Chapter 7
Solution of Triangles

7.1 Solution of Triangles:

A triangle has six parts in which three angles usually denoted by α, β, γ and the three sides opposite to α, β, γ denoted by a, b, c respectively. These are called the elements of Triangle. If any three out of six elements at least one side are given them the remaining three elements can be determined by the use of trigonometric functions and their tables.

This process of finding the elements of triangle is called the solution of the triangle.

First we discuss the solution of right angled triangles i.e. triangles which have one angle given equal to a right angle.

In solving right angled triangle γ denotes the right angle. We shall use the following cases

Case-I:

When the hypotenuse and one Side is given.

Let a & c be the given side and hypotenuse respectively. Then angle α can be found by the relation.

Sin $\alpha = \frac{a}{c}$

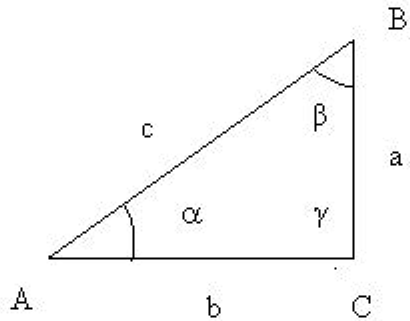

Fig.7.1

Also angle β and side "b" can be obtained by the relations

$\beta = 90^\circ - \alpha$ and Cos $\alpha = \frac{b}{c}$

Case-II:

When the two sides a and b are given. Here we use the following relations to find α, β & c.

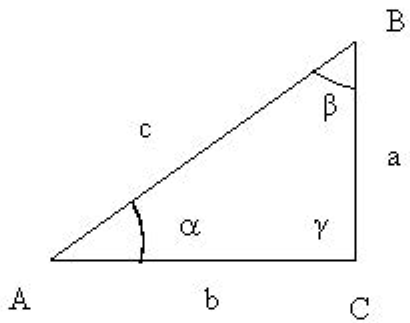

Fig.7.2

Tan $\alpha = \frac{a}{b}$, $\beta = 90^\circ - \alpha$,

$c = \sqrt{a^2 + b^2}$

Case-III:

When an angle α and one of the sides b , is given. The sides a , c and β are found from the following relations.

Tan $\alpha = \frac{a}{b}$ and Cos $\alpha = \frac{b}{c}$, $\beta = 90^o - \alpha$

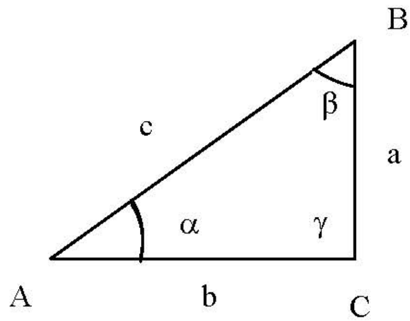

Fig.7.3

Case-IV:

When an angle α and the hypotenuse 'c' is given.The sides a , b and β can be found from the following relations.

Sin $\alpha = \frac{a}{c}$, Cos $\alpha = \frac{b}{c}$ and $\beta = 90^o - \alpha$

Example-1:

Solve the right triangle ABC in which

$\alpha = 34^o\ 17'$, b=31.75, $\gamma = 90^o$

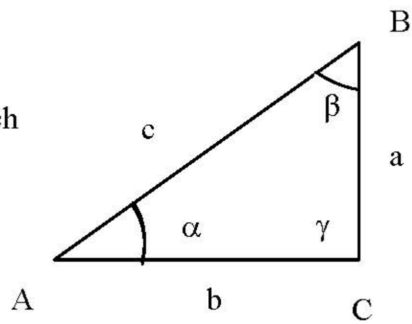

Fig.7.4

Solution:

Given that

$\alpha = 34^o\ 17'$, b = 31.75, $\gamma = 90^o$

We have to find

a = ? c = ? β = ?

$$\text{Tan } \alpha = \frac{a}{b}$$

$$\text{Tan } 34^o\ 17' = \frac{a}{31.75}$$

$\Rightarrow$a = 31.75 tan 34^o 17'

a = 31.75 (0.6817) = 21.64

Also $\cos\alpha = \frac{b}{c}$

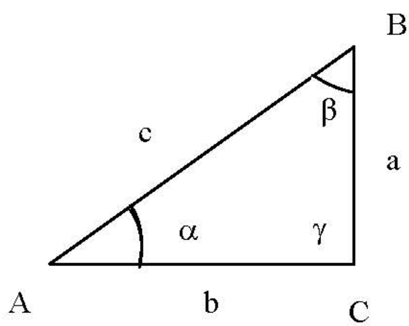

Fig.7.5

$$\Rightarrow \quad \text{Cos } 34^o\ 17' = \frac{31.75}{c}$$

$$C = \frac{31.75}{\text{Cos } 34^o\ 17'} \quad \Rightarrow \quad \beta = 90^o - 34^o\ 17' = 55^o\ 43'$$

Example 2:

Solve the right ΔABC in which $\gamma = 90^o$, a = 450, b = 340

Solution:

$a = 450, \quad b = 340, \quad \gamma = 90^\circ,$

$c = ? = \alpha = ? \quad \beta = ?$

$\text{Tan}\alpha = \frac{a}{b}$

$\text{Tan}\alpha = \frac{450}{340} = 1.3231$

$\Rightarrow \quad \alpha = 52^\circ 56'$

$\beta = 90^\circ - \alpha = 90^\circ - 52^\circ 56' = 37^\circ 4'$

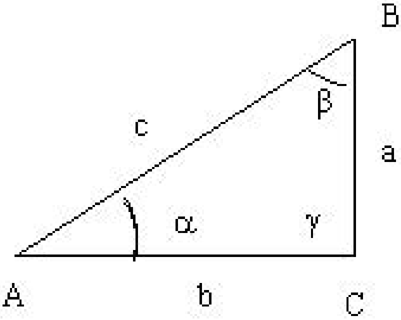

Fig. 7.6

By Pythagoras theorem:

$C^2 = a^2 + b^2 = (450)^2 + (340)^2 = 318100$

$C = 564$

Exercise 7.1

Solve the right triangle ABC in which $\gamma = 90^\circ$

(1)	$a = 250,$	$\alpha = 42^\circ 25'$	(2)	$a = 482,$	$\alpha = 35^\circ 36'$
(3)	$a = 5,$	$c = 13$	(4)	$b = 312,$	$\alpha = 23^\circ 42'$
(5)	$a = 212,$	$\beta = 40^\circ 55'$	(6)	$c = 232,$	$\beta = 52^\circ 46'$
(7)	$c = 540,$	$a = 380$			

Answers 7.1

1.	$\beta = 47^\circ 35'$	$b = 273.63$	$c = 370.64$
2.	$\beta = 54^\circ 24'$	$b = 673.25$	$c = 828.01$
3.	$b = 12$	$\alpha = 22^\circ 37'$	$\beta = 67^\circ 23'$
4.	$a = 136.96$	$c = 340.72$	$\beta = 66^\circ 18'$
5.	$\alpha = 49^\circ 05'$	$b = 183.74$	$c = 280.5$
6.	$a = 184.72$	$b = 140.37$	$\alpha = 37^\circ 14'$
7.	$b = 383.61$	$\alpha = 44^\circ 44'$	$\beta = 45^\circ 16'$

7.2 Application of Right Angled Triangles

(Measurement of Heights and Distances)

Sometimes we deal with problems in which we have to find heights and distances of inaccessible objects.

The solution of these problems are generally the same as that of solving the right triangles.

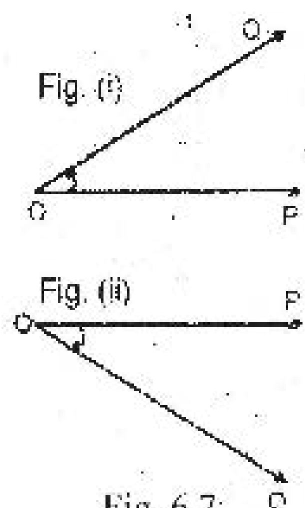

Fig. 6.7