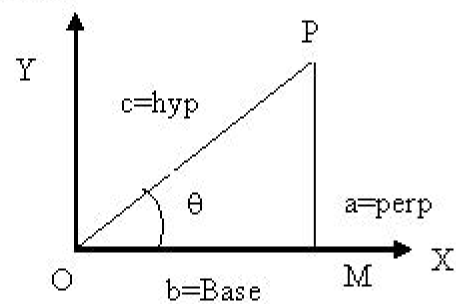
5. A space man land on the moon and observes that the Earth's diameter subtends an angle of $1^\circ 54'$ at his place of landing. If the Earth's radius is 6400km, find the distance between the Earth and the Moon.
6. The sun is about 1.496×10^8 km away from the Earth. If the angle subtended by the sun on the surface of the earth is 9.3×10^{-3} radians approximately. What is the diameter of the sun?
7. A horse moves in a circle, at one end of a rope 27cm long, the other end being fixed. How far does the horse move when the rope traces an angle of 70° at the centre.
8. Lahore is 68km from Gujranwala. Find the angle subtended at the centre of the earth by the road. Joining these two cities, earth being regarded as a sphere of 6400km radius.
9. A circular wire of radius 6 cm is cut straightened and then bend so as to lie along the circumference of a hoop of radius 24 cm. find measure of the angle which it subtend at the centre of the hoop
10. A pendulum 5 meters long swings through an angle of 4.5^0. through what distance does the bob moves ?
11. A flywheel rotates at 300 rev/min. If the radius is 6 cm. through what total distance does a point on the rim travel in 30 seconds ?

Answers 5.1

1	(i) 3.66 rad	(ii) 3π	(iii) 0.74 rad	(iv) 0.42 rad
2.	(i) 225°	(ii) 120°	(iii) $316^\circ 16' 19''$	(iv) $74^\circ 29' 4''$
3.	(i) r = 3cm	(ii) $\theta = 2.443$ rad	(iii) $l = 346.4$ meters	
4.	6704.76 km	5. 386240 km		
6.	1.39×10^6 km	7. 33 m	8. $36' 43''$	9. $\pi/2$ or 90^0
10.	0.39 m	11. 5657 cm		

5.8 Trigonometric Function and Ratios:

Let the initial line OX revolves and trace out an angle θ. Take a point P on the final line. Draw perpendicular PM from P on OX: $\angle XOP = \theta$, where θ may be in degree or radians.



Now OMP is a right angled triangle,
We can form the six ratios as follows:

$\frac{a}{c}, \frac{b}{c}, \frac{a}{b}, \frac{b}{a}, \frac{c}{b}, \frac{c}{a}$

In fact these ratios depend only on the size of the angle and not on the triangle formed. Therefore these ratios called **Trigonometic ratios or**

trigonometric functions of angle θ Fig. 5.10

and defined as below: θ

$$\textbf{Sin } \theta = \frac{a}{c} = \frac{MP}{OP} = \frac{\text{Perpendicular}}{\text{Hypotenuse}}$$

$$\text{Cos } \theta = \frac{b}{c} = \frac{OM}{OP} = \frac{\text{Base}}{\text{Hypotenuse}}$$

$$\tan \theta = \frac{a}{b} = \frac{MP}{OM} = \frac{\text{Perpendicular}}{\text{Base}}$$

$$\text{Cot } \theta = \frac{b}{a} = \frac{OM}{MP} = \frac{\text{Base}}{\text{Perpendicular}}$$

$$\text{Sec } \theta = \frac{c}{b} = \frac{OP}{OM} = \frac{\text{Hypotenuse}}{\text{Perpendicular}}$$

$$\text{Cosec } \theta = \frac{c}{a} = \frac{OP}{PM} = \frac{\text{Hyponenuse}}{\text{Perpendicular}}$$

5.9 Reciprocal Functions:

From the above definition of trigonometric functions, we observe that

(i) $\text{Sin } \theta = \frac{1}{\text{Cosec } \theta}$ or, $\text{Cosec } \theta = \frac{1}{\text{Sin } \theta}$ i.e. Sin θ and Cosec θ are reciprocal of each other.

(ii) $\text{Cos } \theta = \frac{1}{\text{Sec } \theta}$ or, $\text{Sec } \theta = \frac{1}{\text{Cos } \theta}$ i.e. Cos θ and Sec θ are reciprocals of each other.

(iii) $\tan \theta = \frac{1}{\text{Cot } \theta}$ or, $\text{Cot } \theta = \frac{1}{\tan \theta}$ i.e. tan θ and Cot θ are reciprocals of each other.

We can also see that;

$$\tan \theta = \frac{\text{Sin } \theta}{\text{Cos } \theta} \quad \text{and} \quad \text{Cot } \theta = \frac{\text{Cos } \theta}{\text{Sin } \theta}$$

5.10 Rectangular Co-ordinates and Sign Convention:

In plane geometry the position of a point can be fixed by measuring its perpendicular distance from each of two perpendicular called co-ordinate axes. The horizontal line (x-axis) is also called abscissa and the vertical line(y-axis) is called as ordinate.

Distance measured from the point O in the direction OX and OY are regarded as positive, while in the direction of OX′ and OY′ are considered negative.

Thus in the given figure OM_1, OM_4, MP_1 and M_2P_2 are positive, while OM_2, OM_3, M_3P_3 and M_4P_4 are negative.

The terminal line i.e., OP_1, OP_2, OP_3, and OP_4 are positive in all the quadrants.

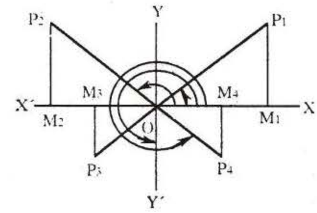
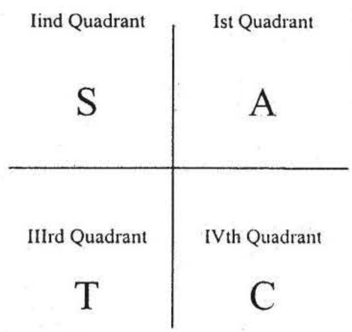


Fig. 4.11

5.11 Signs of Trigonometric Functions:

The trigonometric ratios discussed above have different signs in different quadrants. Also from the above discussion we see that OM and MP changes their sign in different quadrants. We can remember the sign of trigonometric function by "ACTS" Rule or CAST rule. In "CAST" C stands for cosine A stands for All and S stands for Sine and T stands for Tangent.



First Quadrant:

In first quadrant sign of all the trigonometric functions are positive i.e., sin, cos, tan, Cot, Sec, Cosec all are positive.

Second Quadrant:

In second quadrant sine and its inverse cosec are positive. The remaining four trigonometric function i.e., cos, tan, cot, sec are negative.

Third Quadrant:

In third quadrant tan and its reciprocal cot are positive the remaining four function i.e., Sin, cos, sec and cosec are negative.

Fourth Quadrant:

In fourth quadrant cos and its reciprocal sec are positive, the remaining four functions i.e., sin, tan, cot and cosec are negative.

5.12 Trigonometric Ratios of Particular Angles:

1. Trigonometric Ratios of 30^o or $\frac{\pi}{6}$:

Let the initial line OX revolve and trace out an angle of 30^o. Take a point P on the final line. Draw PQ perpendicular from P on OX. In 30^o right angled triangle, the side opposite to the 30^o angle is one-half the length of the hypotenuse, i.e., if PQ = 1 unit then OP will be 2 units.

From fig. OPQ is a right angled triangle

$\therefore$ By Pythagorean theorem, we have

$(OP)^2 = (OQ)^2 + (PQ)^2$

$(2)^2 = (OQ)^2 + (1)^2$

$4 = (OQ)^2 + 1$

$(OQ)^2 = 3$

$(OQ) = \sqrt{3}$

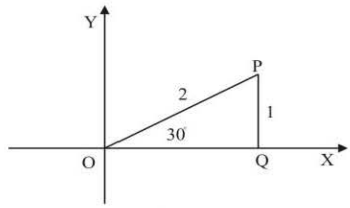


Fig. 4.12

Therefore $\text{Sin } 30^o = \frac{\text{Prep.}}{\text{Hyp}} = \frac{PQ}{OP} = \frac{1}{2}$

$$\text{Cos } 30^o = \frac{\text{Base}}{\text{Hyp}} = \frac{OQ}{OP} = \frac{\sqrt{3}}{2}$$

$$\tan 30^o = \frac{\text{Prep.}}{\text{Base}} = \frac{PQ}{OQ} = \frac{1}{\sqrt{3}}$$

$$\text{Cot } 30^o = \frac{\text{Base}}{\text{Prep.}} = \frac{OQ}{PQ} = \frac{\sqrt{3}}{1} = \sqrt{3}$$

$$\text{Sec } 30^o = \frac{\text{Hyp.}}{\text{Base}} = \frac{OP}{OQ} = \frac{2}{\sqrt{3}}$$

$$\text{Cosec } 30^o = \frac{\text{Hyp.}}{\text{Prep.}} = \frac{OP}{PQ} = \frac{2}{1} = 2$$

2. Trigonometric ratios of 45^o Or $\frac{\pi}{4}$

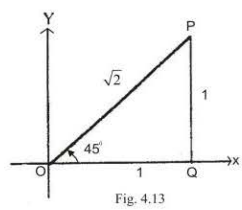


Fig. 4.13

Let the initial line OX revolve and trace out an angle of 45^o. Take a point P on the final line. Draw PQ perpendicular from P on OX . In 45^o right angled triangle the length of the perpendicular is equal to the length of the base

i.e., if PQ = 1 unit. then OQ = 1 unit

From figure by Pythagorean theorem.

$(OP)^2 = (OQ)^2 + (PQ)^2$

$(OP)^2 = (1)^2 + (1)^2 = 1 + 1 = 2$

$OP = \sqrt{2}$

Therefore $\text{Sin } 45^o = \frac{\text{Prep.}}{\text{Hyp}} = \frac{PQ}{OP} = \frac{1}{\sqrt{2}}$

$$\text{Cos } 45^o = \frac{\text{Base}}{\text{Hyp}} = \frac{OQ}{OP} = \frac{1}{\sqrt{2}}$$

$$\tan 45^o = \frac{\text{Prep.}}{\text{Base}} = \frac{PQ}{OQ} = \frac{1}{1} = 1$$

$$\text{Cot } 45^o = \frac{\text{Base}}{\text{Prep.}} = \frac{OQ}{PQ} = \frac{1}{1} = 1$$

$$\text{Sec } 45^o = \frac{\text{Hyp.}}{\text{Base}} = \frac{OP}{OQ} = \frac{\sqrt{2}}{1} = \sqrt{2}$$

$$\text{Cosec } 45^o = \frac{\text{Hyp.}}{\text{Prep.}} = \frac{OP}{PQ} = \frac{\sqrt{2}}{1} = \sqrt{2}$$

3. Trigonometric Ratios of 60^o or $\frac{\pi}{3}$

Let the initial line OX revolve and trace out an angle of 60^o. Take a point P on the final line. Draw PQ perpendicular from P on OX. In 60^o right angle triangle the length of the base is one-half of the Hypotenuse.

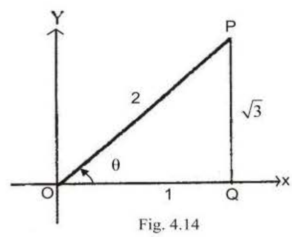


Fig. 4.14

i.e., OQ = Base = 1 unit

then, OP= Hyp = 2 units

from figure by Pythagorean Theorem:

$$(OP)^2 = (OQ)^2 + (PQ)^2$$
$$(2)^2 = (1)^2 + (PQ)^2$$
$$4 = 1 (PQ)^2$$
$$(PQ)^2 = 3$$
$$PQ = \sqrt{3}$$

Therefore
$$\text{Sin } 60^o = \frac{\text{Prep.}}{\text{Hyp}} = \frac{PQ}{OP} = \frac{\sqrt{3}}{2}$$

$$\text{Cos } 60^o = \frac{\text{Base}}{\text{Hyp}} = \frac{OQ}{OP} = \frac{1}{2}$$

$$\tan 60^o = \frac{\text{Prep.}}{\text{Base}} = \frac{PQ}{OQ} = \frac{\sqrt{3}}{1} = \sqrt{3}$$

$$\text{Cot } 60^o = \frac{\text{Base}}{\text{Prep.}} = \frac{OQ}{PQ} = \frac{1}{\sqrt{3}}$$

$$\text{Sec } 60^\circ = \frac{\text{Hyp.}}{\text{Base}} = \frac{OP}{OQ} = \frac{\sqrt{2}}{1} = \sqrt{2}$$

$$\text{Cosec } 60^\circ = \frac{\text{Hyp.}}{\text{Prep.}} = \frac{OP}{PQ} = \frac{2}{\sqrt{3}} =$$

Trigonometric ratios of 0°

Let the initial line revolve and trace out a small angle nearly equal to zero 0°. Take a point P on the final line. Draw PM perpen-dicular on OX.

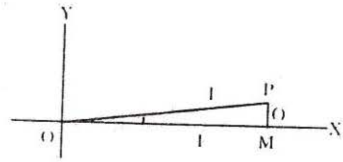


Fig. 4.15

PM = 0

and OP = 1, OM = 1

(Because they just coincide x-axis)

Therefore from figure.

$$\text{Sin } 0^\circ = \frac{\text{Prep.}}{\text{Hyp}} = \frac{PM}{OP} = \frac{0}{1} = 0$$

$$\text{Cos } 0^\circ = \frac{\text{Base}}{\text{Hyp}} = \frac{OM}{OP} = \frac{1}{1} = 1$$

$$\tan 0^\circ = \frac{\text{Prep.}}{\text{Base}} = \frac{PM}{OM} = \frac{0}{1} = 0$$

$$\text{Cot } 0^\circ = \frac{\text{Base}}{\text{Prep.}} = \frac{OM}{PM} = \frac{1}{0} = \infty$$

$$\text{Sec } 0^\circ = \frac{\text{Hyp.}}{\text{Base}} = \frac{OP}{OM} = \frac{1}{1} = 1$$

$$\text{Cosec } 0^\circ = \frac{\text{Hyp.}}{\text{Prep.}} = \frac{OP}{PM} = \frac{1}{0} = \infty$$

Trigonometric Ratio of 90°

Let initial line revolve and trace out an angle nearly equal to 90°. Take a point P on the final line. Draw PQ perpendicular from P on OX.

OQ = 0, OP = 1, PQ = 1 (Because they just coincide y-axis).

Therefore $$\text{Sin } 90^\circ = \frac{\text{Prep.}}{\text{Hyp}} = \frac{PQ}{OP} = \frac{1}{1} = 1$$

$$\text{Cos } 90^\circ = \frac{\text{Base}}{\text{Hyp}} = \frac{OQ}{OP} = \frac{0}{1} = 0$$

$$\tan 90^\circ = \frac{\text{Prep.}}{\text{Base}} = \frac{PQ}{OQ} = \frac{1}{0} = \infty$$

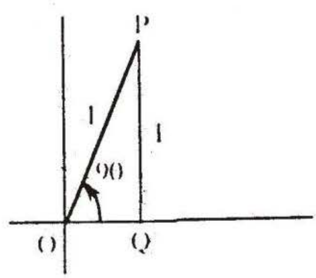


Fig. 4.16

$$\text{Cot } 90^o = \frac{\text{Base}}{\text{Prep.}} = \frac{OQ}{PQ} = \frac{0}{1} = 0$$ Fig. 5.16

$$\text{Sec } 90^o = \frac{\text{Hyp.}}{\text{Base}} = \frac{OP}{OQ} = \frac{1}{0} = \infty$$

$$\text{Cosec } 90^o = \frac{\text{Hyp.}}{\text{Prep.}} = \frac{OP}{PQ} = \frac{1}{1} = 1$$

Table for Trigonometrical Ratios of Special angle

Angles / Ratios	0^o	30^o Or $\frac{\pi}{6}$	45^o Or $\frac{\pi}{4}$	60^o Or $\frac{\pi}{3}$	90^o Or $\frac{\pi}{2}$
Sin θ	$\sqrt{\frac{0}{4}} = 0$	$\sqrt{\frac{1}{4}} = \frac{1}{2}$	$\sqrt{\frac{2}{4}} = \frac{1}{\sqrt{2}}$	$\sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$	$\sqrt{\frac{4}{4}} = 1$
Cos θ	$\sqrt{\frac{4}{4}} = 1$	$\sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$	$\sqrt{\frac{2}{4}} = \frac{1}{\sqrt{2}}$	$\sqrt{\frac{1}{4}} = \frac{1}{2}$	$\sqrt{\frac{0}{4}} = 0$
Tan θ	$\sqrt{\frac{0}{4}} = 0$	$\sqrt{\frac{1}{3}} = \frac{1}{\sqrt{3}}$	$\sqrt{\frac{2}{2}} = 1$	$\sqrt{\frac{3}{1}} = \sqrt{3}$	$\sqrt{\frac{4}{0}} = \infty$

Example 1:

If $\cos\theta = \frac{5}{13}$ and the terminal side of the angle lies in the first quadrant find the values of the other five trigonometric ratio of θ.

Solution:

In this cause $\cos\theta = \frac{5}{13}$

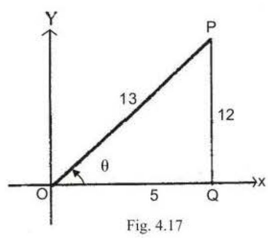


Fig. 4.17

$$\cos\theta = \frac{\text{Base}}{\text{Hyp}} = \frac{5}{13}$$

From Fig. $(OP)^2 = (OQ)^2 + (PQ)^2$

$$(13)^2 = (5)^2 + (PQ)^2$$
$$169 = 25 + (PQ)^2$$
$$(PQ)^2 = 169 - 25$$
$$= 144$$
$$PQ = \pm 12$$

Because θ lies in the first quadrant

i.e., $\sin\theta = \frac{12}{13}$ ∵ All the trigonometric ratios will be positive.

$$\text{Cos } \theta = \frac{5}{13}$$

$$\tan \theta = \frac{12}{5}, \qquad \sec \theta = \frac{13}{5}$$

$$\cot \theta = \frac{5}{12}, \qquad \text{cosec } \theta = \frac{13}{12}$$

Example 2:

Prove that $\cos 90^o - \cos 30^o = -2 \sin 60^o \sin 30^o$

Solution:

$$\text{L.H.S} = \cos 90^o - \cos 30^o$$

$$= 0 - \frac{\sqrt{3}}{2}$$

$$\text{L.H.S} = -\frac{\sqrt{3}}{2}$$

$$\text{R.H.S} = -2 \sin 60^o \sin 30^o$$

$$= -2.\frac{\sqrt{3}}{2} \text{x} \frac{1}{2}$$

$$\text{R.H.S} = -\frac{\sqrt{3}}{2}$$

Hence L.H.S = R.H.S

Example 3:

Verify that $\sin^2 30^o + \sin^2 60^o + \tan^2 45^o = 2$

Solution:

$$\text{L.H.S} = \sin^2 30^o + \sin^2 60^o + \tan^2 45^o$$

$$= \left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 + (1)^2$$

$$= \frac{1}{4} + \frac{3}{4} + 1$$

$$= \frac{1+3+4}{4}$$

$$= \frac{8}{4}$$

L.H.S = 2 = R.H.S

Exercise 5.2

Q.1 If $\sin\theta = \frac{2}{3}$, and the terminal side of the angle lies in the second quadrant, find the remaining trigonometric ratios of θ.

Q.2 If $\sin\theta = \frac{3}{8}$, and the terminal side of the angle lies in the second quadrant, find the remaining trigonometric ratios.

Q.3 If $\cos\theta = -\frac{\sqrt{3}}{2}$, and the terminal side of the angle lies in the third quadrant, find the remaining trigonometric ratios of θ.

Q.4 If $\tan\theta = \frac{3}{4}$, and the terminal side of the angle lies in the third quadrant, find the remaining trigonometric ratios of θ.

Q.5 If $\tan\theta = -\frac{1}{3}$, and the terminal side of the angle lies in the second quadrant, find the remaining trigonometric ratios of θ.

Q.6 If $\cot\theta = \frac{4}{3}$, and the terminal side of the angle is not in the first quadrant, find the trigonometric ratios of 0.

Q.7 If $\cot\theta = \frac{2}{3}$, and the terminal side of the angle does not lies in the first quadrant, find the trigonometric ratios of θ.

Q.8 If $\sin\theta = \frac{4}{5}$,and $\frac{\pi}{2} < \theta < \pi$ find the trigonometric ratios of θ

Q.9 If $\sin\theta = \frac{7}{25}$, find $\cos\theta$, if angle θ is an acute angle.

Q.10 If $\sin\theta = \frac{5}{6}$, find $\cos\theta$, if angle θ is an obtuse angle.

Q.11 Prove that:

(i) $\sin\frac{\pi}{3}\cos\frac{\pi}{6} + \cos\frac{\pi}{3}\sin\frac{\pi}{6} = \sin\frac{\pi}{2}$

(ii) $4\tan 60^\circ \tan 30^\circ \tan 45^\circ \sin 30^\circ \cos 60^\circ = 1$

(iii) $2\sin 45^\circ + \frac{1}{2}\operatorname{cosec} 45^\circ = \frac{3}{\sqrt{2}}$

(iv) $\cos 90^\circ - \cos 30^\circ = -2\sin 60^\circ \sin 30^\circ$

(v) $\sin^2\frac{\pi}{6}+\sin^2\frac{\pi}{3}+\tan^2\frac{\pi}{4}=2$

Q.12 $\sin^2\frac{\pi}{6}:\sin^2\frac{\pi}{4}:\sin^2\frac{\pi}{3}:\sin^2\frac{\pi}{2}=1:2:3:4$

Q.13 Evaluate

(i) $\cos 30^\circ \cos 60^\circ - \sin 30^\circ \sin 60^\circ$

(ii) $\dfrac{\tan 60^\circ - \tan 30^\circ}{1+\tan 60^\circ \tan 30^\circ}$

Answers 5.2

Q.1 $\text{Sin } \theta = \frac{2}{3}$ $\quad$ $\text{Cot } \theta = -\frac{\sqrt{5}}{2}$

$COS\theta = -\frac{\sqrt{5}}{3}$ $\quad$ $\sec\theta = -\frac{3}{\sqrt{5}}$

$\tan\theta = -\frac{2}{\sqrt{5}}$ $\quad$ $\text{Cosec } \theta = \frac{3}{2}$

Q.2 $\text{Sin } \theta = \frac{3}{8}$ $\quad$ $\text{Cot } \theta = -\frac{\sqrt{55}}{3}$

$\text{Cos } \theta = -\frac{\sqrt{55}}{8}$ $\quad$ $\text{Sec } \theta = -\frac{8}{\sqrt{55}}$

$\tan\theta = -\frac{3}{\sqrt{55}}$ $\quad$ $\text{Cosec } \theta = \frac{8}{3}$

Q.3 $\text{Sin } \theta = \frac{-1}{2}$ $\quad$ $\text{Cot } \theta = \sqrt{3}$

$\text{Cos } \theta = -\frac{\sqrt{3}}{2}$ $\quad$ $\text{Sec } \theta = -\frac{2}{\sqrt{3}}$

$\tan\theta = \frac{1}{\sqrt{3}}$ $\quad$ $\text{Cosec } \theta = -2$

Q.4 $\text{Sin } \theta = -\frac{3}{5}$ $\quad$ $\text{Cot } \theta = \frac{4}{3}$

$\text{Cos } \theta = -\frac{4}{5}$ $\quad$ $\text{Sec } \theta = -\frac{5}{4}$

$\tan\theta = \frac{3}{4}$ $\quad$ $\text{Cosec } \theta = -\frac{5}{3}$

Q.5 Sin $\theta = \frac{1}{\sqrt{10}}$ Cot $\theta = -3$

Cos $\theta = -\frac{3}{\sqrt{10}}$ Sec $\theta = -\frac{\sqrt{10}}{3}$

tan $\theta = -\frac{1}{3}$ Cosec $\theta = \sqrt{10}$

Q.6 Sin $\theta = -\frac{3}{5}$ Cot $\theta = \frac{4}{3}$

Cos $\theta = -\frac{4}{5}$ Sec $\theta = -\frac{5}{4}$

tan $\theta = \frac{3}{4}$ Cosec $\theta = -\frac{5}{3}$

Q.7 Sin $\theta = \frac{-3}{\sqrt{10}}$ Cot $\theta = \frac{2}{3}$

Cos $\theta = -\frac{2}{\sqrt{13}}$ Sec $\theta = -\frac{\sqrt{13}}{2}$

tan $\theta = \frac{3}{2}$ Cosec $\theta = -\frac{\sqrt{13}}{3}$

Q.8 Sin $\theta = \frac{4}{5}$ Cot $\theta = -\frac{3}{4}$

Cos $\theta = -\frac{3}{4}$ Sec $\theta = -\frac{5}{3}$

tan $\theta = -\frac{4}{3}$ Cosec $\theta = \frac{5}{4}$

Q.9 cos $\theta = \frac{24}{25}$

Q.10 cos $\theta - \frac{\sqrt{11}}{6}$

Q.13 (i) 0 (ii) $\frac{1}{\sqrt{3}}$