the vectors $\overrightarrow{a}$ and $\overrightarrow{b}$, where $\overrightarrow{a} = i + 2j - k$ and $\overrightarrow{b} = -i + j - 2k$.

Solution:

As $\overrightarrow{a} \cdot \overrightarrow{b} = \left|\overrightarrow{a}\right|\left|\overrightarrow{b}\right| \text{Cos } \theta$

Therefore $\text{Cos } \theta = \dfrac{\overrightarrow{a} \cdot \overrightarrow{b}}{\left|\overrightarrow{a}\right| . \left|\overrightarrow{b}\right|}$, $\overrightarrow{a} \cdot \overrightarrow{b} = -1 + 2 + 2 = 3$

$\left|\overrightarrow{a}\right| = \sqrt{1+4+1} = \sqrt{6}$, $\left|\overrightarrow{b}\right| = \sqrt{1+1+4} = \sqrt{6}$

$$\text{Cos } \theta = \frac{3}{\sqrt{6}\sqrt{6}}$$

$$\text{Cos } \theta = \frac{3}{6} = \frac{1}{2}$$

$$\theta = \text{Cos}^{-1}\frac{1}{2} = 60^{\circ}$$

Example 9:

Consider the points A, B, C, D where coordinates are respectively (1, 1, 0), (– 1, 1, 0), (1, – 1, 0), (0, – 1, 1). Find the direction cosines of AC and BD and calculate the angle between them.

Solution:

Now we have A(1, 1, 0), B(– 1, 1, 0), C(1, – 1, 0), D(0, – 1, 1)

$\overrightarrow{a} = \overline{AC} = (1-1)i + (-1-1)j + (0-0)k = -2j$

$\therefore$ Unit vector along AC $= \dfrac{\overrightarrow{AC}}{|AC|} = \dfrac{-2j}{2} = -j$

$\therefore$ The direction cosines of AC are 0, – 1, 0

Now

$$\vec{b} = \overrightarrow{BD} = (0+1)i + (-1-1)j + (1-0)k = i - 2j + k$$

Unit vector along BD $= \dfrac{\overline{BD}}{|BD|} = \dfrac{i - 2j + k}{\sqrt{1+4+1}} = \dfrac{i - 2j + k}{\sqrt{6}}$

$\therefore$ The direction cosines of BD are:

$$\frac{1}{\sqrt{6}}, \frac{-2}{\sqrt{6}}, \frac{1}{\sqrt{6}}$$

Let, θ be the angle between AC and BD then:

$$\text{Cos } \theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}|\,|\vec{b}|} = \frac{\overrightarrow{AC} \cdot \overrightarrow{BD}}{|\overrightarrow{AC}|\,|\overrightarrow{BD}|}$$

$$= \frac{(-2j).(i - 2j + k)}{(2)\sqrt{6}}$$

$$= \frac{(-2)(-2)}{(2)\sqrt{6}} = \frac{2}{\sqrt{6}}$$

$$\theta = \text{Cos}^{-1}\left(\frac{2}{\sqrt{6}}\right)$$

Example 10:

Show that if $|a + b| = |a - b|$ then a and b are perpendicular.

Solution:

We have $|a + b| = |a - b|$

$\therefore$ $|a + b|^2 = |a - b|^2$ taking square.

$a^2 + b^2 + 2a.b = a^2 + b^2 - 2a.b$

$4a.b = 0$ or $a.b = 0$

Hence $\bar{a}$ and $\bar{b}$ are perpendicular.

2. Vector Product:

If $\bar{a}$ and $\vec{b}$ are non-zero vectors and θ is the angle between a and b, then the vector product of a and b, denoted by $a \times b$, is the vector $\bar{c}$ which is perpendicular to the plane determined by $\bar{a}$ and $\bar{b}$. It is defined by the relation,

$$c = a \times b = (|a||b|\text{Sin }\theta)n$$

Where $|\vec{a}||\vec{b}|\sin\theta$ is the magnitude of $\vec{c}$ and n is the Unit Vector in the direction of $\vec{c}$. The direction of $\vec{c}$ is determined by the right hand rule.

The vector product is also called the 'cross product' or 'Outer product' of the vectors.

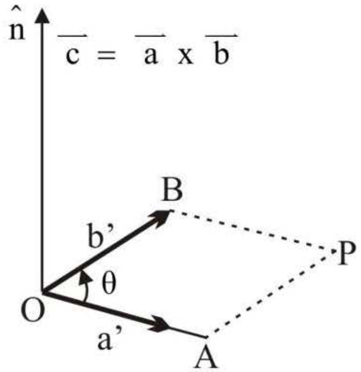

Fig. 17

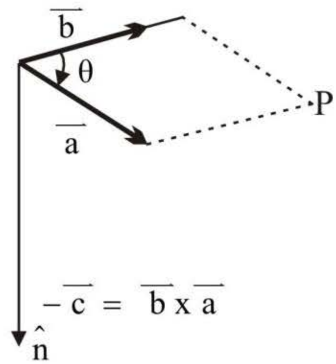

Fig. 18

Remarks:

If we consider $\vec{b} \times \vec{a}$, then $\vec{b} \times \vec{a}$ would be a vector which is opposite in the direction to $\vec{a} \times \vec{b}$.

Hence $\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}$

Which gives that $\vec{a} \times \vec{b} \neq \vec{b} \times \vec{a}$ in general

Hence the vector product is not commutative.

Deductions:

The following results may be derived from the definition.

i. The vector product of two non-zero vectors is zero if $\vec{a}$ and $\vec{b}$ are parallel, the angle between $\vec{a}$ and $\vec{b}$ is zero. Sin $0^{o} = 0$, Hence $\vec{\mathbf{a}} \times \vec{\mathbf{b}} = 0$.

For a x b = 0 implies that $\sin\theta = 0$ which is the condition of parallelism of two vectors. In particular $\vec{a} \times \vec{a} = 0$. Hence for the unit vectors i, j and k,

i x i = j x j = k x k = 0

ii. If $\vec{a}$ and $\vec{b}$ are perpendicular vectors, then $\vec{a} \times \vec{b}$ is a vector whose magnitude is $|\vec{a}||\vec{b}|$ and whose direction is such that the vectors a, b, a x b form a right-handed system of three mutually perpendicular

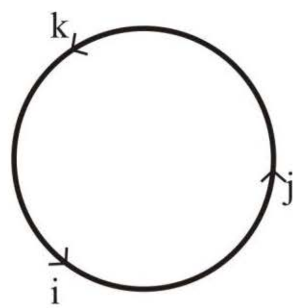

Fig. 19

vectors. In particular i x j = (1) (1) Sin 90^o k (k being perpendicular to i and j) = k

Similarly j x i = −k, i x k = −j, k x j = −i

Hence the cross product of two consecutive unit vectors is the third unit vector with the plus or minus sign according as the order of the product is anti-clockwise or clockwise respectively.

iii. Since $|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}| \text{Sin } \theta$ …… (2)

Which is the area of the parallelogram whose two adjacent sides are $|\vec{a}|$ and $|\vec{b}|$.

Hence, area of parallelogram OABC = $|\vec{a} \times \vec{b}|$

and area of triangle OAB = $\frac{1}{2}|\vec{a} \times \vec{b}|$

If th vertices of a parallelogram are given, then

area of parallelogram OABC = $|\overline{OA} \times \overline{OB}|$

and, area of triangle OAB = $\frac{1}{2}|\overline{OA} \times \overline{OB}|$

iv. If n is the unit vector in the directions of $\vec{c} = \vec{a} \times \vec{b}$ then

$$n = \frac{\vec{c}}{|c|} = \frac{\vec{a} \times \vec{b}}{|a \times b|}$$

or $$n = \frac{\vec{a} \times \vec{b}}{|a||b| \text{Sin } \theta}$$

from equation (2) we also find.

$$\text{Sin } \theta = \frac{|\vec{a} \times \vec{b}|}{|a||b|}$$

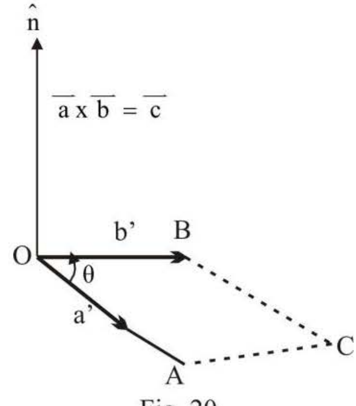

Fig. 20

8.13 Rectangular form of $\vec{a} \times \vec{b}$
(Analytical expression of $\vec{a} \times \vec{b}$)

If $\vec{a} = a_1 i + a_2 j + a_3 k$

and $\vec{b} = b_1 i + b_2 j + b_3 k$

then $\vec{a} \times \vec{b} = (a_1 i + a_2 j + a_3 k) \times (b_1 i + b_2 j + b_3 k)$

$= (a_1 b_2 k - a_1 b_3 j - a_2 b_1 k + a_2 b_3 i + a_3 b_1 j - a_3 b_2 i)$

$= (a_2 b_3 - a_3 b_2) i - (a_1 b_3 - a_3 b_1) j + (a_1 b_2 - a_2 b_1) k$

This result can be expressed in determinant form as

$$\vec{a} \times \vec{b} = \begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

Example 11:

If $\vec{a} = 2i + 3j + 4k$ $\vec{b} = I - j + k$, Find

(i) $\vec{a} \times \vec{b}$

(ii) Sine of the angle between these vectors.

(iii) Unit vector perpendicular to each vector.

Solution:

(i) $\vec{a} \times \vec{b} = \begin{vmatrix} i & j & k \\ 2 & 3 & 4 \\ 1 & -1 & 1 \end{vmatrix}$

$\vec{a} \times \vec{b} = i(3 + 4) - j(2 - 4) + k(-2 - 3)$

$= 7i + 2j - 5k$

(ii) $\text{Sin}\,\theta = \dfrac{|\vec{a} \times \vec{b}|}{|\vec{a}||\vec{b}|} = \dfrac{\sqrt{7^2 + 2^2 + (-5)^2}}{\sqrt{2^2 + 3^2 + 4^2}.\sqrt{1^2 + (-1)^2 + 1^2}}$

$= \dfrac{\sqrt{78}}{\sqrt{29}\sqrt{3}}$

$\text{Sin}\,\theta = \sqrt{\dfrac{26}{29}}$

(iii) If $\hat{n}$ is the unit vector perpendicular to $\vec{a}$ and $\vec{b}$ then

$\hat{n} = \dfrac{\vec{a} \times \vec{b}}{|\vec{a} \times \vec{b}|} = \dfrac{7i + 2j - 5k}{\sqrt{78}}$

Example 12:

$\vec{a} = 3i + 2k$, $\vec{b} = 4i + 4j - 2k$

$\vec{c} = i - 2j + 3k$, $\vec{d} = 2i - j + 5k$

Compute $(\vec{d} \times \vec{c}).(\vec{a} - \vec{b})$

Solution:

$\vec{d} \times \vec{c} = \begin{vmatrix} i & j & k \\ 2 & -1 & 5 \\ 1 & -2 & 3 \end{vmatrix}$

$= i(-3 + 10) - j(6 - 5) + k(-4 + 1)$

$= 7i - j - 3k$

Also $\overrightarrow{a} - \overrightarrow{b} = -i - 4j + 4k$

Hence $(\overrightarrow{d} \times \overrightarrow{c}).(\overrightarrow{a} - \overrightarrow{b}) = (7i - j - 3k).(-i - 4j + 4k)$

$= -7 + 4 - 12$

$= -15$

Example 13:

Find the area of the parallelogram with adjacent sides,

$\overrightarrow{a} = i - j + k$, and $\overrightarrow{b} = 2j - 3k$

Solution:

$$\overrightarrow{a} \times \overrightarrow{b} = \begin{vmatrix} i & j & k \\ 1 & -1 & 1 \\ 0 & 2 & -3 \end{vmatrix}$$

$= i(3 - 2) - j(-3 - 0) + k(2 + 0)$

$= i + 3j + 2k$

Area of parallelogram $= |\overrightarrow{a} \times \overrightarrow{b}| = \sqrt{1 + 9 + 4}$

$= \sqrt{14}$ square unit.

Example 14:

Find the area of the triangle whose vertices are

A(0, 0, 0), B(1, 1, 1) and C(0, 2, 3)

Solution:

Since $\overrightarrow{AB} = (1 - 0, 1 - 0, 1 - 0)$

$= (1, 1, 1)$

and $\overrightarrow{AC} = (0 - 0, 2 - 0, 3 - 0)$

$\overrightarrow{AC} = (0, 2, 3)$

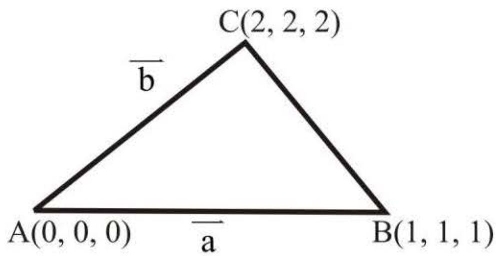

Fig. 21

$$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} i & j & k \\ 1 & 1 & 1 \\ 0 & 2 & 3 \end{vmatrix}$$

$= i(3 - 2) - j(3 - 0) + k(2 - 0)$

$= i - 3j + 2k$

Area of the triangle ABC $= \frac{1}{2} |\overrightarrow{AB} \times \overrightarrow{AC}| = \frac{1}{2}\sqrt{1^2 + (-3)^2 + 2^2}$

$= \frac{\sqrt{14}}{2}$ square unit

Example 15:

Prove by the use of cross-product that the points

A(5, 2, −3), B(6, 1, 4), C(−2, −3, 6) and D(−3, −2, −1) are the vertices of a parallelogram.

Solution:

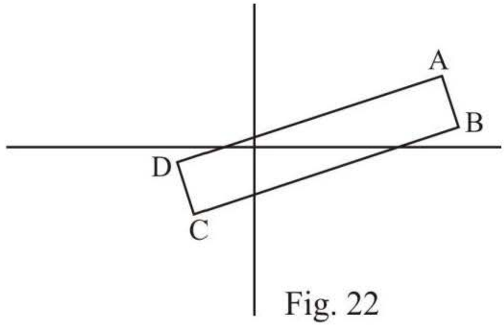

Fig. 22

Since $\overrightarrow{AB} = (1, -1, 7)$

$\overrightarrow{DC} = (+1, -1, +7)$

$\overrightarrow{BC} = (-8, -4, 2)$

and $\overrightarrow{AD} = (-8, -4, 2)$

$$\overrightarrow{AB} \times \overrightarrow{DC} = \begin{vmatrix} i & j & k \\ 1 & -1 & 7 \\ 1 & -1 & 7 \end{vmatrix}$$

$$= i(-7-7) - j(+7-7) + k(1-1)$$

$\overrightarrow{AB} \times \overrightarrow{DC} = 0$, so, $\overrightarrow{AB}$ and $\overrightarrow{DC}$ are parallel.

Also $$\overrightarrow{BC} \times \overrightarrow{AD} = \begin{vmatrix} i & j & k \\ -8 & -4 & 2 \\ -8 & -4 & 2 \end{vmatrix}$$

$$= i(0) - j(0) + k(0)$$

$\overrightarrow{BC} \times \overrightarrow{AD} = 0$, so, $\overrightarrow{BC}$ and $\overrightarrow{AD}$ are parallel.

Hence the given points are the vertices of a parallelogram.

Exercise 8.2

Q.1 Find $\overrightarrow{a} . \overrightarrow{b}$ and $\overrightarrow{a} \times \overrightarrow{b}$

(i) $\overrightarrow{a} = 2i + 3j + 4k$ $\overrightarrow{b} = i - j + k$

(ii) $\overrightarrow{a} = i + j + k$ $\overrightarrow{b} = -5i + 2j - 3k$

(iii) $\overrightarrow{a} = -i - j - k$ $\overrightarrow{b} = 2i + j$

Q.2 Show that the vectors $3i - j + 7k$ and $-6i + 3j + 3k$ are at right angle to each other.

Q.3 Find the cosine of the angle between the vectors:

(i) $\overrightarrow{a} = 2i - 8j + 3k$ $\overrightarrow{b} = 4j + 3k$

(ii) $\overrightarrow{a} = i + 2j - k$ $\overrightarrow{b} = -j - 2k$

(iii) $\overrightarrow{a} = 4i + 2j - k$ $\overrightarrow{b} = 2i + 4j - k$

Q.4 If $\overrightarrow{a} = 3i + j - k$, $\overrightarrow{b} = 2i - j + k$ and $\overrightarrow{c} = 5i + 3k$, find $(2\overrightarrow{a} + \overrightarrow{b}).\overrightarrow{c}$.

Q.5 What is the cosine of the angle between $\overline{P_1P_2}$ and $\overline{P_3P_4}$

If $P_1(2,1,3)$, $P_2(-4, 4, 5)$, $P_3(0, 7, 0)$ and $P_4(-3, 4, -2)$?

Q.6 If $\overline{\mathbf{a}} = [a_1, a_2, a_3]$ and $\overline{\mathbf{b}} = [b_1, b_2, b_3]$, prove that:

$$\vec{a}.\vec{b} = \frac{1}{2}\left[\left|\vec{a}+\vec{b}\right|^2 - \left|\vec{a}\right|^2 - \left|\vec{b}\right|^2\right]$$

Q.7 Find $(\vec{a}+\vec{b}).(\vec{a}-\vec{b})$ if $\vec{a} = i + 2j + 3k$ and $\vec{b} = 2i - j + k$.

Q.8 Prove that for every pair of vectors $\vec{a}$ and $\vec{b}$

$$(\vec{a}+\vec{b}).(\vec{a}-\vec{b}) = \left|\vec{a}\right|^2 - \left|\vec{b}\right|^2$$

Q.9 Find x so that $\vec{a}$ and $\vec{b}$ are perpendicular,

(i) $\overline{\mathbf{a}} = 2i + 4j - 7k$ and $\overline{\mathbf{b}} = 2i + 6j + xk$

(ii) $\vec{a} = xi - 2j + 5k$ and $\vec{b} = 2i - j + 3k$

Q.10 If $\overline{\mathbf{a}} = 2i - 3j + 4k$ and $\overline{\mathbf{b}} = 2j + 4k$

Find the component or projection of $\vec{a}$ along $\overline{b}$.

Q.11 Under what condition does the relation $(\vec{a}.\vec{b})^2 = \vec{a}^2\,\vec{b}^2$ hold for two vectors $\vec{a}$ and $\vec{b}$.

Q.12 If the vectors $3i + j - k$ and $\lambda i - 4j + 4k$ are parallel, find value of λ.

Q.13 If $\overline{\mathbf{a}} = i - 2j + k$, $\overline{\mathbf{b}} = i + 2j - 4k$, $\overline{\mathbf{c}} = 2i - 3j + k$ Evaluate:

(i) $(\vec{a} \times \vec{b}).(\vec{a} \times \vec{c})$ (ii) $(\vec{a} \times \vec{b}) \times (\vec{a} \times \vec{c})$

Q.14 If $\vec{a} = i + 3j - 7k$ and $\vec{b} = 5i - 2j + 4k$. Find:

(i) $\overline{a}.\overline{b}$ (ii) $\overline{a} \times \vec{b}$

(iii) Direction cosines of $\overline{a} \times \overline{b}$

Q.15 Prove that for the vectors $\vec{a}$ and $\vec{b}$

(i) $\left|\vec{a} \times \vec{b}\right|^2 + \left|\vec{a}.\vec{b}\right|^2 = \left|\vec{a}\right|^2 \left|\vec{b}\right|^2$

(ii) $(\vec{a} - \vec{b}) \times (\overline{a} \times \overline{b}) = 2(\overline{a} \times \overline{b})$

Q.16 Prove that for vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$

$$[\vec{a} \times (\vec{b} + \vec{c})] + [\vec{b} \times (\vec{c} + \vec{a})] + [\vec{c} \times (\vec{a} + \vec{b})] = 0$$

Q.17 Find a vector perpendicular to both the lines, $\overrightarrow{AB}$ and $\overline{CD}$

where A is (0, 2, 4), B is (3, –1, 2), C is (2, 0, 1) and D is (4, 2, 0).

Q.18 Find $\left|(\vec{a} \times \vec{b}) \times c\right|$ if $\vec{a} = i - 2j - 3k$, $\vec{b} = 2i + j - k$,

$\vec{c} = i + 3j - 2k$.

Q.19 Find the sine of the angle and the unit vector perpendicular to each:

(i) $\vec{a} = i + j + k$ and $\vec{b} = 2i + 3j - k$

(ii) $\vec{a} = 2i - j + k$ and $\vec{b} = 3i + 4j - k$

Q.20 Given $\vec{a} = 2i - j$ and $\vec{b} = j + k$, if $\left|\vec{c}\right| = 12$ and $\vec{c}$ is perpendicular to both $\vec{a}$ and $\vec{b}$, write the component form of $\vec{c}$.

Q.21 Using cross product, find the area of each triangle whose vertices have the following co-ordinates:

(i) (0, 0, 0),(1, 1, 1),(0, 0, 3)

(ii) (2, 0, 0), (0, 2, 0), (0, 0, 2)

(iii) (1, -1, 1),(2, 2, 2), (4, -2, 1)

Q.22 Find the area of parallelogram determined by the vectors $\vec{a}$ and $\vec{b}$ $\vec{a} = i + 2j + 3k$ and $\vec{b} = -3i - 2j + k$.

Answers 8.2

Q.1 **(i)** 3; 7i – 2j – 5k **(ii)** –6, –5i – 2i + 7k **(iii)** –3; –i –2j + k

Q.3 **(i)** $-\frac{33}{5\sqrt{77}}$ **(ii)** 0 **(iii)** $\frac{17}{21}$ **Q.4** 37

Q.5 $\frac{5}{7\sqrt{22}}$ **Q.7** 8 **Q.9(i)**4 **(ii)** $-\frac{17}{2}$ **Q.10** $\sqrt{5}$

Q.11 [0, 11] **Q.12** $\lambda = -12$ **Q.13** (i) 15 (ii)i – 2j + k

Q.14 **(i)** –29 **(ii)** –2i – 39j – 17k **(iii)** $-\frac{2}{\sqrt{1814}}, \frac{-39}{\sqrt{1814}}, \frac{-17}{\sqrt{1814}}$

Q.17 7i – j + 12k **Q.18** $5\sqrt{26}$

Q.19 **(i)** $\sqrt{\frac{13}{21}}, \frac{-4i + 3j + k}{\sqrt{26}}$ **(ii)** $\sqrt{\frac{155}{156}}, \frac{-3i + 5j + 11k}{\sqrt{155}}$

Q.20 –4i – 8j + 8k

Q.21 **(i)** $\frac{3\sqrt{2}}{2}$ sq. unit. **(ii)** $2\sqrt{3}$ sq. unit.

(iii) $\frac{\sqrt{110}}{2}$ sq. unit. Q.22 $\sqrt{180}$ sq. unit

Summary

A vector is a quantity which has magnitude as well as direction while scalar is a quantity which has only magnitude. Vector is denoted as $\overrightarrow{AB}$ or $\overrightarrow{OP}$.

1. If P (x, y, z) be a point in space, then the position vector of P relative to 0 = $\overrightarrow{OP}$.
2. Unit coordinator vectors x, j, k are taken as unit vector s along axis $\overrightarrow{OP} = xi + yj + zk$.
3. Magnitude of a vector. i.e. $|\overrightarrow{OP}| = \sqrt{x^2 + y^2 + z^2}$
4. Unit vector of a (non-zero vector), then $\hat{a} = \frac{\overrightarrow{a}}{|a|}$
5. Direction cosines of $\overrightarrow{OP} = xi + yj + zk$ then,

$$\cos\alpha = \frac{x}{|\overrightarrow{OP}|}.\ \mathrm{Cos}\beta = \frac{y}{|\overrightarrow{OP}|}.\mathrm{Cos}\gamma = \frac{z}{|\overrightarrow{OP}|}$$

Scalar product:

The scalar product of two vector $\overrightarrow{a}$ and $\overrightarrow{b}$ is defined as $\overrightarrow{a}\ .\ \overrightarrow{b} = |\overrightarrow{a}||\overrightarrow{b}|\ \mathrm{Cos}\,\theta$

1. If a.b = 0, vectors are perpendicular.
2. i.j = j.k = i.k = zero while i.i = j.j = k.k = 1
3. $\overrightarrow{a}\ .\ \overrightarrow{b} = (a_1i + a_2j + a_3k).(b_1i + b_2j + b_3k) = a_1b_1 + a_2b_2 + a_3b_3$

Vector product:

The vector or cross product of two vectors $\overline{a}$ and $\overline{b}$ denoted $\overrightarrow{a}\ x\ \overrightarrow{b}$ and is defined as: $\overrightarrow{a}\ x\ \overrightarrow{b} = |\overrightarrow{a}||\overrightarrow{b}|\ \mathrm{Sin}\theta\ n$, $\mathrm{Sin}\theta = \frac{|a\ x\ b|}{|a||b|}$

1. $n = \frac{\overline{a}\ x\ \overline{b}}{|a\ x\ b|}$ unit vector.
2. $\overrightarrow{a}\ x\ \overrightarrow{b} = 0$. $\overrightarrow{a}$ and $\overrightarrow{b}$ are parallel or collinear.
3. i x j = j x j = k x k = 0 and i x j = k.j x k = i, k x i = j
4. $\overrightarrow{a}\ x\ \overrightarrow{b} = -\overrightarrow{b}\ x\ \overrightarrow{a}$
5. $\overrightarrow{a}\ x\ \overrightarrow{b} = (a_1i + a_2j + a_3k)\ x\ (b_1i + b_2j + b_3k)$

$$= \begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$