Q.7 $2+\frac{31}{4(x+3)}+\frac{1}{12(x-1)}-\frac{137}{6(x+5)}$

Q.8 $\frac{1}{2(1-x)}-\frac{4}{(1-2x)}+\frac{9}{2(1-3x)}$

Q.9 $\frac{3}{x}+\frac{4}{2x-3}+\frac{2}{2x+3}$

Q.10 $\frac{2}{x-1}-\frac{3}{2x-1}+\frac{4}{x+12}$

Q.11 $x+6+\frac{1}{2(x-1)}-\frac{16}{x-2}+\frac{81}{2(x-3)}$

Q.12 $1+\frac{1}{x}-\frac{1}{5(x-1)}-\frac{8}{5(2x+3)}$

4.9 Type II:

When the factors of the denominator are all linear but some are repeated.

Example 1:

Resolve into partial fractions: $\frac{x^2-3x+1}{(x-1)^2(x-2)}$

Solution:

$$\frac{x^2-3x+1}{(x-1)^2(x-2)}=\frac{A}{x-1}+\frac{B}{(x-1)^2}+\frac{C}{x-2}$$

Multiplying both sides by L.C.M. i.e., $(x-1)^2(x-2)$, we get

$x^2-3x+1=A(x-1)(x-2)+B(x-2)+C(x-1)^2$ (I)

Putting $x-1=0 \Rightarrow x=1$ in (I), then

$(1)^2-3(1)+1=B(1-2)$

$1-3+1=-B$

$-1=-B$

$\Rightarrow B=1$

Putting $x-2=0 \Rightarrow x=2$ in (I), then

$(2)^2-3(2)+1=C(2-1)^2$

$4-6+1=C(1)^2$

$\Rightarrow -1=C$

Now $x^2-3x+1=A(x^2-3x+2)+B(x-2)+C(x^2-2x+1)$

Comparing the co-efficient of like powers of x on both sides, we get

$A+C=1$

$A=1-C$

$= 1 - (-1)$
$= 1 + 1 = 2$

$\Rightarrow \quad A = 2$

Hence the required partial fractions are

$$\frac{x^2 - 3x + 1}{(x-1)^2(x-2)} = \frac{2}{x-1} + \frac{1}{(x-1)^2} + \frac{1}{x-2}$$

Example 2:

Resolve into partial fraction $\frac{1}{x^4(x+1)}$

Solution

$$\frac{1}{x^4(x+1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^3} + \frac{D}{x^4} + \frac{E}{x+1}$$

Where A, B, C, D and E are constants. To find these constants multiplying both sides by L.C.M. i.e., $x^4 (x + 1)$, we get

$1 = A(x^3)(x + 1) + Bx^2 (x + 1) + Cx (x + 1) + D(x + 1) + Ex^4$ (I)

Putting $x = -1$ in Eq. (I)

$1 = E(-1)^4$

$\Rightarrow \quad E = 1$

Putting x = 0 in Eq. (I), we have

$1 = D(0 + 1)$
$1 = D$

$\Rightarrow \quad D = 1$

$1 = A(x^4 + x^3)+B(x^3 + x^2)+C(x^2 + x)+D(x + 1) + Ex$

Comparing the co-efficient of like powers of x on both sides.

Co-efficient of x^3 : $A + B = 0$ .... (i)

Co-efficient of x^2 : $B + C = 0$ .... (ii)

Co-efficient of x : $C + D = 0$ .... (iii)

Putting the value of D = 1 in (iii)

$C + 1 = 0$

$\Rightarrow \quad C = -1$

Putting this value in (ii), we get

$B - 1 = 0$

$\Rightarrow \quad B = 1$

Putting B = 1 in (i), we have

$A + 1 = 0$

$\Rightarrow \quad A = -1$

Hence the required partial fraction are

$$\frac{1}{x^4(x+1)} = \frac{-1}{x} + \frac{1}{x^2} - \frac{1}{x^3} + \frac{1}{x^4} + \frac{1}{x+1}$$

Example 3:

Resolve into partial fractions $\frac{4+7x}{(2+3x)(1+x)^2}$

Solution:

$$\frac{4+7x}{(2+3x)(1+x)^2} = \frac{A}{2+3x} + \frac{B}{1+x} + \frac{C}{(1+x)^2}$$

Multiplying both sides by L.C.M. i.e., $(2 + 3x)(1 + x)^2$

We get $4 + 7x = A(1 + x)^2 + B(2 + 3x)(1 + x) + C(2 + 3x)$ …. (I)

Put $2 + 3x = 0 \quad \Rightarrow \quad x = -\frac{2}{3}$ in (I)

Then $4 + 7\left(-\frac{2}{3}\right) = A\left(1 - \frac{2}{3}\right)^2$

$$4 - \frac{14}{3} = A\left(-\frac{1}{3}\right)^2$$

$$-\frac{2}{3} = \frac{1}{9}A$$

$\Rightarrow \quad A = \frac{-2}{3} x \frac{9}{1} = -6$

$A = -6$

Put $1 + x = 0 \quad \Rightarrow \quad x = -1$ in eq. (I), we get

$4 + 7(-1) = C(2 - 3)$

$4 - 7 = C(-1)$

$-3 = -C$

$\Rightarrow \quad C = 3$

$4 + 7x = A(x^2 + 2x + 1) + B(2 + 5x + 3x^2) + C(2 + 3x)$

Comparing the co-efficient of x^2 on both sides

$A + 3B = 0$

$-6 + 3B = 0$

$3B = 6$

$\Rightarrow \quad B = 2$

Hence the required partial fraction will be

$$\frac{-6}{2+3x} + \frac{2}{1+x} + \frac{3}{(1+x)^2}$$

Exercise 4.2

Resolve into partial fraction:

Q.1 $\frac{x+4}{(x-2)^2(x+1)}$

Q2. $\frac{1}{(x+1)(x^2-1)}$

Q.3 $\frac{4x^3}{(x+1)^2(x^2-1)}$

Q.4 $\frac{2x+1}{(x+3)(x-1)(x+2)^2}$

Q.5 $\frac{6x^2-11x-32}{(x+6)(x+1)^2}$

Q.6 $\frac{x^2-x-3}{(x-1)^3}$

Q.7 $\frac{5x^2+36x-27}{x^4-6x^3+9x^2}$

Q.8 $\frac{4x^2-13x}{(x+3)(x-2)^2}$

Q.9 $\frac{x^4+1}{x^2(x-1)}$

Q.10 $\frac{x^3-8x^2+17x+1}{(x-3)^3}$

Q.11 $\frac{x^2}{(x-1)^3(x+2)}$

Q.12 $\frac{2x+1}{(x+2)(x-3)^2}$

Answers4.2

Q.1 $-\frac{1}{3(x-2)}+\frac{2}{(x-2)^2}+\frac{1}{3(x+1)}$

Q.2 $\frac{1}{4(x-1)}-\frac{1}{4(x+1)}-\frac{1}{2(x+1)^2}$

Q.3 $\frac{1}{2(x-1)}+\frac{7}{2(x+1)}-\frac{5}{(x+1)^2}+\frac{2}{(x+1)^3}$

Q.4 $\frac{5}{4(x+3)}+\frac{1}{12(x-1)}-\frac{4}{3(x+2)}+\frac{1}{(x+2)^2}$

Q.5 $\frac{10}{x+6}-\frac{4}{x+1}-\frac{3}{(x-1)^2}$

Q.6 $\frac{1}{x-1}+\frac{1}{(x-1)^2}-\frac{3}{(x-1)^3}$

Q.7 $\frac{2}{x}-\frac{3}{x^2}-\frac{2}{(x-3)}+\frac{14}{(x-3)^2}$

Q.8 $\frac{3}{x+3}+\frac{1}{x-2}-\frac{2}{(x-2)^2}$

Q.9 $x+1-\frac{1}{x}-\frac{1}{x^2}+\frac{2}{x-1}$

Q.10 $1+\frac{1}{x-3}-\frac{4}{(x-3)^2}+\frac{7}{(x-3)^3}$

Q.11 $\frac{4}{27(x-1)}+\frac{5}{9(x-1)^2}+\frac{1}{3(x-1)^3}-\frac{4}{27(x+2)}$

Q.12 $-\frac{3}{25(x+2)}+\frac{3}{25(x-3)}+\frac{7}{5(x-3)^2}$

4.10 Type III:

When the denominator contains ir-reducible quadratic factors which are non-repeated.

Example 1:

Resolve into partial fractions $\frac{9x-7}{(x+3)(x^2+1)}$

Solution:

$$\frac{9x-7}{(x+3)(x^2+1)}=\frac{A}{x+3}+\frac{Bx+C}{x^2+1}$$

Multiplying both sides by L.C.M. i.e., $(x + 3)(x^2 + 1)$, we get

$9x - 7 = A(x^2 + 1) + (Bx + C)(x + 3)$ (I)

Put $x + 3 = 0 \Rightarrow x = -3$ in Eq. (I), we have

$9(-3) -7 = A((-3)^2 + 1) + (B(-3) + C)(-3 + 3)$

$-27 -7 = 10A + 0$

$A = -\frac{34}{10} \Rightarrow \boxed{A=-\frac{17}{5}}$

$9x - 7 = A(x^2 + 1) + B(x^2 + 3x) + C(x + 3)$

Comparing the co-efficient of like powers of x on both sides

$A + B = 0$

$3B + C = 9$

Putting value of A in Eq. (i)

$-\frac{17}{5}+B=0 \Rightarrow \boxed{B=\frac{17}{5}}$

From Eq. (iii)

$C = 9 - 3B = 9-3\left(\frac{17}{4}\right)$

$= 9-\frac{51}{5} \Rightarrow \boxed{C=-\frac{6}{5}}$