Q.9 $x + 1 - \frac{1}{x} - \frac{1}{x^2} + \frac{2}{x-1}$

Q.10 $1 + \frac{1}{x-3} - \frac{4}{(x-3)^2} + \frac{7}{(x-3)^3}$

Q.11 $\frac{4}{27(x-1)} + \frac{5}{9(x-1)^2} + \frac{1}{3(x-1)^3} - \frac{4}{27(x+2)}$

Q.12 $-\frac{3}{25(x+2)} + \frac{3}{25(x-3)} + \frac{7}{5(x-3)^2}$

4.10 Type III:

When the denominator contains ir-reducible quadratic factors which are non-repeated.

Example 1:

Resolve into partial fractions $\frac{9x-7}{(x+3)(x^2+1)}$

Solution:

$$\frac{9x-7}{(x+3)(x^2+1)} = \frac{A}{x+3} + \frac{Bx+C}{x^2+1}$$

Multiplying both sides by L.C.M. i.e., $(x + 3)(x^2 + 1)$, we get

$$9x - 7 = A(x^2 + 1) + (Bx + C)(x + 3) \qquad \text{(I)}$$

Put $x + 3 = 0 \Rightarrow x = -3$ in Eq. (I), we have

$$9(-3) - 7 = A((-3)^2 + 1) + (B(-3) + C)(-3 + 3)$$

$$-27 - 7 = 10A + 0$$

$$A = -\frac{34}{10} \Rightarrow \boxed{A = -\frac{17}{5}}$$

$$9x - 7 = A(x^2 + 1) + B(x^2 + 3x) + C(x + 3)$$

Comparing the co-efficient of like powers of x on both sides

$$A + B = 0$$

$$3B + C = 9$$

Putting value of A in Eq. (i)

$$-\frac{17}{5} + B = 0 \Rightarrow \boxed{B = \frac{17}{5}}$$

From Eq. (iii)

$$C = 9 - 3B = 9 - 3\left(\frac{17}{4}\right)$$

$$= 9 - \frac{51}{5} \Rightarrow \boxed{C = -\frac{6}{5}}$$

Hence the required partial fraction are

$$\frac{-17}{5(x+3)}+\frac{17x-6}{5(x^2+1)}$$

Example 2:

Resolve into partial fraction $\dfrac{x^2+1}{x^4+x^2+1}$

Solution:

Let $\dfrac{x^2+1}{x^4+x^2+1}=\dfrac{x^2+1}{(x^2-x+1)(x^2+x+1)}$

$$\frac{x^2+1}{(x^2-x+1)(x^2+x+1)}=\frac{Ax+B}{(x^2-x+1)}+\frac{Cx+D}{(x^2+x+1)}$$

Multiplying both sides by L.C.M. i.e., $(x^2-x+1)(x^2+x+1)$

$x^2+1=(Ax+B)(x^2+x+1)+(Cx+D)(x^2-x+1)$

Comparing the co-efficient of like powers of x, we have

Co-efficient of x^3 : $A+C=0$ ………… (i)

Co-efficient of x^2 : $A+B-C+D=1$ ……….. (ii)

Co-efficient of x : $A+B+C-D=0$ ……….. (iii)

Constant $B+D=1$ ……….. (iv)

Subtract (iv) from (ii) we have

$A-C=0$ …………… (v)

$A=C$ …………… (vi)

Adding (i) and (v), we have

$A=0$

Putting $A=0$ in (vi), we have

$C=0$

Putting the value of A and C in (iii), we have

$B-D=0$ …………… (vii)

Adding (iv) and (vii)

$2B=1 \quad \Rightarrow \quad B=\dfrac{1}{2}$

from (vii) $B=D$, therefore

$D=\dfrac{1}{2}$

Hence the required partial fraction are

$$\frac{0x+\frac{1}{2}}{(x^2-x+1)}+\frac{0x+\frac{1}{2}}{(x^2+x+1)}$$

i.e., $$\frac{1}{2(x^2-x+1)}+\frac{1}{2(x^2+x+1)}$$

Exercise 4.3

Resolve into partial fraction:

Q.1 $\dfrac{x^2+3x-1}{(x-2)(x^2+5)}$ Q.2 $\dfrac{x^2-x+2}{(x+1)(x^2+3)}$

Q.3 $\dfrac{3x+7}{(x+3)(x^2+1)}$ Q.4 $\dfrac{1}{(x^3+1)}$

Q.5 $\dfrac{1}{(x+1)(x^2+1)}$ Q.6 $\dfrac{3x+7}{(x^2+x+1)(x^2-4)}$

Q.7 $\dfrac{3x^2-x+1}{(x+1)(x^2-x+3)}$ Q.8 $\dfrac{x+a}{x^2(x-a)(x^2+a^2)}$

Q.9 $\dfrac{x^5}{x^4-1}$ Q.10 $\dfrac{x^2+x+1}{(x^2-x-2)(x^2-2)}$

Q.11 $\dfrac{1}{x^3-1}$ Q.12 $\dfrac{x^2+3x+3}{(x^2-1)(x^2+4)}$

Answers 4.3

Q.1 $\dfrac{1}{x-2}+\dfrac{3}{x^2+5}$ Q.2 $\dfrac{1}{x+1}-\dfrac{1}{x^2+3}$

Q.3 $-\dfrac{1}{5(x+3)}+\dfrac{x+12}{5(x^2+1)}$ Q.4 $\dfrac{1}{3(x+1)}-\dfrac{(x-2)}{3(x^2-x+1)}$

Q.5 $\dfrac{1}{2(x+1)}-\dfrac{x-1}{2(x^2+1)}$

Q.6 $\dfrac{13}{28(X-2)}-\dfrac{1}{12(X+2)}-\dfrac{8X+31}{21(X^2+X+1)}$

Q.7 $\dfrac{1}{x+1}+\dfrac{2x-2}{x^2-x+3}$

Q.8 $\dfrac{1}{a^3}\left[\dfrac{1}{X-a}+\dfrac{x}{X^2+a^2}-\dfrac{2}{X}-\dfrac{a}{X^2}\right]$

Q.9 $x + \dfrac{1}{4(x-1)} + \dfrac{1}{4(x+1)} - \dfrac{x}{2(x^2+1)}$

Q.10 $\dfrac{1}{3(x+1)} + \dfrac{7}{6(x-2)} - \dfrac{3x+2}{2(x^2-2)}$

Q.11 $\dfrac{1}{3(x-1)} - \dfrac{x+2}{3(x^2+x+1)}$

Q.12 $\dfrac{7}{10(x-1)} - \dfrac{1}{10(x+1)} - \dfrac{3x-1}{5(x^2+4)}$

4.11 Type IV: Quadratic repeated factors

When the denominator has repeated Quadratic factors.

Example 1:

Resolve into partial fraction $\dfrac{x^2}{(1-x)(1+x^2)^2}$

Solution:

$$\frac{x^2}{(1-x)(1+x^2)^2} = \frac{A}{1-x} + \frac{Bx+C}{(1+x^2)} + \frac{Dx+E}{(1+x^2)^2}$$

Multiplying both sides by L.C.M. i.e., $(1-x)(1+x^2)^2$ on both sides, we have

$x^2 = A(1 + x^2)^2 + (Bx + C)(1 - x)(1 + x^2)+(Dx + E)(1 - x)$ ……(i)

$x^2 = A(1 + 2x^2 + x^4) + (Bx + C)(1 - x + x^2 - x^3)+(Dx + E)(1 - x)$

Put $1 - x = 0 \Rightarrow x = 1$ in eq. (i), we have

$(1)^2 = A(1 + (1)^2)^2$

$1 = 4A \Rightarrow \boxed{A = \frac{1}{4}}$

$x^2 = A(1 + 2x^2 + x^4) + B(x - x^2 + x^3 - x^4)+C(1 - x + x^2 - x^3)$
$+ D(x-x^2)+ E(1 - x)$ …………………… (ii)

Comparing the co-efficients of like powers of x on both sides in Equation (II), we have

Co-efficient of x^4 : $A - B = 0$ ……………. (i)
Co-efficient of x^3 : $B - C = 0$ ……………. (ii)
Co-efficient of x^2 : $2A - B + C - D = 1$ ……. (iii)
Co-efficient of x : $B - C + D - E = 0$ ……. (iv)
Co-efficient term : $A + C + E = 0$ ……………. (v)
from (i), $B = A$

$\Rightarrow B = \frac{1}{4} \quad \because \quad A = \frac{1}{4}$