7.3   Angle of Elevation and Depression:

If O be the eye of the observer, Q the position of the object and OP a horizontal line through O then:

i.   If Q be above OP, then <POQ is called angle of elevation is shown in Figure (1)

ii.   If Q be below OP, then <POQ is called angle of depression is shown in Figure (2)

Example 1:

Find the distance of man from the foot of tower 100m high if the angle of elevation of its top as observed by the man is $52^\circ 30'$.

Solution:

Let, A be the position of man and B be the foot of tower BC. Height of tower = BC = 100m in right $\triangle AB$

$$\text{Tan } 52^\circ 32' \quad = \quad \frac{BC}{AB}$$

$$1.3032 = \frac{100}{AB} \Rightarrow AB = \frac{100}{1.3032} = 78.73m$$

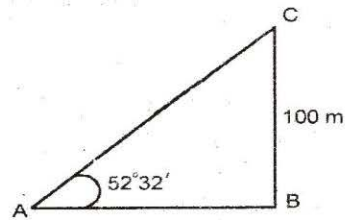AB = distance of man from the foot of tower = 76.73m

Fig. 6.8

Example 2:

From the two successive positions on the straight road 1000 meters apart man observes that the angle of elevation of the top a directly ahead of him are of $12^\circ 10'$ and $42^\circ 35'$. How high is the tower above the road.

Solution:

Let, A and D be the two successive positions of a man on the road.

AD = 1000m (Given)

Let BC = height of tower = h = ?

And DB = xm

In $\triangle ABC$

$$\tan 12^\circ 10' = \frac{BC}{AB}$$

$$0.2156 = \frac{h}{(x + 1000)}$$

Fig. 6.9
Fig. 7.9

$$h = 0.2156 (x + 1000) \ldots\ldots\ldots\ldots(1)$$

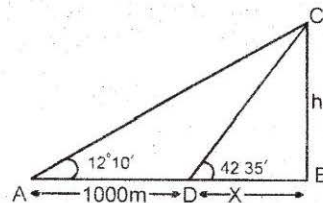In $\triangle DBC$

$$\tan 42^\circ 35' = \frac{BC}{DB} = \frac{h}{x}$$

$$0.9190 = \frac{h}{x}$$

$$x = \frac{h}{0.9190}$$

Put in (1)

$$h = 0.2156 \left[\frac{h}{0.9190} + 100 \right]$$

$$h = \frac{0.2156}{0.9190} h + \frac{(0.2156)(100)}{0.9190}$$

$$h = 0.2346h + 215.6$$
$$h = 0.2346h = 2156$$
$$0.7654h = 215.6$$
$$h = \frac{2156}{.7654} = 28.168$$

Example 3:

Measure of the angle of elevation of the top of a flag staff observed from a point 200 meters from its foot is ?

Solution:

Let height of flog staff $= BC = h = ?$
A = point of observation
In right $\triangle ABC$

$$\tan 30° = \frac{h}{200} \Rightarrow \quad h = 200 \,(0.577)$$
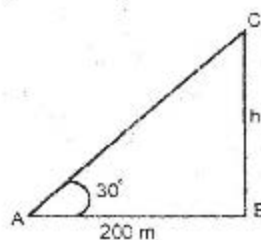$$h \quad = \quad 115.4m$$

Fig. 6.10

Example 4:

Find the measures of the angle of elevation of the top of a tree 400 meters high, when observed from a point 250 meters away from the foot of the base.

Solution: Given that:

Height of tree = $BC = 400m$
AB   =   250m
Let
$\angle BAC = \alpha = ?$
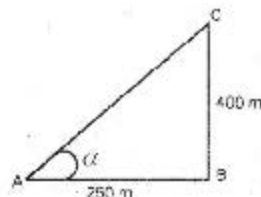

$\angle BAC =$ angle of elevation of top of the tree

Fig. 6.11

$$\tan \alpha = \frac{BC}{AB} = \frac{400}{250} = 1.6$$
$$\alpha = \tan^{-1}(1.6) = 58°$$

Example 5:

The measure of the angle of depression of an airport as observed by a pilot while flying at a height of 5000 meters is $40^\circ 32'$. How far is the pilot from a point directly over the airport?

Solution:

The pilot is at the height of C

BC = 5000m

From right $\angle ABC$

$$\tan 40^\circ 32' = \frac{5000}{x}$$

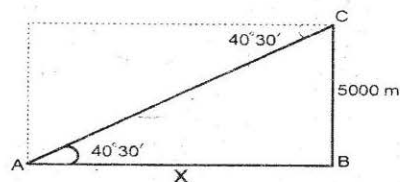$$x = \frac{5000}{\tan 40^\circ 32'} = \frac{5000}{0.8551} = 584736m$$

Fig. 6.12
Fig. 7.12

Example 6: From a point on the ground the measure of angle of elevation of the top of tower is 30°. On walking 100 meters towards the tower the measure of the angle is found to be of 45°. Find the height of the tower.

Solution:

Let BC = height of tower

= h   =   ?

And DB = x m

AD = 100 m

AB = 100 + x

In right $\triangle ABC$

$$\text{Tan}30^\circ = \frac{BC}{AB}$$

$$\frac{1}{\sqrt{3}} = \frac{h}{100+x}$$

$$100 + x = \sqrt{3}\,h \quad \dots\dots\dots\dots\dots\dots (1)$$

In right $\triangle BDC$

$$\tan 45^\circ = \frac{h}{x}$$

$$1 = \frac{h}{x}$$

$$x = h \quad \dots\dots\dots\dots\dots (2)$$

Put x = h in (1)

$$100 + h = \sqrt{3}\,h$$
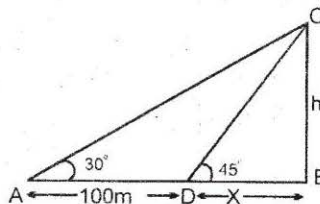
$$1.7321h - h = 100$$

Fig. 6.13

$$h = \frac{100}{0.7321} = 136.60m$$

Example 7:

A pole being broken by the wind, its top struck ground at an angle of 30^0 and at a distance of 10m from the foot of the pole. Find the whole height of the pole.

Solution:

Let BC = h = height of pole = ?

AD = CD

In right $\triangle ABD$

$Tan30^0 = \dfrac{BD}{10}$

BD = 10tan30^0 = 10(0.5774) = 5.77m

Also
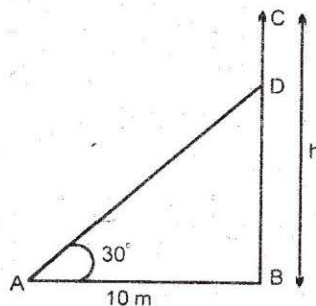

Fig. 6.14

$Cos30^o = \dfrac{AB}{AD} \Rightarrow AD = \dfrac{10}{Cos30^o} = \dfrac{10}{0.8660} = 11.55m$

Height of pole = h = BD + AD

$\therefore$ AD = CD

h = 11.55 + 5.77 = 17.32m

Exercise 7.2

Q1.   How far is a man from the foot of tower 150 meters high, if the measure of the angle of elevation of its top as observed by him is 40° 30′.

Q2.   The shadow of a building is 220 meters when the measure of the angle of elevation of the sun is 35°. Find the height of the building.

Q3.   The measure of the angle of elevation of a kite is 35. The string of the kite is 340 meters long. If the sag in the string is 10 meters. Find the height of the kite.

Q4.   A man 18dm. tall observes that the angle of elevation of the top of a tree at a distance of 12m from the man is 32^0. What is the height of the tree?

Q5.   On walking 300 meters towards a tower in a horizontal line through its base, the measure of the angle of elevation of the top changes from 30° to 60°. Find the height of the tower.

Q6.   The measure of the angle of elevation of the top of a cliff is 25. On walking 100 meters straight towards the cliff, the measure of the angle of elevation of the top is 48°. Find the height of the cliff.

Q7.   From two points A and B. 50 meters apart and in the line with a tree, the measures of the angles of elevation of the top of the tree are 30° and 40° respectively. Find the height of the tree.

Q8.   Two men on the opposite sides of a tower observe that the measures of the angles of elevation of the tower as observed by them separately are 15° and 25° respectively. If the height of the tower is 150 meters. Find the distance between the observers.

Q9.   From a light-house, angles of depression of two ships on opposite of the light-house are observed to be 30° and 45°. If the height of the light house be 300m. Find the distance between the ships of the line joining them passes through foot of light-hosue.

Q10.   The measure of angle elevation of the top of a tower is 30° from a point on the ground. On retreating 100 meters, the measure of the angle of elevation is found to be 15°. Find the height of the tower.

Q11.   From the top of a hill 200 meters high, the angles of depression of the top and bottom of a tower are observed to be 30° and 60° respectively. Find the height of the tower.

Q12.   A television antenna is on the roof of a building. From a point on the ground 36m from the building, the angle of elevation of the top and the bottom of the antenna are 51° and 42° respectively. How tall is the antenna?

Q13.   A ladder 20 meter long reaches the distance of 20 meters, from the top of a building. At the foot of the ladder the measure of the angle of elevation of the top of the building is 60°. Find the height of the building.

Q14.   A man standing on the bank of a canal observes that the measure of the angle of elevation of a tree is 60^0.On retreating 40m from the bank , he finds the measure the angle of elevation of the tree as 30^0.Find the height of the tree and the width of the canal.

Q15.   Two buildings A and B are 100m apart. The angle of elevation from the top of the building A to the top of the building B is 20^0. The angle of elevation from the base of the building B to the top of the building A is 50^0. Find the height of the building B.

Answers 7.2

(1)	175.63m	(2)	154.05m	(3)	h = 189.29m
(4)	9.6m	(5)	h = 259.81m	(6)	h = 80.37m
(7)	h = 17.10m	(8)	881.58m	(9)	819.6m
(10)	49.98m	(11)	133.3m	(12)	12.1m
(13)	h = 30m	(14)	34.64m ; 20m	(15)	155.5 m