(iv) $\frac{3(3^n - 1)}{2}$

2. n = 6

3. (i) $\frac{1}{3}\left[n - \frac{1}{9}\left(1 - \frac{1}{10^n}\right)\right]$ (ii) $\frac{1}{3}\left[\frac{10(10^n - 1)}{9} - n\right]$

(iii) $\frac{1}{(1-x)}\left[\frac{1-r^n}{1-r} - \frac{x(1-r^n x^n}{1-rx}\right]$

4. 62 5. 26.76 6. $\frac{4085}{2}$ 7. $\sqrt{3}; \frac{\sqrt{3}(3\sqrt{3}-1)}{\sqrt{3}-1}$

2.14 Infinite Geometric Sequence:

A geometric sequence in which the number of terms are infinite is called as infinite geometric sequence.

For example:

(i) $2, \frac{4}{3}, \frac{8}{9}, \frac{16}{27}$, - - - - - -

(ii) 2, 4, 8, 16, 32, - - - - - - -

Infinite Series:

Consider a geometric sequence a, ar, ar^2, - - - - - - - to n terms.

Let S_n denote the sum of n terms then $S_n = a + ar + ar^2 +$ - - - - - - to n terms.

Formula $S_n = \frac{a(1-r^n)}{1-r} |r| < 1$

Taking limit as n → ∞ on both sides

$$\lim_{n\to\infty} S_n = \lim_{n\to\infty} a\frac{(1-r^n)}{1-r}$$

$$= \lim_{n\to\infty} a\left[\frac{1}{1-r} - \frac{r^n}{1-r}\right]$$

$$= \lim_{n\to\infty}\left(\frac{a}{1-r}\right) - \lim_{n\to\infty}\frac{ar^n}{1-r}$$

as $n \to \infty, r^n \to 0$

Therefore $S\infty = \frac{a}{1-r} - 0$

$$\boxed{S\infty = \frac{a}{1-r}}$$

$\therefore$ the formula for the sum of infinite terms of G.P.

Convergent Series:

An infinite series is said to be the convergent series when its sum tends to a finite and definite limit.

For example:

$\frac{2}{3}+\frac{1}{3}+\frac{1}{6}+\frac{1}{12}+$ - - - - - - - is a series

Here $a = \frac{2}{3}$, $r = \frac{1}{3} + \frac{2}{3} = \frac{1}{2} < 1$

$$S\infty = \frac{a}{1-r}$$

$$= \frac{\frac{2}{3}}{1-\frac{1}{3}} = \frac{\frac{2}{3}}{\frac{1}{2}}$$

$$= \frac{2}{3} \text{ x } \frac{1}{2} = \frac{4}{3}$$

Hence the series is convergent.

Divergent Series:

When the sum of an infinite series is infinite, it is said to be the Divergent series.

For example:

2 + 4 + 8 + 16 + 32 + - - - - - - - -

Here $a = 2$, $r = , 2 > 1$

Therefore we use formula

$$S_n = \frac{a(r^n-1)}{r-1} = \frac{2(2^n-1)}{2-1}$$

$$S_n = 2^{n+1} - 2$$

$$\lim_{n\to\infty} S_n = \lim_{n\to\infty} (2^{n+1} - 2)$$

$$S\infty = 2^{\infty+1} - 2$$

$$= \infty \text{ as } n \to \infty, 2^{n+1} \to \infty$$

Hence the series is a divergent series.

2.14 Recurring Decimals:

When we attempt to express a common fraction such as $\frac{3}{8}$ or as $\frac{4}{11}$ as a decimal fraction, the decimal always either terminates or ultimately repeats.

Thus $\frac{3}{8} = 0.375$ (Decimal terminate)

$\frac{4}{11} = 0.363636$ (Decimal repeats)

We can express the recurring decimal fraction $0.\overline{36}$ (or $0.\dot{3}\dot{6}$) as a common fraction.

The bar ($0.\overline{36}$) means that the numbers appearing under it are repeated endlessly. i.e. $0.\overline{36}$ means 0.363636 - - - - - - -

Thus a non-terminating decimal fraction in which some digits are repeated again and again in the same order in its decimal parts is called a recurring decimal fraction.

Example 1:

Find the fraction equivalent to the recurring decimals $0.\overline{123}$.

Solution:

Let $S = 0.\overline{123}$

$= 0.123\ 123\ 123 - - - - - - - - - - - \infty$

$= 0.123 + 0.000123 + 0.000000123 - - - - - - \infty$

$= \frac{123}{1000} \frac{123}{1000\ 000} \frac{123}{1000\ 000\ 000} + - - - - - - \infty$

$= \frac{123}{10^3} + \frac{123}{10^6} + \frac{123}{10^9} + - - - - - - \infty$

Here $a = \frac{123}{10^3}$, $r = \frac{1}{10^3}$

$S = \frac{a}{1-r}$

$= \frac{\frac{123}{10^3}}{1-\frac{1}{10^3}} = \frac{\frac{123}{1000}}{1-\frac{1}{1000}} = \frac{\frac{123}{1000}}{\frac{1000-1}{1000}}$

$=$

$$= \frac{123}{1000} \times \frac{1000}{999} = \frac{123}{999}$$

$$= \frac{41}{333}$$

Example 2:

Find the sum of infinite geometric series in which a = 128,

$$r = -\frac{1}{2}.$$

Solution:

$$a = 128,\ r = -\frac{1}{2}$$

Using

$$S\infty = \frac{a}{1-r}$$

$$S\infty = \frac{128}{1-\left(\frac{1}{2}\right)} = \frac{128}{1+\frac{1}{2}}$$

$$= \frac{128}{\frac{3}{2}} = 128 \times \frac{2}{3}$$

$$S\infty = \frac{256}{3}$$

Exercise 2.7

Q.1 Find the sum of the following infinite geometric series

(i) $\frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \text{- - - - - - -}$

(ii) $2 + \sqrt{2} + 1 + \text{- - - - - - -}$

Q.2 Find the sum of the following infinite geometric series

(i) $a = 3,\ r = \frac{2}{3}$ (ii) $a = 3,\ r = \frac{3}{4}$

Q.3 Which of the following series are (i) divergent (ii) convergent

(i) $1 + 4 + 16 + 64 + \text{- - - - - - - - -}$

(ii) $6 + 3 + \frac{3}{2} + \frac{3}{4} + \frac{3}{8} + \text{- - - - - - -}$

(iii) $6 + 12 + 24 + 48 + \text{- - - - - - - - -}$

Q.4 Find the fractions equivalent to the recurring decimals.

(i) $0.\overline{36}$ (ii) $2.\overline{43}$ (iii) $0.\overline{836}$

Q.5 Find the sum to infinity of the series $1 + (1 + k)r + 1 + k + k^2)r^2 + (1 + k + k^2 + k^3)r^3 + \text{- - - - - - -}$ r and k being proper fraction.

Q.6 If $y = x + x^2 + x^3 + \text{- - - - - - -} \infty$ and if x is positive and less than unity show that $x = \frac{y}{1+y}$

Q.7 What distance a ball travel before coming to rest if it is dropped from a height of 6 dm and after each fall it rebounds $\frac{2}{3}$ of the distance it fell.

Q.8 The sum of an infinite geometric series in 15 and the sum of the squares of its terms is 45. Find the series.

Answers 2.7

Q.1 (i) $S\infty = \frac{1}{4}$ (ii) $S\infty = \frac{2\sqrt{2}}{\sqrt{2}-1}$

Q.2 (i) 9 (ii) 12

Q.3 (i) Divergent (ii) Convergent (iii) Divergent

Q.4 (i) $\frac{4}{11}$ (ii) $\frac{241}{99}$ (iii) $\frac{5}{6}$

Q.5 $\frac{1}{(1-r)(1-Kr)}$

Q.7 30 dm.

Q.8 $5 + \frac{10}{5} + \frac{20}{9} + \text{- - - - - - -}$