5.	c = 31.06	6.	b= 181.89
7.	c = 12.68	8.	b = 449.22
9.	a = 69.13	10.	0.7319
11.	1578.68, 1654.46m		

7.5 The Law of Cosines:

This law states that "the square of any sides of a triangle is equal to the sum of the squares of the other two sides minus twice their product times the cosine of their included angle. That is

$$a^2 = b^2 + c^2 - 2bc\cos\alpha$$

$$b^2 = c^2 + a^2 - 2ac\cos\beta$$

and $$c^2 = a^2 + b^2 - 2ab\cos\gamma$$

Proof:

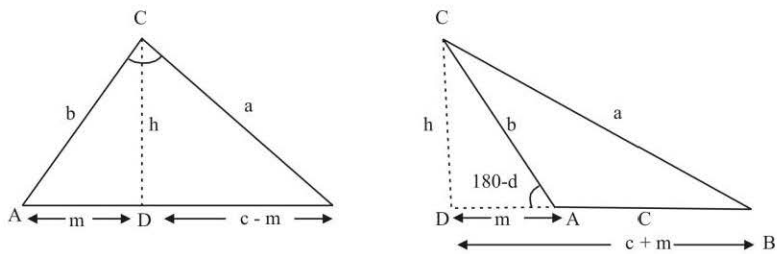

Fig. (I) Fig. (II)

Fig. 6.17

Let β be an acute angle of ΔABC, draw $CD \perp AB$

Let AD = m and CD = h

In right triangle BCD, we have

$$(BC)^2 = (BD)^2 + (CD)^2$$

$$a^2 = (BD)^2 + h^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

(i) If α is an acute angle, then from (i)

In right triangle ACD,

$$\operatorname{Sin}\alpha = \frac{h}{b} \quad \Rightarrow h = b \operatorname{Sin}\alpha$$

and $$\operatorname{Cos}\alpha = \frac{m}{b} \quad \Rightarrow m = b\operatorname{Cos}\alpha$$

So, $\boxed{BD = c - m = c - b\cos\alpha}$

Putting the values of h and BD in equation (1)

$$\begin{aligned} a^2 &= (c - b\cos\alpha)^2 + (b\sin\alpha)^2 \\ &= c^2 - 2bc\cos\alpha + b^2\cos^2\alpha + b^2\sin^2\alpha \\ &= c^2 - 2bc\cos\alpha + b^2(\cos^2\alpha + \sin^2\alpha) \end{aligned}$$

$$= c^2 - 2bc\cos\alpha + b^2$$

$$\boxed{a^2 = b^2 + c2 - 2bc\cos\alpha}$$

(ii) If α in an obtuse angle, then from fig (ii)

In right triangle ACD,

$$\text{Sin}\,(180 - \alpha) = \frac{h}{b}$$

$$\text{Sin}\,\alpha = \frac{h}{b} \quad \Rightarrow h = b\sin\alpha$$

and $$\cos\,(180 - \alpha) = \frac{m}{b}$$

$$-\cos\alpha = \frac{m}{b} \quad \Rightarrow m = -b\cos\alpha$$

So, $\boxed{BD = c + m = c - b\cos\alpha}$

Putting the values of h and BD in equation (1)

$$a^2 = (c - b\cos\alpha)^2 + (b\sin\alpha)^2$$

we get, $a^2 = b^2 + c^2 - 2bc\cos\alpha$

Similarly we obtain

$$b^2 = a^2 + c^2 - 2ac\cos\beta$$

and $c^2 = b^2 + c^2 - 2bc\cos\alpha$

Also when three sides are given, we find

$$\text{Cos}\,\alpha = \frac{b^2+c^2-a^2}{2bc},\ \text{Cos}\,\beta = \frac{a^2+c^2-b^2}{2ac} \text{ and}$$

$$\text{Cos}\,\gamma = \frac{a^2+b^2-c^2}{2ab}$$

Note: we use the cosine formula, when

(i) Two sides and their included angle are given.

(ii) When the three sides are given.

Example 1: In any by using the law of cosines

$a = 7$, $c = 9$, $\beta = 112^0$ Find b

Solution: By law of cosines

$b^2 = a^2 + c^2 - 2ac\cos\beta$

$b^2 = (7)^2 + (9)^2 - 2(7)(9)\,\text{Cos}\,112^o$

$= 49 + 81 - 126(.3746)$

$b^2 = 130 + 47.20 = 177.2$

$b = 13.31$

Example 2: Two man start walking at the same time from a cross road, both walking at 4 km/hour. The roads make an angle of measure 80^o with each other. How far apart will they be at the end of the two hours?

Solution: Let, A be the point of starting of two man V = 4 km/hour

Distance traveled by two men after 2 hours = vt

= 4 x 2 = 8km

Thus, we have to find BC = a = ?

By law of cosine:

$a^2 = b^2 + c^2 - 2bc \cos \alpha$

$a^2 = (8)^2 + (8)^2 - 2(8)(8) \text{ Cos } 80^o = 128 - 128\ (0.1736)$

$a^2 = 105.77 \quad \Rightarrow \quad a = 10.28\text{km}$

Thus, two men will apart 10.28 km after two hours.

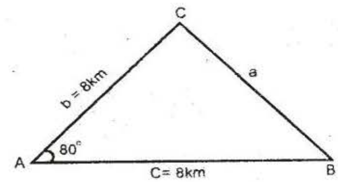

Fig. 6.18

Fig. 7.18

Exercise 7.4

In any triangle ABC by using the law of cosines:

1. a = 56 c = 30 $\beta = 35^o$ Find b
2. b = 25 c = 37 $\alpha = 65^o$ Find a
3. b = 5 c = 8 $\alpha = 60^o$ Find a
4. a = 212 c = 135 $\beta = 37^o\ 15'$ Find b
5. a = 16 b = 17 $\gamma = 25^o$ Find c
6. a = 44 b = 55 $\gamma = 114^o$ Find c
7. a = 13 b = 10 c = 17 Find α and β
8. Three villages P, Q and R are connected by straight roads. Measure PQ is 6 km and the measure QR is 9km. The measure of the angle between PQ and QR is 120^o. Find the distance between P and R.
9. Two points A and B are at distance 55 and 32 meters respectively from a point P. The measure of angle between AP and BP is 37^o. Find the distance between B and A.
10. Find the cosine of the smallest measure of an angle of a triangle with 12, 13 and 14 meters as the measures of its sides.

Answers 7.4

1.	b = 35.83	2.	a = 34.83	3.	a =7
4.	b = 132.652	5.	c = 7.21	6.	c = 83.24
7.	$\alpha = 49^o\ 40'\ 47''$		$\beta = 35^o\ 54'\ 30''$		
8.	13.08km	9.	35.18m	10.	$52^o\ 37'$