7.6 Solution of Oblique Triangles:

Definition:

The triangle in which have no right angle is called oblique triangle.

A triangle has six elements (i.e. three sides and three angles) if any three of a triangle are given, provided at least one of them is a side, the remaining three can be found by using the formula discussed in previous articles i.e. law of sines and law of cosines.

There are four important cases to solve oblique triangle.

Case I: Measure of one side and the measures of two angles.

Case II: Measure of two sides and the measures of the angle included by them.

Case III: When two sides and the angle opposite to one of them is given.

Case IV: Measure of the three sides.

Example 1:

Solve the ABC with given data.

$a = 850,\qquad \alpha = 65^{o},\qquad \beta = 40^{o}$

Solution:

Given that:

$a = 850,\qquad \alpha = 65^{o},\qquad \beta = 40^{o}$

$b = ?\qquad c = ?\qquad \gamma = ?$

Since, $\alpha + \beta + \gamma = 180^{o}$

$65^{o} + 40^{o} + \gamma = 180\ \gamma \qquad \Rightarrow\ = 75^{o}$

By law of sines to find b:

$$\frac{a}{\text{Sin}\,\alpha} = \frac{b}{\text{Sin}\,\beta}$$

$$\frac{850}{\text{Sin}\,65^{o}} = \frac{b}{\text{Sin}\,40^{o}} \Rightarrow\ b = \frac{850\ \text{Sin}\,40^{o}}{\text{Sin}\,65^{o}}$$

$$b = \frac{850(0.64.28)}{0.9063} = 602.85$$

To find c, by law of Sines

$$\frac{b}{\text{Sin}\,\beta} = \frac{a}{\text{Sin}\,\alpha} \Rightarrow c = \frac{b\ \text{Sin}\,\gamma}{\text{Sin}\,\beta}$$

$$c = \frac{602.85\ \text{Sin}\,75^{o}}{\text{Sin}\,40^{o}} = \frac{602.85(0.9659)}{0.6428}$$

$= \quad 905.90$

Example2:

Solve the triangle with given data:

$a = 45 \qquad b = 34 \qquad \gamma = 52^{o}$

Solution:

Given $a = 45$ $b = 34$ $\gamma = 52^o$

$c = ?$ $\beta = ?$ $\gamma = ?$

To find c, we use law of cosines

$c^2 = a^2 + b^2 - 2ab \text{ Cos } \gamma$

$c^2 = (45)^2 + (34)^2 - 2(45)(34) \text{ Cos } 52^o$

$c^2 = 2025 + 1156 - (3060)(0.6157)$

$c^2 = 1297 \quad \Rightarrow \quad \boxed{c = 36.01}$

To find α, we use law of sines

$$\frac{a}{\text{Sin}\alpha} = \frac{b}{\text{Sin}\gamma} \Rightarrow \frac{45}{\text{Sin}\alpha} = \frac{36.01}{\text{Sin}52^o}$$

$$\text{Sin}\alpha = \frac{45 \text{ Sin}52^o}{36.01} = \frac{45(0.7880)}{36.01} = 0.9847$$

$\alpha = \text{Sin}^{-1}(0.9847) = 97^o\ 58'\ 39''$

To find β, we use $\alpha + \beta + \gamma = 180^o$

$79^o\ 58'\ 39'' + \beta + 52^o = 180^o$

$\beta = 48^o\ 01'\ 20''$

Exercise 7.5

Solve the triangle ABC with given data.

Q1. $c = 4$ $\alpha = 70^o$ $\gamma = 42^o$

Q2. $a = 464$ $\beta = 102^o$ $\gamma = 23^o$

Q3. $b = 85$ $\beta = 57^o 15'$ $\gamma = 78^o 18'$

Q4. $b = 56.8$ $\alpha = 79^o 31'$ $\beta = 44^o 24'$

Q5. $b = 34.57$ $\alpha = 62^o 11'$ $\beta = 63^o 22'$

Q 6. Find the angle of largest measure in the triangle ABC where:

(i) $a = 224$ $b = 380$ $c = 340$

(ii) $a = 374$ $b = 514$ $c = 425$

Q7. solve the triangle ABC where:

(i) $a = 74$ $b = 52$ $c = 47$

(ii) $a = 7$ $b = 9$ $c = 7$

(iii) $a = 2.3$ $b = 1.5$ $c = 2.7$

Answers 7.4

Q1.	$a = 5.62$	$b = 5.54$	$\beta = 68^o$	
Q2.	$\alpha = 55^o$	$b = 454$	$c = 221.31$	
Q3.	$\alpha = 44^o27'$	$a = 70.78$	$c = 98.97$	
Q4.	$a = 79.82$	$c = 67.37$	$\gamma = 56^o\,0'$	
Q5.	$a = 34.20$	$c = 31.47$	$\gamma = 54^o\,2'$	
Q6.	(i) $81^o\,55'\,57''$	(ii) $79^o\,47'\,53''$		
Q7.	(i)	$\alpha = 96^o37'$	$\beta = 44^o16'$	$\gamma = 39^o\,07'$
	(ii)	$\alpha = 50^o$	$\beta = 80^o$	$\gamma = 50^o$
	(iii)	$\alpha = 58^o21'$	$\beta = 33^o45'$	$\gamma = 87^o\,55'$

Summary

1. **Right Triangle:**
   A triangle which has one angle given equal to a right angle.
2. **Oblique Triangle:**
   The triangle in which have no right angle is called oblique triangle.
3. **Law of Sines**
   In any ΔABC, the measures of the sides are proportional to the sines of the opposite angles.
   i.e. $\dfrac{a}{\text{Sin}\alpha} = \dfrac{b}{\text{Sin}\beta} = \dfrac{c}{\text{Sin}\gamma}$
4. **Law of Cosines**
   (i) $a^2 = b^2 + c^2 - 2bc\,\text{Cos}\,\alpha$
   (ii) $b^2 = a^2 + c^2 - 2ac\,\text{Cos}\,\beta$
   (iii) $c^2 = a^2 + b^2 - 2ab\,\text{Cos}\,\gamma$
   (iv) $\text{Cos}\,\alpha = \dfrac{b^2 + c^2 - a^2}{2bc}$
   (v) $\text{Cos}\,\beta = \dfrac{a^2 + c^2 - b^2}{2ac}$
   (vi) $\text{Cos}\,\gamma = \dfrac{a^2 + b^2 - c^2}{2ab}$

Angle of Elevation:

The angle AOP which the ray from an observer's eye at O to an object P at O to an object P at a higher level, makes with horizontal ray OA through O is called the angle of elevation.

Angle of Depression:

The angle AOP which the ray from an observer's eye at O to an object at P at a lower level makes with the horizontal ray OA through O is called the angle of depression.