Chapter 6
General Identities

6.1 Introduction:

In the previous chapters, we have dealt with functions of one angle. In this chapter we will discuss the trigonometrical ratios of the sum and difference of any two angles in terms of the ratios of these angles themselves. We will also derives several formulas for this purpose and point out some of their more elementary uses.

6.2 Distance formula:

Let P(x_1 , y_1) and Q(x_2 , y_2) be two points . If "d" denotes the distance between them, then

$$d = |PQ| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

i.e., sum of the square of the difference of x-coordinates and y-coordinates and then the square roots.

Examplel: Find distance between the points P(5 , 7) and Q(-3 , 4)

Solution: $d = |PQ| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$$= \sqrt{(3 - 5)^2 + (4 - 7)^2}$$

$$= \sqrt{(-2)^2 + (-3)^2}$$

$$= \sqrt{4 + 9} \quad = \sqrt{13}$$

6.3 Fundamental law of trigonometry

Let α and β be any two angles (real numbers), then

$$\text{Cos}(\alpha - \beta) = \cos\alpha \cos\beta \; - \; \sin\alpha \sin\beta$$

Which is called the **Fundamental law of trigonometry**

Proof: for convenience, let us assume that $\alpha > \beta > 0$

Consider a unit circle with centre at origin O.

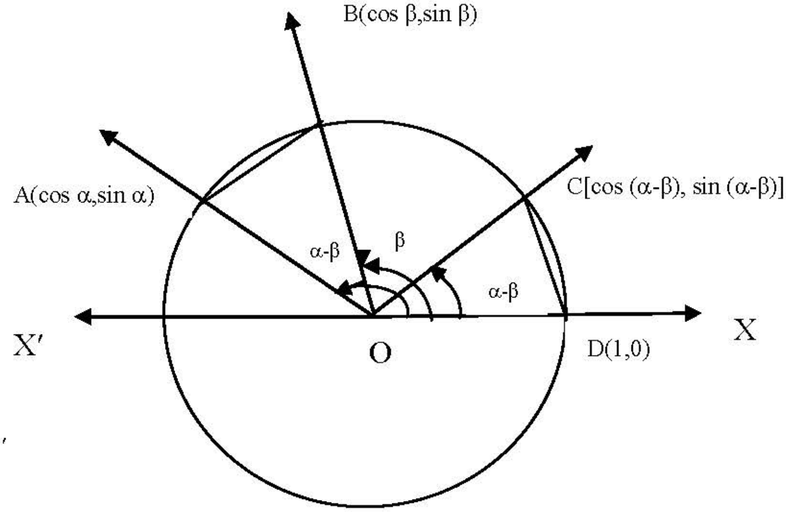
Let the terminal side of angles α and β

cut the unit circle at A and B respectively.

Evidently $\angle AOB = \alpha - \beta$. Take a point C

on the unit circle so that $\angle XOC = \angle AOB = \alpha - \beta$

Join A , B and C , D.



Now angle α , β and $\alpha - \beta$ are in standard position.

$\therefore$ The coordinates of A are $(\cos\alpha, \sin\alpha)$

The coordinates of B are $(\cos\beta, \sin\beta)$

The coordinates of C are $[\cos(\alpha-\beta), \sin(\alpha-\beta)]$

and the coordinates D are $(1, 0)$

Now ΔAOB and ΔCOD are congruent.

$\therefore \quad |AB| = |CD|$

$\Rightarrow \quad |AB|^2 = |CD|^2$

Using the distance formula , we have:

$$(\cos\alpha - \cos\beta)^2 + (\sin\alpha - \sin\beta)^2 = [\cos(\alpha-\beta) - 1]^2 + [\sin(\alpha-\beta) - 0]^2$$

$$\Rightarrow \cos^2\alpha + \cos^2\beta - 2\cos\alpha\cos\beta + \sin^2\alpha + \sin^2\beta - 2\sin\alpha\sin\beta$$

$$= \cos^2(\alpha-\beta) + 1 - 2\cos(\alpha-\beta) + \sin^2(\alpha-\beta)$$

$$\Rightarrow 2 - 2(\cos\alpha\cos\beta + \sin\alpha\sin\beta) = 2 - 2\cos(\alpha-\beta)$$

Hence,

$$\boldsymbol{\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta \quad \ldots\ldots\ldots\ldots\ldots(1)}$$

Note: Although we have proved this law for $\alpha > \beta > 0$, it is true for all values of α and β

Suppose we know the values of sin and cos of two angles α and β, we can find

$\cos(\alpha - \beta)$ using this law as explained in the following example:

Example 1:

Find the value of Cos 15^o.

Solution:

$$\text{Cos } 15^o = \text{Cos } (45^o - 30^o)$$
$$= \text{Cos } 45^o \text{ Cos } 30^o + \text{Sin } 45^o \text{ Sin } 30^o$$
$$= \frac{1}{\sqrt{2}}.\frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}}.\frac{1}{2} = \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}} = \frac{\sqrt{3}+1}{2\sqrt{2}}$$

6.4 Deductions from fundamental law:

(i) Prove that : $\text{Cos}(-\beta) = \text{Cos}\,\beta$

Put $\alpha = 0$ in above equation (1) , then

$$\text{Cos } (0 - \beta) = \text{Cos } 0 \text{ Cos } \beta + \text{Sin } 0 \text{ Sin } \beta$$
$$\text{Cos}(-\beta) = 1.\text{ Cos } \beta + 0.\text{ Sin } \beta \qquad \because \text{Cos } 0 = 1$$
$$\text{Cos}(-\beta) = \text{Cos } \beta \qquad \because \text{Sin } 0 = 0$$

(ii) Prove that : $\text{Cos}\left(\frac{\pi}{2} - \beta\right) = \text{Sin }\beta$

Putting $\alpha = \pi/2$ in equation

$$\text{Cos}(\alpha - \beta) = \text{Cos}\alpha \text{ Cos}\beta + \text{Sin}\alpha \text{ Sin}\beta$$
$$\text{Cos}\left(\frac{\pi}{2} - \beta\right) = \text{Cos}\frac{\pi}{2} \text{ Cos}\beta + \text{Sin}\frac{\pi}{2} \text{ Sin}\beta$$
$$= 0 \text{ . Cos}\beta + 1 \text{ . Sin}\beta$$

$$\boxed{\text{Cos}\left(\frac{\pi}{2} - \beta\right) = \text{Sin }\beta}$$

(iii) Prove that : $\cos(\alpha + \frac{\pi}{2}) = -\sin\alpha$

Put $\beta = -\frac{\pi}{2}$ in equation

$$\text{Cos}(\alpha - \beta) = \text{Cos}\alpha\,\text{Cos}\beta - \text{Sin}\alpha\,\text{Sin}\beta$$

$$\text{Cos}[(\alpha - (-\frac{\pi}{2})] = \cos\alpha \,.\, \cos(-\frac{\pi}{2}) + \sin\alpha \,.\, \sin(-\frac{\pi}{2})$$

$$\cos(\alpha + \frac{\pi}{2}) = \cos\alpha \,.\, 0 + \sin\alpha \,.\, (-1)$$

$$\boxed{\cos(\alpha + \frac{\pi}{2}) = -\sin\alpha}$$

(iv) Prove that: $\sin(-\beta) = -\sin\beta$

By (iii) we have $\cos(\frac{\pi}{2} + \beta) = -\sin\beta$

replace β by $-\beta$

$$\cos(\frac{\pi}{2} - \beta) = -\sin(-\beta)$$

$\sin\beta = -\sin(-\beta)$ [by (ii)]

$$\boxed{\mathbf{\sin(-\beta) = -\sin\beta}}$$

(v) Prove that : $\sin(\frac{\pi}{2} + \alpha) = \cos\alpha$

we know that $\mathbf{Cos}\left(\frac{\pi}{2} - \beta\right) = \mathbf{Sin}\ \beta$

putting $\beta = \frac{\pi}{2} + \alpha$ in above equation, we get

$$\text{Cos}[\frac{\pi}{2} - (\frac{\pi}{2} + \alpha)] = \sin(\frac{\pi}{2} + \alpha)$$

$$\Rightarrow \quad \cos(-\alpha) = \sin\left(\frac{\pi}{2} + \alpha\right)$$

$$\cos\alpha = \sin\left(\frac{\pi}{2} + \alpha\right)$$

$$\boxed{\sin\left(\frac{\pi}{2} + \alpha\right) = \cos\alpha}$$

(vi) Prove that : $\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

Since $\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

replacing β by $-\beta$, we get

$\cos[\alpha - (-\beta)] = \cos\alpha\cos(-\beta) + \sin\alpha\sin(-\beta)$

{because $\cos(-\beta) = \cos\beta$,
$\sin(-\beta) = \sin\beta$ }

$$\boxed{\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta}$$

(vii) Prove that : $\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

We know that

$$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

replace α by $\frac{\pi}{2} + \alpha$, we get

$$\cos\left[\left(\frac{\pi}{2} + \alpha\right) + \beta\right] = \cos\left(\frac{\pi}{2} + \alpha\right)\cos\beta - \sin\left(\frac{\pi}{2} + \alpha\right)\sin\beta$$

$$\cos\left[\frac{\pi}{2} + (\alpha + \beta)\right] = -\sin\alpha\cos\beta - \cos\alpha\sin\beta$$

$$-\sin(\alpha + \beta) = -[\sin\alpha\cos\beta + \cos\alpha\sin\beta]$$

$$\boxed{\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta}$$

(viii) Prove that: $\sin(\alpha - \beta) = \sin\alpha \cos\beta - \cos\alpha \sin\beta$

We know that

$$\sin(\alpha + \beta) = \sin\alpha \cos\beta + \cos\alpha \sin\beta$$

replacing β by $-\beta$, we get

$$\sin(\alpha - \beta) = \sin\alpha \cos(-\beta) + \cos\alpha \sin(-\beta)$$

{because $\cos(-\beta) = \cos\beta$,
$\sin(-\beta) = \sin\beta$}

$$\boxed{\sin(\alpha - \beta) = \sin\alpha \cos\beta - \cos\alpha \sin\beta}$$

(ix) Prove that: $\text{Sin}\left(\frac{\pi}{2} + \beta\right) = -\text{Cos}\ \beta$

Put $\alpha = \frac{\pi}{2}$ in

$$\text{Sin}(\alpha + \beta) = \text{Sin}\alpha\ \text{Cos}\beta + \text{Cos}\alpha\ \text{Sin}\beta$$

$$\text{Sin}\left(\frac{\pi}{2} + \beta\right) = \text{Sin}\frac{\pi}{2}\ \text{Cos}\beta + \text{Cos}\frac{\pi}{2}\ \text{Sin}\beta$$

$$= 0\ .\ \text{Cos}\beta + 0\ .\ \text{Sin}\beta$$

$$\boxed{\text{Sin}\left(\frac{\pi}{2} + \beta\right) = \text{Cos}\ \beta}$$

(x) Prove that: $\text{Sin}\left(\frac{\pi}{2} - \beta\right) = \text{Cos}\beta$

Put $\alpha = \frac{\pi}{2}$ in

$$\text{Sin}(\alpha - \beta) = \text{Sin}\alpha\ \text{Cos}\beta - \text{Cos}\alpha\ \text{Sin}\beta$$

$$\text{Sin}\left(\frac{\pi}{2}-\beta\right) = \text{Sin}\frac{\pi}{2}\ \text{Cos}\beta - \text{Cos}\frac{\pi}{2}\text{Sin}\beta$$

$$\text{Sin}\left(\frac{\pi}{2}-\beta\right) = 1.\text{Cos}\beta - 0.\text{Sin}\beta$$

$$\boxed{\text{Sin}\left(\frac{\pi}{2}-\beta\right) = \text{Cos}\beta}$$

(xi) $\tan(\alpha+\beta) = \dfrac{\text{Sin}\,(\alpha+\beta)}{\text{Cos}(\alpha+\beta)}$

$$=\frac{\text{Sin}\ \alpha\ \text{Cos}\beta + \text{Cos}\ \alpha\ \text{Sin}\ \beta}{\text{Cos}\alpha\ \text{Cos}\beta - \text{Sin}\ \alpha\ \text{Sin}\ \beta}$$

Divide numerator and denominator by $\text{Cos}\alpha\ \text{Cos}\beta$, we get

$$\tan(\alpha+\beta) = \frac{\dfrac{\text{Sin}\ \alpha\ \text{Cos}\beta}{\text{Cos}\alpha\ \text{Cos}\beta} + \dfrac{\text{Cos}\ \alpha\ \text{Sin}\ \beta}{\text{Cos}\alpha\ \text{Cos}\beta}}{1 - \dfrac{\text{Sin}\ \alpha\ \text{Cos}\beta}{\text{Cos}\alpha\ \text{Cos}\beta}}$$

$$\boxed{\mathbf{tan}(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1-\tan\alpha\tan\beta}}$$

(xii) $\text{Cot}\,(\alpha+\beta) = \dfrac{1}{\tan(\alpha+\beta)}$

$$= \frac{1-\tan\alpha\ \tan\beta}{\tan\alpha + \tan\beta}$$

$$= \frac{1-\dfrac{1}{\text{Cot}\alpha\ \text{Cot}\beta}}{\dfrac{1}{\text{Cot}\alpha} + \dfrac{1}{\text{Cot}\beta}}$$

$$\text{Cot}(\alpha+\beta)=\frac{\text{Cot}\alpha\ \text{Cot}\beta-1}{\text{Cot}\alpha+\text{Cot}\beta}$$

Similarly, $\tan(\alpha-\beta)=\dfrac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}$

and $\cot(\alpha-\beta)=\dfrac{\cot\alpha\ \cot\beta+1}{\cot\alpha-\cot\beta}$

($\tan\theta$ and $\text{Cot}\ \theta$ are odd functions).

Note :

(1) If θ is added or subtracted from odd multiple of right angle ($\pi/2$), the trigonometric ratios change into co-ratios and vice-versa.

e.g.,

$\text{Sin}\ (\frac{\pi}{2}-\theta)=\cos\theta$, $\cos\ (\frac{\pi}{2}-\theta)=\sin\theta$, $\tan\ (\frac{\pi}{2}-\theta)=\cot\theta$

$\text{Sin}(\frac{3\pi}{2}-\theta)=-\cos\theta$, $\cos(\frac{3\pi}{2}-\theta)=-\sin\theta$, $\tan(\frac{3\pi}{2}-\theta)=\cot\theta$

(2) If θ is added or subtracted from an even multiple of right angle ($\pi/2$), the trigonometric ratios shall remain the same.

e.g.,

$\text{Sin}\ (\pi-\theta)=\sin\theta$, $\cos\ (\pi-\theta)=-\cos\theta$, $\tan\ (\pi-\theta)=-\tan\theta$

$\text{Sin}(2\pi-\theta)=-\sin\theta$, $\cos(2\pi-\theta)=\cos\theta$, $\tan(2\pi-\theta)=-\tan\theta$

Example 2:

Show that $\text{Cos}\ (180^\circ+\theta)=-\text{Cos}\ \theta$

Solution:

$$\begin{aligned}\text{L.H.S} &= \text{Cos}\ (180^\circ+\theta)\\ &= \text{Cos}\ 180^\circ\ \text{Cos}\,\theta-\text{Sin}\ 180^\circ\ \text{Sin}\ \theta\\ &= (-1)\ \text{Cos}\,\theta-(0)\ \text{Sin}\,\theta\\ &= \text{Cos}\,\theta-0\\ &= -\text{Cos}\,\theta=\text{R.H.S}\end{aligned}$$

Example 3:

If $\text{Sin}\,\alpha=\dfrac{4}{5}$ and $\text{Sin}\,\beta=\dfrac{12}{13}$, neither terminal ray of α nor β is in the first quadrant, find $\text{Sin}(\alpha+\beta)$.

Solution:

Because $\text{Cos}\alpha = \sqrt{1-\text{Sin}^2\alpha}$

$$= \sqrt{1-\left(\frac{4}{5}\right)^2} = \sqrt{1-\frac{16}{25}}$$

$$\text{Cos}\alpha = \sqrt{\frac{25-16}{25}} = \sqrt{\frac{9}{25}} = \pm\frac{3}{5}$$

Since α and β does not lie in 1st quadrant and Sin α and Sin β is positive, therefore α, β lies is 2nd quadrant and in 2nd quadrant Cos α is negative is

$$\text{Cos}\alpha = -\frac{3}{5}$$

Then $\text{Cos}\beta = \sqrt{1-\text{Sin}^2\beta}$

$$= \sqrt{1-\left(\frac{12}{13}\right)^2} = \sqrt{1-\frac{144}{169}}$$

$$= \sqrt{\frac{169-144}{169}} = \sqrt{\frac{25}{169}} = \pm\frac{5}{13}$$

$$\therefore\ \text{Cos}\beta = -\frac{5}{13} \qquad \because \beta \text{ lies in } 2^{nd} \text{ quadrant}$$

Now $\text{Sin}(\alpha+\beta) = \text{Sin}\alpha\ \text{Cos}\beta + \text{Cos}\alpha\ \text{Sin}\beta$

$$= \left(\frac{4}{5}\right)\left(-\frac{5}{13}\right) + \left(-\frac{3}{5}\right)\left(\frac{12}{13}\right)$$

$$= -\frac{20}{65} - \frac{36}{65} = \frac{-20-36}{65} = -\frac{56}{65}$$

$$\text{Sin}(\alpha+\beta) = -\frac{56}{65}$$

Example 4:

Express $4\text{Sin}\,\theta + 7\,\text{Cos}\,\theta$ in the form $r\ \text{Sin}\ (\theta + \phi)$, where the terminal side of θ and ϕ are in the first quadrant.

Solution: Multiplying and Dividing the expression by

$$r = \sqrt{(4)^2+(7)^2} = \sqrt{16+49} = \sqrt{65}$$

$$4\ \text{Sin}\,\theta + 7\text{Cos}\,\theta = \sqrt{65}\left[\frac{4}{\sqrt{65}}\text{Sin}\,\theta + \frac{7}{\sqrt{65}}\text{Cos}\,\theta\right]$$

$$= \sqrt{65}\left[\operatorname{Sin}\theta\left(\frac{4}{\sqrt{65}}\right) + \operatorname{Cos}\theta\left(\frac{7}{\sqrt{65}}\right)\right] \quad ..\ (1)$$

Since $\quad r\operatorname{Sin}(\theta + \phi) = r(\operatorname{Sin}\theta\operatorname{Cos}\phi + \operatorname{Cos}\theta\operatorname{Sin}\phi) \quad ..\ (2)$

Where $\quad 0 < \phi < \frac{\pi}{2}$

Let $\quad \operatorname{Cos}\phi = \frac{4}{\sqrt{65}}$ and $\operatorname{Sin}\phi = \frac{4}{\sqrt{65}}$, then

$$4\operatorname{Sin}\theta + 7\operatorname{Cos}\theta = \sqrt{65}[\operatorname{Sin}\theta\operatorname{Cos}\phi + \operatorname{Cos}\theta\operatorname{Sin}\phi]$$

$$= \sqrt{65}[\operatorname{Sin}(\theta + \phi)]$$

Where $\operatorname{Cos}\phi = \frac{4}{\sqrt{65}}$ and $\operatorname{Sin}\phi = \frac{4}{\sqrt{65}}$

i.e., $\quad \tan\phi = \frac{7}{4}$

$$\phi = \tan^{-1}\frac{7}{4}$$

Example 5:

Find the value of Sin 75°.

Solution: $\operatorname{Sin}(75^o) = \operatorname{Sin}(45^o + 30^o)$

$$= \operatorname{Sin}45^o\operatorname{Cos}30^o + \operatorname{Cos}45^o\operatorname{Sin}30^o$$

$$= \frac{1}{\sqrt{2}}.\frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}}.\frac{1}{2} = \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}$$

$$= \frac{\sqrt{3}+1}{2\sqrt{2}}$$

Example 6:

$$\operatorname{Sin}(\alpha+\beta).\operatorname{Sin}(\alpha-\beta) = \operatorname{Cos}^2\beta - \operatorname{Cos}^2\alpha$$

Solution: L.H.S $= \operatorname{Sin}(\alpha+\beta).\operatorname{Sin}(\alpha-\beta)$

$$= [\operatorname{Sin}\alpha\operatorname{Cos}\beta + \operatorname{Cos}\alpha\operatorname{Sin}\beta]$$

$$[\operatorname{Sin}\alpha\operatorname{Cos}\beta - \operatorname{Cos}\alpha\operatorname{Sin}\beta]$$

$$= \operatorname{Sin}^2\alpha\operatorname{Cos}^2\beta - \operatorname{Cos}^2\alpha\operatorname{Sin}^2\beta$$

$$= (1 - \cos^2\alpha)\cos^2\beta - \cos^2\alpha(1 - \cos^2\beta)$$

$$= \operatorname{Cos}^2\beta - \operatorname{Cos}^2\alpha\operatorname{Cos}^2\beta - \operatorname{Cos}^2\alpha + \operatorname{Cos}^2\alpha\operatorname{Cos}^2\beta$$

$$= \operatorname{Cos}^2\beta - \operatorname{Cos}^2\alpha$$

= R.H.S

Exercise 6.1

Q.1 Find the value of (i) $\cos 75^0$ (ii) $\sin 15^0$ (iii) $\sin 105^0$ (iv) $\cos 105^0$ (v) $\tan 105^0$.

Q.2 Prove that:

(i) $\sin(180^0 - \theta) = \sin\theta$ (ii) $\cos(270^0 + \theta) = \sin\theta$

(iii) $\tan(180^0 + \theta) = \tan\theta$ (iv) $\sin(360^0 - \theta) = -\sin\theta$

(v) $\cot(360^0 + \theta) = \cot\theta$ ((vi) $\tan(90^0 + \theta) = -\cot\theta$

Q.3 Show that:

(i) $\sin(x - y)\cos y + \cos(x - y)\sin y = \sin x$

(ii) $\cos(x + y)\cos y + \sin(x + y)\sin y = \cos x$

(iii) $\cos(A + B)\sin(A - B) = \sin A\cos A - \sin B\cos B$

(iv) $\dfrac{\tan(x+y) - \tan x}{1 + \tan(x+y)\tan x} = \dfrac{\sin x}{\cos y}$

Q.4 Suppose that A , B and C are the measure of the angles of a triangle such that $A + B + C = \pi$, prove that

$\tan A + \tan B + \tan C = \tan A\tan B\tan C$

Q.5 Prove that:

(i) $\text{Sin}\,\theta + \cos\theta = \sqrt{2}\sin(\theta + \frac{\pi}{4})$

(ii) $\sqrt{3}\cos\theta - \sin\theta = 2\cos(\theta + 30^0)$

(iii) $\text{Tan}(45^0 - \theta) = \dfrac{1 - \tan 6}{1 + \tan 6}$

(iv) $\text{Tan}(45^0 + \theta) = \dfrac{\cos\theta - \sin 6}{\cos\theta - \sin 6}$

(v) $\dfrac{\tan(\alpha + \beta)}{\cot(\alpha - \beta)} = \dfrac{\tan^2\alpha - \tan^2\beta}{1 - \tan^2\alpha\,\tan^2\beta}$

(vi) $\cot(\alpha + \beta) = \dfrac{\cot\alpha\cot\beta - 1}{\cot\alpha + \cot\beta}$

(vii) $\dfrac{\sin(\alpha + \beta)}{\cos\alpha\cos\beta} = \tan\alpha + \tan\beta$

(viii) $\dfrac{\tan\alpha + \tan\beta}{\tan\alpha - \tan\beta} = \dfrac{\sin(\alpha+\beta)}{\sin(\alpha-\beta)}$

(ix) $\tan(x + \frac{\pi}{4}) - \tan\left(x - \frac{3\pi}{4}\right) = 0$

(x) $\text{Cos}(\frac{\pi}{3} + x) - \sin\left(\frac{\pi}{6} - x\right) = 0$

Q.6 Prove that:

(i) $\cos(\alpha+\beta)\cos(\alpha-\beta) = \cos^2\alpha - \sin^2\beta$

(ii) $\sin(x+y)\sin(x-y) = \sin^2 x - \sin^2 y$

Q.7 If $\text{Sin}\,\alpha = \frac{4}{5}$ and $\text{Sin}\,\beta = \frac{12}{13}$, both α and β are in the 1^{st} quadrant find:

(i) $\sin(\alpha-\beta)$ (ii) $\cos(\alpha+\beta)$

Q.8 If $\cos A = \frac{1}{5}$ and $\cos B = \frac{1}{2}$ A and B be acute angles, find the value of: (i) $\sin(A+B)$ (ii) $\cos(A-B)$

Q.9 If $\tan\alpha = \frac{3}{4}$ and $\sec\beta = \frac{13}{5}$ and neither α nor β is in the 1^{st} quaderant, find $\sin(\alpha+\beta)$

Q.10 Prove that: $\dfrac{\sin\alpha}{\sec 4\alpha} + \dfrac{\cos\alpha}{\text{cosec}\,4\alpha} = \sin 5\alpha$

Q.11 If $\tan\beta = \dfrac{n\sin\alpha\cos\alpha}{1 - n\sin^2\alpha}$, prove that $\tan(\alpha-\beta) = (1-n)\tan\alpha$

Q.12 If α, β and γ are the angle of triangle ABC, then prove that

(i) $\sin(\alpha+\beta) = \sin\gamma$ (ii) $\cos(\alpha+\beta) = -\sin\gamma$

(iii) $\tan(\alpha+\beta) + \tan\gamma = 0$

Q.13 (i)If $\cos\alpha = \frac{1}{7}$, $\cos\beta = \frac{13}{14}$, then prove that $\alpha - \beta = 60^0$, where the terminal rays of α and β are in 1^{st} quadrants.

(ii)If If $\tan\alpha = \frac{5}{6}$ and $\tan\beta = \frac{1}{11}$, then prove that $\alpha + \beta = 45^0$, where the terminal rays of α and β are in 1^{st} quadrants.

Q.14 Express the following in the form of $r\sin(\theta + \phi)$, where the terminal rays of θ is in the 1^{st} quadrants. Be sure to specify ϕ :

(i) $4\sin\theta + 3\cos\theta$ (ii) $\sqrt{3}\sin\theta + \sqrt{7}\cos\theta$

(iii) $5\sin\theta - 4\cos\theta$ (iv) $\sin\theta + \cos\theta$

Answers 6.1

Q.1 (i) $\frac{\sqrt{3}-1}{2\sqrt{2}}$ (ii) $\frac{\sqrt{3}-1}{2\sqrt{2}}$ (iii) $\frac{\sqrt{3}+1}{2\sqrt{2}}$

(iv) $\frac{1-\sqrt{3}}{2\sqrt{2}}$ (v) $\frac{\sqrt{3}+1}{1-\sqrt{3}}$

Q.7 (i) $-\frac{16}{25}$ (ii) $-\frac{33}{65}$

Q.8 (i) $\frac{\sqrt{24}+\sqrt{3}}{10}$ (ii) $\frac{1+6\sqrt{2}}{10}$

Q.9 $\frac{33}{65}$

Q.14 (i) $5\sin(\theta+\phi)$, $\phi = \tan^{-1}\left(\frac{3}{4}\right)$

(ii)$10\sin(\theta+\phi)$, $\phi = \tan^{-1}\left(\sqrt{\frac{7}{3}}\right)$