Q.9 $x+\frac{1}{4(x-1)}+\frac{1}{4(x+1)}-\frac{x}{2(x^2+1)}$

Q.10 $\frac{1}{3(x+1)}+\frac{7}{6(x-2)}-\frac{3x+2}{2(x^2-2)}$

Q.11 $\frac{1}{3(x-1)}-\frac{x+2}{3(x^2+x+1)}$

Q.12 $\frac{7}{10(x-1)}-\frac{1}{10(x+1)}-\frac{3x-1}{5(x^2+4)}$

4.11 Type IV: Quadratic repeated factors

When the denominator has repeated Quadratic factors.

Example 1:

Resolve into partial fraction $\frac{x^2}{(1-x)(1+x^2)^2}$

Solution:

$$\frac{x^2}{(1-x)(1+x^2)^2}=\frac{A}{1-x}+\frac{Bx+C}{(1+x^2)}+\frac{Dx+E}{(1+x^2)^2}$$

Multiplying both sides by L.C.M. i.e., $(1-x)(1+x^2)^2$ on both sides, we have

$x^2 = A(1+x^2)^2 + (Bx+C)(1-x)(1+x^2)+(Dx+E)(1-x)$ ……(i)

$x^2 = A(1+2x^2+x^4) + (Bx+C)(1-x+x^2-x^3)+(Dx+E)(1-x)$

Put $1-x=0 \Rightarrow x=1$ in eq. (i), we have

$(1)^2 = A(1+(1)^2)^2$

$1 = 4A \Rightarrow \boxed{A=\frac{1}{4}}$

$x^2 = A(1+2x^2+x^4) + B(x-x^2+x^3-x^4)+C(1-x+x^2-x^3)$
$+ D(x-x^2)+ E(1-x)$ …………………… (ii)

Comparing the co-efficients of like powers of x on both sides in Equation (II), we have

Co-efficient of x^4 : $A-B=0$ ……………. (i)
Co-efficient of x^3 : $B-C=0$ ……………. (ii)
Co-efficient of x^2 : $2A-B+C-D=1$ ……. (iii)
Co-efficient of x : $B-C+D-E=0$ ……. (iv)
Co-efficient term : $A+C+E=0$ ……………. (v)
from (i), $B=A$

$\Rightarrow B=\frac{1}{4} \quad \because \quad A=\frac{1}{4}$

from (i) $B = C$

$\Rightarrow \quad C = \frac{1}{4} \quad \because \quad C = \frac{1}{4}$

from (iii) $D = 2A - B + C - 1$

$$= 2\left(\frac{1}{4}\right) - \frac{1}{4} + \frac{1}{4} - 1$$

$\Rightarrow \quad \boxed{D = -\frac{1}{2}}$

from (v) $E = -A - C$

$$E = -\frac{1}{4} - \frac{1}{4} = -\frac{1}{2}$$

Hence the required partial fractions are by putting the values of A, B, C, D, E,

$$\frac{\frac{1}{4}}{1-x} + \frac{\frac{1}{4}x + \frac{1}{4}}{1+x^2} + \frac{-\frac{1}{2}x - \frac{1}{2}}{(1+x^2)^2}$$

$$\frac{1}{4(1-x)} + \frac{(x+1)}{4(1+x^2)} - \frac{x+1}{2(1+x^2)^2}$$

Example 2:

Resolve into partial fractions $\frac{x^2 + x + 2}{x^2(x^2+3)^2}$

Solution:

Let $\frac{x^2 + x + 2}{x^2(x^2+3)^2} = \frac{A}{x} + \frac{B}{x^2} + \frac{Cx + D}{x^2+3} + \frac{Ex + F}{(x^2+3)^2}$

Multiplying both sides by L.C.M. i.e., $x^2(x^2+3)^2$, we have

$$x^2 + x + 2 = Ax(x^2+3)^2 + B(x^2+3)^2$$
$$+ (cx + D)x^2(x^2+3) + (Ex + F)(x^2)$$

Putting x = 0 on both sides, we have

$2 = B(0+3)^2$

$2 = 9B \quad \Rightarrow \quad \boxed{B = \frac{2}{9}}$

Now $x^2 + x + 2 = Ax(x^4 + 6x^2 + 9) + B(x^4 + 6x^2 + 9)$
$+ C(x^5 + 3x^2) + D(x^4 + 3x^2) + E(x^3) + Fx^2$

$x^2 + x + 2 = (A+C)x^5 + (B+D)x^4 + (6A + 3C + E)x^3$

$$+(6B+3D+F)x^2 + (x+9B)$$

Comparing the co-efficient of like powers of x on both sides of Eq. (I), we have

Co-efficient of x^5 : $A + C = 0$ ……… (i)

Co-efficient of x^4 : $B - D = 0$ ……… (ii)

Co-efficient of x^3 : $6A + 3C + E = 0$ ……… (iii)

Co-efficient of x^2 : $6B + 3D + F = 1$ ……… (iv)

Co-efficient of x : $9A = 1$ ……… (v)

Co-efficient term : $9B = 1$ ……… (vi)

from (v) $9A = 1$

$\Rightarrow$ $\boxed{A = \frac{1}{9}}$

from (i) $A + C = 0$

$C = -A$

$\Rightarrow$ $\boxed{C = -\frac{1}{9}}$

from (i) $B + D = 0$

$D = -B$

$\Rightarrow$ $\boxed{D = -\frac{2}{9}}$

from (iii) $6A + 3C + E =$

$$6\left(\frac{1}{9}\right) + 3\left(-\frac{1}{9}\right) + E = 0$$

$$E = \frac{3}{9} - \frac{6}{9}$$

$\Rightarrow$ $\boxed{E = -\frac{1}{3}}$

from (iv) $6B + 3D + F = 1$

$$F = 1 - 6B - 3D$$

$$= 1 - 6\left(\frac{2}{9}\right) - 3\left(\frac{2}{9}\right)$$

$$= 1 - \frac{12}{9} + \frac{6}{9}$$

$$\Rightarrow \boxed{F = \frac{1}{3}}$$

Hence the required partial fractions are

$$\frac{\frac{1}{9}}{x} + \frac{\frac{2}{9}}{x^2} + \frac{-\frac{1}{9}x - \frac{2}{9}}{x^2+3} + \frac{-\frac{1}{3}x + \frac{1}{3}}{(x^2+3)^2}$$

$$= \frac{1}{9x} + \frac{2}{9x^2} - \frac{x+2}{9(x^2+3)} - \frac{x-1}{3(x^2+3)^2}$$

Exercise 4.4

Resolve into Partial Fraction:

Q.1 $\frac{7}{(x+1)(x^2+2)^2}$

Q.2 $\frac{x^2}{(1+x)(1+x^2)^2}$

Q.3 $\frac{5x^2+3x+9}{x(x^2+3)^2}$

Q.4 $\frac{4x^4+3x^3+6x^2+5x}{(x-1)(x^2+x+1)^2}$

Q.5 $\frac{2x^4-3x^2-4x}{(x+1)(x^2+2)^2}$

Q.6 $\frac{x^3-15x^2-8x-7}{(2x-5)(1+x^2)^2}$

Q.7 $\frac{49}{(x-2)(x^2+3)^2}$

Q.8 $\frac{8x^2}{(1-x^2)(1+x^2)^2}$

Q.9 $\frac{x^4+x^3+2x^2-7}{(x+2)(x^2+x+1)^2}$

Q.10 $\frac{x^2+2}{(x^2+1)(x^2+4)^2}$

Q.11 $\frac{1}{x^4+x^2+1}$

Answers 4.4

Q.1 $\frac{7}{9(X+1)} - \frac{7X-7}{9(X^2+2)} - \frac{7X-7}{3(X^2+2)^2}$

Q.2 $\frac{1}{4(1+x)} - \frac{x-1}{4(1+x^2)} + \frac{x-1}{2(1+x^2)^2}$

Q.3 $\frac{1}{x} - \frac{x}{x^2+3} + \frac{2x+3}{(x^2+3)^2}$

Q.4 $\frac{2}{X-1} + \frac{2X-1}{X^2+X+1} + \frac{3}{(X^2+X+1)^2}$

Q.5 $\frac{1}{3(x+1)}+\frac{5(x-1)}{3(x^2+2)}-\frac{2(3x-1)}{(x^2+1)^2}$

Q.6 $-\frac{2}{2x-5}+\frac{x+3}{1+x^2}+\frac{x-2}{(1+x^2)^2}$

Q.7 $\frac{1}{x-2}-\frac{x+2}{x^2+3}-\frac{7x+14}{(x^2+3)^2}$

Q.8 $\frac{1}{1-x}+\frac{1}{1+x}+\frac{2}{1+x^2}-\frac{4}{(1+x^2)^2}$

Q.9 $\frac{1}{x+2}+\frac{2x-3}{(x^2+x+1)^2}-\frac{1}{x^2+x+1}$

Q.10 $\frac{1}{9(x^2+1)}-\frac{1}{9(x^2+4)}+\frac{2}{3(x^2+4)^2}$

Q.11 $-\frac{(x-1)}{2(x^2-x+1)}+\frac{(x+1)}{2(x^2+x+1)}$

Summary

Let N(x) ≠ and D(x) ≠ 0 be two polynomials. The $\frac{N(x)}{D(x)}$ is called a proper fraction if the degree of N(x) is smaller than the degree of D(x).

For example: $\frac{x-1}{x^2+5x+6}$ is a proper fraction.

Also $\frac{N(x^1)}{D(x)}$ is called an improper fraction of the degree of N(x) is greater than or equal to the degree of D(x).

For example: $\frac{x^5}{x^4-1}$ is an improper fraction.

In such problems we divide N(x) by D(x) obtaining a quotient Q(x) and a remainder R(x) whose degree is smaller than that of D(x).

Thus $\frac{N(x)'}{D(x)} = Q(x) + \frac{R(x)}{D(x)}$ where $\frac{R(x)'}{D(x)}$ is proper fraction.

Types of proper fraction into partial fractions.

Type 1:

Linear and distinct factors in the D(x)

$$\frac{x-a}{(x+a)(x+b)} = \frac{A}{x+a} + \frac{B}{x+b}$$

Type 2:

Linear repeated factors in D(x)

$$\frac{x-a}{(x+a)(x^2+b^2)} = \frac{A}{x+a} + \frac{Bx+C}{x^2+b^2}$$

Type 3:

Quadratic Factors in the D(x)

$$\frac{x-a}{(x+a)(x^2+b)^2} = \frac{A}{x+a} + \frac{Bx+C}{x^2+b^2}$$

Type 4:

Quadratic repeated factors in D(x):

$$\frac{x-a}{(x^2+a^2)(x^2+b^2)} = \frac{Ax+B}{x^2+a^2} + \frac{Cx+D}{x^2+b^2} + \frac{Ex+F}{(x^2+b^2)^2}$$