7.4 Law of Sines:

In any triangle, the length of the sides are proportional to the sines of measures of the angle opposite to those sides. It means

$$\frac{a}{\text{Sin}\alpha} = \frac{b}{\text{Sin}\beta} = \frac{c}{\text{Sin}\gamma}$$

Proof: Let one angle of the triangle say β be acute, then γ will be either acute, obtuse or right as in figure 1, 2, 3.

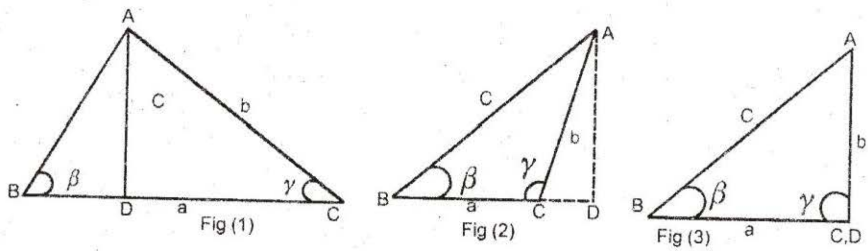


Fig. 6.15

Draw $AD \perp BC$ or BC produced.

Then from ΔABC (for all figures)

$$\frac{AD}{AB} = \text{Sin}\beta \qquad \therefore\ AD = c\text{Sin}\beta \ldots\ldots\ldots\ldots\ldots \quad (1)$$

If γ is acute in figure (1) $\frac{AD}{AC} = \text{Sin}\ \gamma \qquad \Rightarrow AD = b\ \text{Sin}\ \gamma$

If γ is obtuse in figure (2) $\frac{AD}{AC} = \text{Sin}\ (180 - \gamma) = \text{Sin}\ \gamma$

$$\Rightarrow AD = b\ \text{Sin}\ \gamma$$

If γ is right in figure (3) $\frac{AD}{AC} = 1 = \text{Sin}\ 90^o = \text{Sin}\ \gamma$

$$AD = b\ \text{Sin}\ \gamma$$

In each case we have

$$AD = b\ \text{Sin}\ \gamma \ldots\ldots\ldots\ldots \quad (2)$$

From (1) & (2), we have

It can similarly be proved that:

$$\frac{a}{\text{Sin}\alpha} = \frac{b}{\text{Sin}\beta}, \text{ Similarly, } \frac{a}{\text{Sin}\alpha} = \frac{c}{\text{Sin}\ \gamma}$$

Hence ,

$$\frac{a}{\text{Sin}\alpha} = \frac{b}{\text{Sin}\beta} = \frac{c}{\text{Sin}\gamma}$$

This is known as law of sines.

Note: we use sine formula when

i. one side and two angles are given

ii. two sides and the angle opposite one of them are given

Example 1:

In any ΔABC

$a = 12,\ b = 7,\ \alpha = 40^o$ Find β

Solution:

By law of sines $\dfrac{a}{\text{Sin}\alpha} = \dfrac{b}{\text{Sin}\beta} = \dfrac{c}{\text{Sin}\gamma}$

$$\Rightarrow \frac{a}{\text{Sin}\alpha} = \frac{b}{\text{Sin}\beta} = \frac{12}{\text{Sin}40^o} = \frac{7}{\text{Sin}\beta}$$

$$\Rightarrow \text{Sin}\beta = \frac{7\text{Sin}40^o}{12} = \frac{7(0.6429)}{12}$$

$$\text{Sin }\beta = 0.3750$$

$$\Rightarrow \beta = \text{Sin}^{-1}(.3750) \qquad \Rightarrow \beta = 22^o1'$$

Example 2:

In any ΔABC, $b = 24$, $c = 16$

Find the ratio of $\text{Sin}\,\beta$ to $\text{Sin}\,\gamma$

Solution:

By law of sines

$$\frac{a}{\text{Sin}\alpha} = \frac{b}{\text{Sin}\beta} = \frac{c}{\text{Sin}\gamma}$$

$$\Rightarrow \quad \frac{b}{\text{Sin}\beta} = \frac{c}{\text{Sin}\gamma} \Rightarrow \frac{\text{Sin}\beta}{\text{Sin}\gamma} = \frac{b}{c} = \frac{24}{16} = \frac{3}{2}$$

Example 3:

A town B is 15 km due North of a town A. The road from A to B runs North 27°, East to G, then North 34°, West to B. Find the distance by road from town A to B.

Solution:

Given that: $c = 15$ km $\quad \alpha = 24^o, \quad \beta = 34^o$

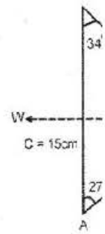
We have to find

Distance from A to B by road.

Since $\alpha + \beta + \gamma = 180^o \quad \Rightarrow \quad 27^o + 34^o + \gamma = 180^o$

$$\gamma = 119^o$$

By law of sines:



$$\frac{a}{\text{Sin}\alpha} = \frac{b}{\text{Sin}\beta} = \frac{c}{\text{Sin}\gamma}$$

$$\Rightarrow \quad \frac{a}{\text{Sin}\alpha} = \frac{c}{\text{Sin}\gamma} \quad \Rightarrow \quad a = \frac{\text{Sin}\alpha}{\text{Sin}\gamma}$$

$$a = \frac{15\text{Sin}27^o}{\text{Sin}119^o} = \frac{15(0.4539)}{0.8746} = 7.78$$

Fig. 7.16

$$\text{Also } \frac{b}{\text{Sin}\beta} = \frac{c}{\text{Sin}\gamma} \quad \Rightarrow \quad \frac{b}{\text{Sin}34^o} = \frac{15}{\text{Sin}119^o}$$

$$b = \frac{15\text{Sin}34^o}{\text{Sin}119^o} = \frac{15(0.5592)}{0.8746} = 9.59$$

Thus distance from A to B by road:

= b + a = 9.59 + 7.78 = 17.37km

Exercise7.3

In any triangle ABC if:

Q1.	$a = 10$	$b = 15$	$\beta = 50^o$ Find α
Q2.	$a = 20$	$c = 32$	$\gamma = 70^o$ Find α
Q3.	$a = 3$	$b = 7$	$\beta = 85^o$ Find α
Q4.	$a = 5$	$c = 6$	$\alpha = 45^o$ Find γ
Q5.	$a = 20\sqrt{3}$	$\alpha = 75^o$	$\gamma = 60^o$ Find c
Q6.	$a = 211.3$	$\beta = 48^o 16'$	$\gamma = 71^o 38'$ Find b
Q7.	$a = 18$	$\alpha = 47^o$	$\beta = 102^o$ Find c
Q8.	$a = 475$	$\beta = 72^o 15'$	$\gamma = 43^o 30'$ Find b
Q9.	$a = 82$	$\beta = 57^o$	$\gamma = 78^o$ Find a
Q10.	$\alpha = 60^o$	$\beta = 45^o$ Find the ratio of b to c	

Q11. Two shore batteries at A and B, 840 meters apart are firing at a target C. The measure of angle ABC is 80^o and the measure of angle BAC is 70^o. Find the measures of distance AC and BC.

Answers 7.3

1.	$\alpha = 30^o 42' 37''$	2.	$\alpha = 35^o 37' 58''$
3.	$\alpha = 25^o 16' 24''$	4.	$\gamma = 58^o 3'$